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Left Semi derivations in Prime near-rings

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Abstract:

Let N be a 2-torsion free prime near-ring.If N admits a non zero left semi derivation d with g such that *(i)*$d[x,y]=0$ *(ii)* $d[x,y]=[x,y]$ *(iii)* $d[x,y]=-[x,y]$ *(iv)* $d(x\,o\,y)=(x\,o\,y)$ *(v)* $d(x\,o\,y)=-(x\,o\,y)$ *(vi)* $d[x,y]\in Z(N)$ *(vii)* $[d(x),y]\in Z(N)$ *(viii)* $d(x)oy=xoy$ *(ix)* $d(x\,o\,y)\in Z(N)$ *(x)* $d(x)\,o\,y\in Z(N)$ *(xi)* $d(x\,o\,y)=[x,y]$ *(xii)* $d[x,y]=(x\,o\,y)$, for all $x,y\in N$, then N is a commutative ring.

Keywords: Prime near-ring, semiderivation, left semiderivation, commutativity.

1. Introduction

In this paper N will denote a zero symmetric right near –ring (i.e., a right near ring N satisfying the property $x.0=0$ for all $x\in N$). Note that right distributivity in N gives $0.x=0$ for all $x\in N$. For any $x,y\in N$ the symbol $[x,y]$ will denote the commutator $xy-yx$. While the symbol $x\,o\,y$ will stands for the anti-commutator $xy+yx$. The symbol $Z(N)$ will represent the multiplicative center of N, that is $Z(N)=\{x\in N/\ xy=yx \text{ for all } y\in N\}$. An additive mapping $d: N\rightarrow N$ is said to be a derivation if $d(xy)=xd(y)+d(x)y$ for all $x,y\in N$, or equivalently, as noted in [17] that $d(xy)=d(x)y+xd(y)$ for all $x,y\in N$. An additive mapping $d: N\rightarrow N$ is said to be a left derivation if $d(xy)=xd(y)+y\,d(x)$ for all $x,y\in N$.

In [2] J.Bergen has introduced the notion of semiderivations of a ring R which extends the notion of derivations of a ring R. An additive mapping $d: R\rightarrow R$ is called a semiderivation if there exists a function $g: R\rightarrow R$ such that (i) $d(xy)=d(x)g(y)+x\,d(y)=d(x)y+g(x)d(y)$ and (ii)$d(g(x))=g(d(x))$ holds for all $x,y\in R$. In case g is an identity map of R then all semiderivations associated with g are merely ordinary derivations. On the other hand , if g is a homomorphism of R such that $g\neq 1$ then $d=g-1$ is a semiderivation which is not a derivation. In case R is a Prime and $d\neq 0$, it has been shown by Chang [3] that g must necessarily be a ring endomorphism.

An additive mapping $d: N\rightarrow N$ is called a semiderivation if there exists a surjective function $g: N\rightarrow N$ such that (i) $d(xy)=d(x)g(y)+x\,d(y)=d(x)y+g(x)d(y)$ and (ii) $d(g(x))=g(d(x))$ holds for all $x,y\in N$ An additive mapping $d: N\rightarrow N$ is called a left semiderivation if there exists a surjective function $g: N \rightarrow N$ such that (i) $d(xy) = xd(y) + g(y)d(x) = g(x)d(y) + y\,d(x)$ and (ii) $d(g(x)) = g(d(x))$ holds for all $x, y \in N$.

According to [10] , a near-ring N is said to be prime if $x\,N\,y = \{0\}$ for all $x , y \in N$ implies $x = 0$ or $y = 0$. Recently there has been a great deal of wok concerning commutativity of prime and semi-prime rings with derivations satisfying certain differential identities (see[4,9,11,16] for reference where further references can be found). In view of these results many authors have investigated commutativity of prime near-rings satisfying certain polynomial conditions(see[5-8, 10-15, 17], etc.). In [1] authors investigated on Semiderivations and commutativity in prime rings. In the present paper it is shown that near-rings with left semiderivations satisfying certain identities are commutative rings.

