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Performance Analysis Of A System Having One Main Unit And Two Supporting Units

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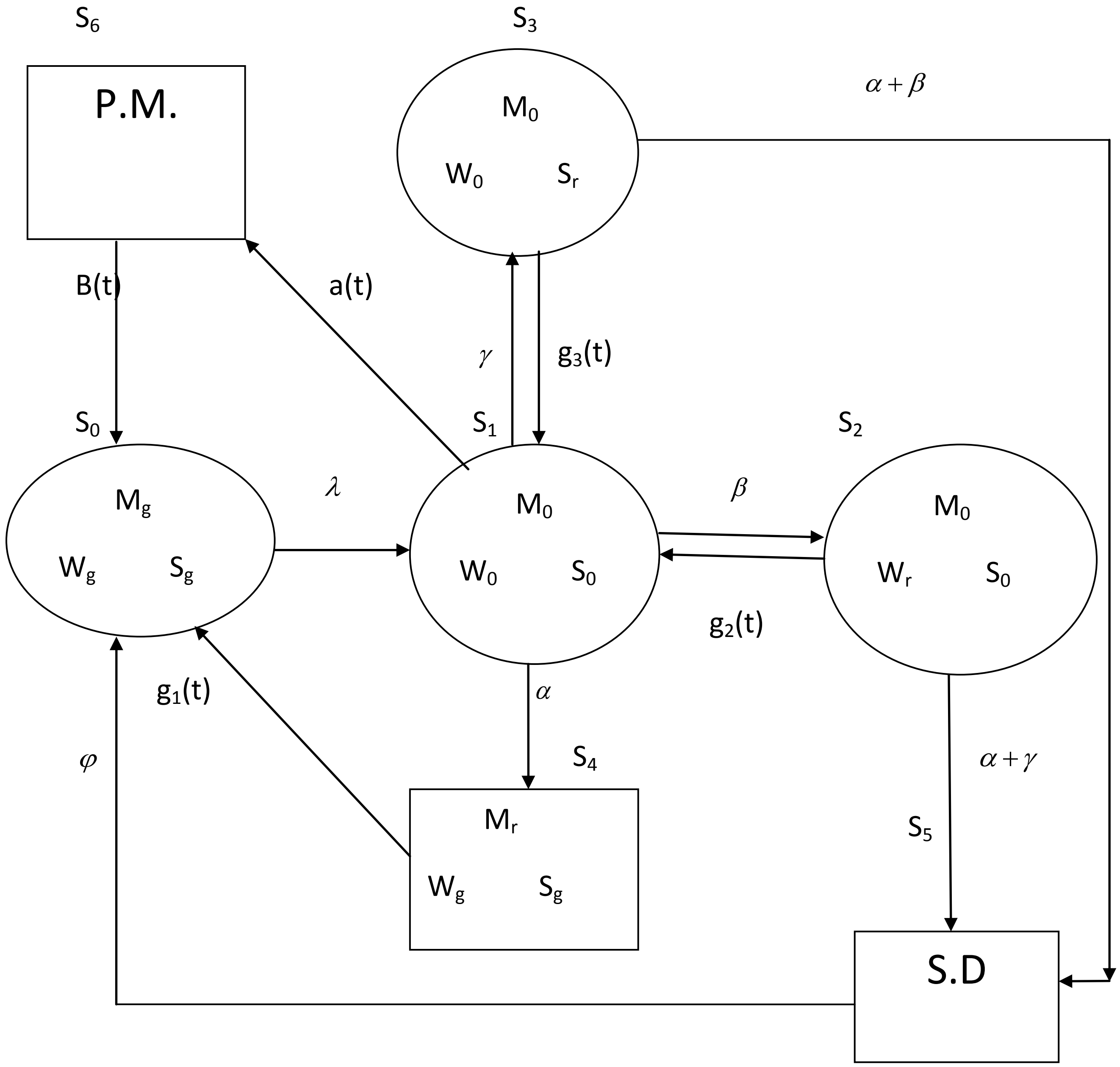
Abstract