2. Main result

Lemma 1:- Let N be a 2-torsion free prime near-ring, and d a non zero left semi derivation with g of N and $a \in N$. If $ad(N)=0$, then $a = 0$.

Proof:

Suppose that $ad(N)=0$.

For arbitrary $x , y \in N$ we have

$$ad(xy) = 0$$

$$a\,xd(y) + ag(y)d(x) = 0$$

Replace y by x in the above equation and g is on to we get

$$2axd(x)=0$$

Since N is a 2-torsion free near –ring , we get

$$axd(x) = 0 \quad \text{for all } x , y \in N$$

Since N is prime near ring and $d \neq 0$, we get $a = 0$.

Lemma 2:- Let N be a 2-torsion free prime near-ring, and d a non zero left semiderivation with g of N . If $d^2 = 0$, then $d=0$.

Proof:

For arbitrary $x , y \in N$ we have

$$d^2(xy)= 0$$

$$d(d(xy)) =0$$

$$d(xd(y) + g(y)\,d(x)) = 0$$

$$xd^2(y) + g(d(y))d(x) + g(y)d^2(x) + g(d(x))d(g(y)) = 0 \text{ for all } x , y \in N$$

By hypothesis,

$$g(d(y))d(x) + g(d(x))d(g(y)) = 0$$

and g is on to we have

$$d(y)d(x) + d(x)d(y) = 0$$

Replace y by x in the above equation

$$2d(x)d(x) = 0 , \qquad \text{for all } x , y \in N$$

Since N is a2-torsion free prime near-ring ,we get

$$d(x)d(N)= 0, \text{ for all } x \epsilon N$$

Using Lemma 1 we get $d = 0$.

Lemma 3:- Let N be a prime near-ring, and d a non zero left semi derivation with g of N. If $d(N) \subset Z(N)$, then $(N, +)$ is Abelian. Moreover , if N is 2-torsion free, then N is commutative ring.

Proof:

Suppose that $a \epsilon N$ such that $d(a) \neq 0$, So , $d(a) \epsilon Z(N) \setminus \{0\}$ and $d(a) + d(a) \epsilon Z(N) \setminus \{0\}$.

For all $x , y \epsilon N$, we have

$$(d(a) + d(a)) (x + y) = (x + y) (d(a) + d(a))$$

That is,

$$d(a)x+d(a)x+d(a)y+d(a)y = xd(a)+yd(a)+xd(a)+yd(a)$$

Since $d(a) \epsilon Z(N)$, we get

$$xd(a) + yd(a) =yd(a) + xd(a)$$

$$(x, y)d(a) = 0 \text{ for all } x , y \epsilon N$$

Since $d(a) \epsilon Z(N) \setminus \{0\}$ and N is a prime near-ring , we get $(x ,y) =0$, for all $x , y \epsilon N$

Thus $(N , +)$ is Abelian.

Now using hypothesis, for any $a,b,c \epsilon N$,

$$cd(ab) = d(ab)c$$

$$cad(b) + cg(b)d(a) = ad(b)c + g(b)d(a)c$$

Using $d(N) \subset Z(N)$ and $(N, +)$ is Abelian, we obtain that

$$cad(b) + cg(b)d(a) = acd(b) + g(b)cd(a)$$

$$[c,a] d(b) = [g(b) , c] d(a) \qquad \text{for all } a,b,c \epsilon N$$

Suppose now that N is not commutative. Choosing $b ,c \epsilon N$ such that $[g(b) , c] \neq 0$ and replacing a by $d(a) \epsilon Z(N)$, we get

$$[g(b) , c]d^2(a) = 0 \text{ for all } a,b,c \epsilon N$$

Since g is on to we have

$$[b , c] d^2(a) = 0 \text{ for all } a,b,c \epsilon N$$

$d^2(a) \epsilon Z(N)$, we conclude that $d^2(a) = 0$, for all $a \epsilon N$, and so $d=0$ by lemma 2.

But it contradicts $d \neq 0$. Which implies $[b , c]= 0$

Thus N is a commutative ring.

Theorem 1: Let N be a 2-torsion free prime near-ring.If N admits a non zero left semi derivation d with g such that $d[x , y] = 0$, for all $x,y \epsilon N$, then N is a commutative ring.

Proof:

Suppose $d[x ,y] = 0$ for all $x,y \epsilon N$ (1)

Replace y by yx in (1), weget

$$d[x , yx] =0$$

$$d([x , y] x) = 0$$

$$[x , y]d(x) + g(x) d[x , y] = 0$$

Using (1) implies $[x , y]d(x) = 0$ for all $x,y \epsilon N$ (2)

Replace y by yt in (2), we get

$[x , yt]d(x) = 0$ for all $x,t \epsilon N$

$y [x , t]d(x) + [x , y] t d(x) = 0$ for all $x, y ,t \epsilon N$

Using (2) implies $[x , y] t d(x) = 0$ for all $x, y , t \epsilon N$

$[x , y] N d(x) = 0$ for all $x,y \epsilon N$ (3)

Since N is prime near-ring equation (3) reduces to

$[x , y] = 0$ or $d(x) = 0$ for all $x, y \epsilon N$ (4)

From equation (4) it follows that for each fixed $x \epsilon N$ we have

$d(x) = 0$ or $x \epsilon Z(N)$ (5)

But $x \epsilon Z(N)$ also implies that $d (x) \epsilon Z(N)$ for all $x \epsilon N$ (6)

In light of (6), $d(N) \subset Z(N)$ and using lemma 3

We conclude that N is a commutative ring.

Theorem 2 :- Let N be a 2-torsion free prime near-ring.If N admits a non zero left semi derivation d with g such that

$d[x , y] = [x , y]$, for all $x,y \epsilon N$

$d[x , y] = - [x , y]$, for all $x,y \epsilon N$, then N is a commutative ring.

Proof:

(i) By hypothesis $d[x, y] = [x, y]$ for all $x, y \epsilon N$ (7)

Replace x by yx in (7)

$$d[yx, y] = [yx, y]$$

$$d(y[x, y]) = y[x, y]$$

$$y\, d[x, y] + g[x, y]\, d(y) = y[x, y]$$

Using (7) in the above equation, we get

$$g[x, y]\, d(y) = 0$$

Since g is on to we have $[x, y]\, d(y) = 0$ for all $x, y \epsilon N$ (8)

Replace x by xt in (8), we get

$[xt, y]d(y) = 0$ for all $x, t \epsilon N$

$x[t, y]d(y) + [x, y]\, t\, d(y) = 0$ for all $x, y, t \epsilon N$

Using (8) implies $[x, y]\, t\, d(y) = 0$ for all $x, y, t \epsilon N$

$[x, y]\, N\, d(y) = 0$ for all $x, y \epsilon N$ (9)

Since N is prime near-ring equation (9) reduces to

$[x, y] = 0$ or $d(y) = 0$ for all $x, y \epsilon N$ (10)

From equation (10) it follows that for each fixed $y \epsilon N$ we have

$d(y) = 0$ or $y \epsilon Z(N)$ (11)

But $y \epsilon Z(N)$ also implies that $d(y) \epsilon Z(N)$ for all $y \epsilon N$ (12)

In light of (12), $d(N) \subset Z(N)$ and using lemma 3

We conclude that N is a commutative ring.

(ii) By hypothesis $d[x, y] = -[x, y]$ for all $x, y \epsilon N$ (13)

Replace x by yx in (13)

$$d[yx, y] = -[yx, y]$$

$$d(y[x, y]) = -y[x, y]$$

$$y\, d[x, y] + g[x, y]\, d(y) = -y[x, y]$$

Using (13) in the above equation, we get

$$g\{x, y]\, d(y) = 0$$

Since g is on to we have $[x, y]\, d(y) = 0$ for all $x, y \epsilon N$ (14)

The rest of the proof is as in the proof of theorem 2(i).

Theorem 3: Let N be a 2-torsion free prime near-ring.If N admits a non zero left semi derivation d with g such that

$$d(x\, o\, y) = (x\, o\, y)\text{, for all } x,y \in N$$

$$d(x\, o\, y) = -(x\, o\, y)\text{, for all } x,y \in N\text{, then } N \text{ is a commutative ring.}$$

Proof:

(i) By hypothesis $d(x\, o\, y) = (x\, o\, y)$, for all $x,y \in N$ (15)

Replace y by xy in (15)

$$d(x\, o\, xy) = (x\, o\, xy)$$

$$d(x(x\, o\, y)) = x(x\, o\, y)$$

$$x\, d(x\, o\, y) + g(x\, o\, y)\, d(x) = x\,(x\, o\, y)$$

Using (15) in the above equation , we get

$$g(x\, o\, y)\, d(x) = 0 \text{ for all } x,y \in N$$

$$g(xy)\, d(x) = -g(yx)d(x) \tag{16}$$

Replace y by yz in (16)

$$g(xyz)\, d(x) = -\, g(yzx)\, d(x) \text{ for all } x,y,z \in N$$

$$= -\, g(y)g(zx)d(x$$

$$= -\, g(y)\,(-g(xz)d(x))$$

$$= g(yxz)\, d(x)$$

$$g(xy\text{-}yx)\, g(z)d(x) = 0$$

$$g[\, x , y\,]\, g(z)\, d(x) = 0 \text{ for all } x,y,z \in N$$

$$g[\, x , y\,]\, N\, d(x) = 0 \text{ for all } x,y \in N$$

Since g is on to we have, $[\, x , y\,]\, N\, d(x) = 0$ for all $x,y \in N$ (17)

Since N is prime , equation (17) yields,

$$d(x) = 0 \quad \text{or} \quad [\, x , y\,] = 0 \text{ for all } x,y \in N \tag{18}$$

from (18) it follows that for each fixed $x \in N$ we have

$$d(\, x\,) = 0 \text{ or } x \in Z(N) \tag{19}$$

But $x \in Z(N)$ also implies that $d\,(x) \in Z(N)$ for all $x \in N$ (20)

In light of (20), $d(N) \subset Z(N)$ and using lemma 3

We conclude that N is a commutative ring.

(ii) By hypothesis $d(x\, o\, y) = -(x\, o\, y)$, for all $x,y \in N$ (21)

Replace y by xy in (15)

$$d(x \circ xy) = -(x \circ xy)$$

$$d(x(x \circ y)) = -x(x \circ y)$$

$$x\, d(x \circ y) + g(x \circ y)\, d(x) = -x(x \circ y)$$

Using (21) in the above equation , we get

$g(x \circ y)\, d(x) = 0$ for all $x,y \in N$

The rest of the proof is as in the proof of theorem 3(i).

Theorem 4: Let N be a 2-torsion free prime near-ring which admits a non zero left semi derivation d with g such that $d[x, y] \in Z(N)$, for all $x,y \in N$, then either $d(Z(N)) = 0$ or N is a commutative ring.

Proof:

Given that $d[x, y] \in Z(N)$ for all $x,y \in N$ (22)

(a) If $Z(N) = \{0\}$, it follows that $d[x, y] = 0$, for all $x,y \in N$

By Theorem 1 we conclude that N is a commutative ring.

(b) If $Z(N) \neq \{0\}$, replace y by yz in (22), where $z \in Z(N)$ weget

$d[x, yz] \in Z(N)$, for all $x,y \in N$, $z \in Z(N)$

$d([x, y]z) + d(y[x, z]) \in Z(N)$,

Since $z \in Z(N)$ implies $d([x, y]z) \in Z(N)$

$[x, y]\, d(z) + g(z)\, d[x, y] \in Z(N)$, for all $x,y \in N$, $z \in Z(N)$ (23)

Since $d[x, y] \in Z(N)$ and $z \in Z(N)$, equation (23) reduces to

$[x, y]\, d(z) \in Z(N)$, for all $x,y \in N$, $z \in Z(N)$

Accordingly $[[x, y]\, d(z), t] = 0$ for all $t \in N$

$[x, y]\, [d(z), t] + [[x, y], t]\, d(z) = 0$ for all $x,y, t \in N$, $z \in Z(N)$

$[[x, y], t]\, d(z) = 0$ for all $x,y, t \in N$, $z \in Z(N)$ (24)

Replace t by tr , for all $t,r \in N$, we get

$[[x, y], t]\, r\, d(z) + r\, [[x, y], t]\, d(z) = 0$ for all $x,y, t \in N$, $z \in Z(N)$

Using (24) in the above equation, we get $[[x, y], t]\, r\, d(z) = 0$ for all $x,y, t \in N$, $z \in Z(N)$

$[[x, y], t]\, N\, d(z) = 0$ for all $x,y, t \in N$, $z \in Z(N)$ (25)

Using primeness of N, from (25) it follows that

$d(Z(N)) = \{0\}$ or $[[x , y], t] = 0$ for all $x,y, t \in N$

Assume that $[[x , y], t] = 0$ for all $x,y, t \in N$, substituting yx for y, we get

$[[x , y]x, t] = 0$ and therefore $[x , y][x, t] = 0$ for all $x,y, t \in N$

As $[x , y] \in Z(N)$, hence

$$[x , y] N [x, y] = 0 \text{ for all } x,y \in N \qquad (26)$$

In light of the primeness of N , Eq.(26) shows that

$[x , y] =0$ and hence $x \in Z(N)$

Accordingly , $d(x) \in Z(N)$, for all $x \in N$ (27)

Once again using lemma 3, we get N is a commutative ring.

Theorem 5:- Let N be a prime near-ring which admits a non zero left semi derivation d with g, if $[d(x) , y] \in Z(N)$, for all $x,y \in N$, then N is a commutative ring.

Proof:

Assume that $[d(x) , y] \in Z(N)$, for all $x,y \in N$ (28)

Hence $[[d(x) , y] , t] = 0$, for all $x,y, t \in N$ (29)

Replacing y by $yd(x)$ in (29) we find that

$[[d(x) , y]d(x) , t] = 0$, for all $x,y, t \in N$ (30)

In view of (28), Eq.(30) assures that

$[d(x), y] N [d(x) , y] = \{0\}$, for all $x,y \in N$ (31)

By primeness of N Equation (31) shows that

$[d(x), y] = 0$, for all $x,y \in N$

Hence $d(N) \subset Z(N)$ and application of lemma 3 assures that N is a commutative ring.

Theorem 6: Let N be a 2-torsion free prime near-ring then there exists a non zero left semi derivation d with g of N such that $d(x) \circ y = x \circ y$ for all $x,y \in N$, then N is a commutative ring.

Proof:

Suppose that $d(x) \circ y = x \circ y$ for all $x,y \in N$ (32)

Replacing x by yx in (32) we obtain

$d(yx) \circ y = yx \circ y$

$$d(yx) \circ y = y\,(x \circ y)$$

Using eq.(32) implies

$$d(yx) \circ y = y\,(d(x) \circ y)$$

$$d(yx)\,y + y\,d(yx) = y\,d(x)\,y + y^2\,d(x)$$

$$y\,d(x)\,y + g(x)d(y)y + y^2\,d(x) + y\,g(x)d(y) = y\,d(x)\,y + y^2\,d(x)$$

$$g(x)d(y)y + y\,g(x)d(y) = 0$$

Since g is on to we have $x\,d(y)\,y + yx\,d(y) = 0$

$$yx\,d(y) = -x\,d(y)\,y \text{ for all } x,y \in N \qquad (33)$$

Replacing x by xz in (33), we find that

$$yxzd(y) = -xzd(y)y$$

$$= -x(zd(y)y) = -x(-yzd(y)) = -x(-y)zd(y) \text{ for all } x,y,z \in N$$

The last expression reduced to

$$yxzd(y) = -x(-y)zd(y) \text{ for all } x,y,z \in N \qquad (34)$$

Since $-yxzd(y) = (-y)xzd(y)$, (34) becomes

$$(-y)xzd(y) = x(-y)zd(y), \text{ for all } x,y,z \in N \qquad (35)$$

Taking $-y$ instead of y in (35) we obtain

$$yxzd(-y) = xyzd(-y) \text{ for all } x,y,z \in N$$

So that $(yx-xy)zd(-y) = 0$ and therefore

$$[y, x]\,N\,d(-y) = \{0\} \text{ for all } x,y \in N \qquad (36)$$

By primeness , Eq.(36) assures that for each $y \in N$, either $y \in Z(N)$ or $d(-y) = 0$.

Accordingly , $d(y) = 0$ or $y \in Z(N)$ for all $y \in N$ (37)

Since Eq.(37) is the same as Eq.(11), arguing as in the proof of Theorem 2 we conclude that N is a commutative ring.

Theorem 7: Let N be a 2-torsion free prime near-ring which admits a non zero left semi derivation d with g such that $d(x \circ y) \in Z(N)$, for all $x,y \in N$, then N is a commutative ring.

Proof:

Suppose that $d(x \circ y) \in Z(N)$, for all $x,y \in N$ (38)

(a) If $Z(N) = \{0\}$, then $d(x \circ y) = 0$ and replacing y by yx we obtain

$$d(x \circ yx) = 0$$

$$d((x \circ y)x) = 0$$

$(xoy)d(x) + g(x)d(xoy) = 0$, since $d(xoy) = 0$ implies

$(xoy)d(x) = 0$ for all $x,y \in N$ and thus

$$xyd(x) = -yxd(x) \text{ for all } x,y \in N \qquad (39)$$

Substituting yz for y in (39), we have

$xyxd(x) = -yzxd(x) = -y(-xzd(x)) = -y(-x)zd(x)$ for all $x,y,z \in N$

this means that

$$xyxd(x) = -y(-x)zd(x) \text{ for all } x,y,z \in N \qquad (40)$$

Since $-xyzd(x) = (-x)yzd(x)$, then (40) becomes

$$(-x)yzd(x) = y(-x)zd(x) \text{ for all } x,y,z \in N \qquad (41)$$

Then $[-x , y] z d(x) = 0$ for all $x,y,z \in N$ (42)

Taking $-x$ instead of x in (42) gives

$$[x , y] z d(-x) = 0 \text{ for all } x,y,z \in N \qquad (43)$$

Accordingly, $[x , y] N d(-x) = 0$ for all x,y,z (44)

Since Eq.(44) is the same as Eq.(36), arguing as in the proof of Theorem 6, we conclude that N is a commutative ring.

(b) If $Z(N) \neq \{0\}$, replace y by yz in (38), where $z \in Z(N)$ weget

$$d(x \circ yz) \in Z(N)$$

$$d((xoy)z - y[x , z]) \in Z(N)$$

Since $z \in Z(N)$ implies, $d((xoy)z) \in Z(N)$

$$(xoy)d(z) + g(z)d(xoy) \in Z(N), \text{ for all } x,y \in N, z \in Z(N) \qquad (45)$$

Using $d[x , y] \in Z(N)$ and $z \in Z(N)$, equation (45) reduces to

$$(xoy)\, d(z) \in Z(N), \text{ for all } x,y \in N, z \in Z(N) \qquad (46)$$

Since $d(z) \in Z(N)$, (46) yields that

$[(xoy)\, d(z) , t] = 0$ for all $x,y,t \in N$, $z \in Z(N)$

$$(xoy)\,[d(z) , t] + [(xoy) , t]\, d(z) = 0,$$

Since $d(z) \in Z(N)$ implies $[(xoy) , t]\, d(z) = 0$, so

$$[(xoy) , t]\, N\, d(z) = 0 \text{ for all } x,y,t \in N, z \in Z(N) \qquad (47)$$

By primeness of N Eq.(47) forces

either $d(Z(N)) = \{0\}$ or $xoy \in Z(N)$ for all $x,y \in N$ (48)

Suppose that $d(Z(N)) = 0$. If $0 \neq y \in Z(N)$

Since $d(xoy) = d(xy+yx) \in Z(N) = xd(y) + g(y)d(x) + yd(x) + g(x)d(y) \in Z(N)$,

Since $y \in Z(N)$, g is onto we have

$d(x o y) = d(x)y + d(x)y \in Z(N)$, then $d(d(x)y+d(x)y) = 0$ and hence

$$(d^2(x) + d^2(x)) y = 0 \text{ for all } x \in N \quad (49)$$

Using the fact that $0 \neq y \in Z(N)$, Eq.(49) leads to $d^2(x) = 0$ for all $x \in N$.

So that $d^2 = 0$ and lemma 2 forces $d = 0$, a contradiction. Accordingly, we have

$x \ o\ y \in Z(N)$ for all $x , y \in N$.

Let $0 \neq y \in Z(N)$, from $x o y = y (x + x)$.

$x^2 o y = y (x^2 + x^2)$ it follows , because of the primeness, that

$(x + x) xt = (x^2 + x^2)t = t(x^2 + x^2) = t(x + x)x = (x + x)tx$ for all $x , t \in N$.

and therefore

$$(x + x) N [x , t] = \{0\} \text{ for all } x ,t \in N. \quad (50)$$

Once again using the primeness hypothesis, Eq.(50) yields

$x \in Z(N)$ or $2x = 0$ in which case 2-torsion freeness forces $x= 0$.

Consequently , in both cases we arrive at $x \in Z(N)$ for all $x \in N$.

Hence $d(Z) \subset Z(N)$ and lemma 3 assures that N is a commutative ring.

Theorem 8: Let N be a 2-torsion free prime near-ring which admits a non zero left semi derivation d with g such that $d(x) o y \in Z(N)$, for all $x,y \in N$, then N is a commutative ring.

Proof:

$$\text{Assume that } d(x) o y \in Z(N), \text{ for all } x,y \in N \quad (51)$$

(a) If $Z(N) = 0$, then Eq.(51) reduces to

$$d(x)y = - yd(x) \quad \text{for all } x,y \in N \quad (52)$$

Substituting yz for y in (52) we obtain

$d(x)yz = - yzd(x) = (-y)zd(x) = (-y)(-d(x)z) = (-y)d(-x)z$ for all $x,y, z \in N$

in such a way

$$(d(x)y+yd(-x))z = 0 \text{ for all } x,y, z \in N \quad (53)$$

Taking $-x$ instead of x in (53) we get

$$(-d(x)y + yd(x)) N z = \{0\} \text{ for all } x,y, z \in N \quad (54)$$

Since N is prime , Eq.(54) forces $d(N) \subset Z(N)$ and lemma 3 it follows that N is a commutative ring.

(b) Suppose that $Z(N) \neq \{0\}$. If $0 \neq z \in Z(N)$, then since $d(x) \circ z \in Z(N)$, we find that $d(x)z + zd(x) \in Z(N)$

Since $z \in Z(N)$ implies $d(x) + d(x) \in Z(N)$ for all $x \in N$ (55)

More over from (51) it follows

$d(x + x)y + y\, d(x+ x) \in Z(N)$ which, because of (55) yields that

$(d(x + x) + d(x+ x))\, y \in Z(N)$ for all $x,y, z \in N$

and therefore, for all $t, x, y \in N$ we have

$$(d(x + x) + d(x+ x))\, ty = y\, (d(x + x) + d(x+ x))t$$

$$= (d(x + x) + d(x+ x))\, yt \text{ for all } x , t \in N$$

So that

$$(d(x + x) + d(x+ x))\, N\, [t , y] = \{0\} \text{ for all } t, x, y \in N \tag{56}$$

In view of the primeness of N Eq.(56) implies that either $d(x + x) + d(x + x) = 0$ and thus $d= 0$, a contradiction, or $N \subset Z(N)$ in which case $d(N) \subset Z(N)$,then by lemma 3, N is a commutating ring.

Theorem 9: Let N be a 2-torsion free prime near-ring which admits a non zero left semi derivation d with g such that $d(x \circ y) = [x , y]$ for all $x,y \in N$, then N is a commutative ring.

Proof:

$$\text{We have } d(x \circ y) = [x , y] \text{ for all } x,y \in N \tag{57}$$

Replacing y by xy in (57), we get

$$d(x(x \circ y)) = x[x , y]$$

$$x\, d(x \circ y) + g(x \circ y)d(x) = x[x , y]$$

Since g is on to and using (57) we get

$$(x \circ y)\, d(x) = 0 \text{ for all } x,y \in N \tag{58}$$

Replacing y by zy in (58) we find that

$$(x(zy) + (zy)x)d(x) = 0 \text{ for all } x,y,z \in N$$

Now application of (58) yields that $yxd(x) = -xyd(x)$

Combining this fact together with the latter relation we arrive at

$$(xz + z(-x))yd(x) = 0 \text{ for all } x,y,z \in N$$

$$[x , z] \, y \, d(x) = 0 \text{ for all } x,y,z \in N$$

$$[x , z] \, N \, d(x) = 0 \text{ for all } x,z \in N \qquad (59)$$

But since N is a prime near-ring ,for which fixed $x \in N$ either

$$d(x) = 0 \quad \text{or} \quad x \in Z(N) \text{ for all } x \in N$$

Hence using similar arguments as used after Eq.(5) we find that N is a commutative ring.

In this case (57) and 2-torsion freeness implies that

$$d(xy) = 0 \text{ for all } x,y \in N \qquad (60)$$

this means that $xd(y) + g(y)d(x) = 0$ for all $x,y \in N$ (61)

putting x by xz in (61) and using(60) , we get

$x \, z \, d(y) = 0$ which implies $x \, N \, d(y) = \{0\}$ for all $x,y \in N$

Since N is a prime and $d \neq 0$, then $x = 0$ for $x \in N$, a contradiction.

Theorem 10: Let N be a 2-torsion free prime near-ring which admits a non zero left semi derivation d with g such that $d[x , y] = (x \circ y)$ for all $x,y \in N$, then N is a commutative ring.

Proof:

We have $d[x , y] = (x \circ y)$ for all $x,y \in N$ (62)

Replacing y by xy in (62), we get

$$d(x[x , y]) = x(x \circ y)$$

$$x \, d[x , y] + g[x , y]d(x) = x(x \circ y)$$

Since g is on to and using (61) we get

$$[x , y] \, d(x) = 0 \text{ for all } x,y \in N \qquad (63)$$

Replacing y by yz in (63) , we get

$$[x , y] \, N \, d(x) = 0 \text{ for all } x,y \in N \qquad (64)$$

Since Eq.(64) is the same as Eq.(59) , arguing as in the proof of Theorem 9 we get the required result.

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