Reliability for functioning units is essential, be it on the parts of memory chips of computers, software or hardware or it can be on the part of heavy machinery in any industrial system. Reliability analysis can give excellent results to improve the maintainability and portability of management design for existing and future product. Extensive reviews of two-component repairable system models have been presented by Lie et al(1977) and Yearout et al. (1986). In all these models, it is assumed that failure times and repair times of the components are independent. In this paper, Reliability analysis of a system having one main unit and two supporting units is proposed, assuming that the system fails whenever the main unit fails and system shuts down whenever either both the supporting units fail or main unit and one of the supporting units fail. To improve the reliability of the system, concept of preventive maintenance is also added. Using regenerative point technique various system parameters such as Transition Probabilities, Mean sojourn times, Mean time to system failure, Availability, Busy period of repairman in repairing the failed units etc. are calculated. At last profit analysis is also done. In this paper, failure time distributions are taken to be negative exponential whereas the repair time distributions are arbitrary.

Key words : mean sojourn times, mtsf, availability, busy period.

Introduction : In recent years many reliability models have been studied and evaluated. Engineers and Managers of Industries continuously make some modifications in the configuration and assumptions in the existing model in order to get the estimation of the various parameters such as mean time to system failure, steady state availability, busy period analysis and expected profit etc. which are responsible for making predictions about the production in their industries. In this competitive world of manufacturing there is immense pressure of the manufacturers to improve the quality of their product, to make them foolproof and modify the product frequently, if needed. Gopalan et [6] al have before this, carried out cost benefit analysis of single server n-unit imperfect switch system with delayed repair. Switching devices play an important role in the cold standby systems. Rander et al [5] have studied the idea of major and minor failures. Singh et al[7] have performed cost benefit analysis of a two unit warm standby system with inspection, repair and post repair. In this particular paper, we have considered one main unit with two supporting units which act like helping partner in the proper functioning of the whole system with the assumption that the whole system stops functioning when the main unit fails. The system also fails when either both the supporting units or main unit and one of the supporting unit fails. It is also proposed that after a random period of time the whole system goes for preventive maintenance. This approach can significantly improve the reliability of the working unit with limited overheads. Another way of improving reliability can be adopting an additional main unit in standby mode. But ,the idea here is to propose a system which can reliable and cost less at the same time.



<u>STATE TRANSITION DIAGRAM</u>

1. Assumptions :

1. The system consists of three units namely, One main unit and two associate units.
2. Any unit can fail when it is put to work.
3. After repair, a unit works as a new.
4. Switching devices are perfect and instantaneous.
5. If main unit fails, the system goes down.
6. If main and any of the associate units fails, the system shuts down.
7. If both the associate units fail, the system again shuts down.
8. There is a single repairman , who repairs the failed unit on the priority basis.
9. The failure time distributions of all the units are taken to be negative exponential whereas the repair time distributions are arbitrary.

2. Symbols and Notations: M is main unit and W and S are Associate units

E_0 =State of the system at epoch t=0

E=set of regenerative states $S_0 - S_6$

λ = Job arrival Rate

$q_{ij}(t) =$ Probability density function of transition time from $S_i \; to \, S_j$

$Q_{ij}(t) =$ Cumulative distribution function of time to transition time from $S_i \; to \, S_j$

$\pi_i(t) =$ Cumulative distribution function of time to system failure when starting from $E_0 = S_i \in E$ state

$\mu_i(t) =$Mean Sojourn time in the state $E_0 = S_i \in E$

$B_i(t) =$Repairman is busy in the repair at time t / $E_0 = S_i \in E$

$r_1 / r_2 / r_3 =$Constant repair rate of Main Unit / Associate units respectively.

$\alpha / \beta / \gamma / \Psi$ =Failure rate of Main Unit / Associate units respectively.

$g_1(t)/g_2(t)/g_3(t)$ = Probability density function of repair time of Main Unit / Associate units respectively.

$G_1(t)/G_2(t)/G_3(t)$ = Cumulative distribution function of repair time of Main Unit / Associate units respectively.

a(t) = Probability density function of preventive maintenance .

b(t) = Probability density function of preventive maintenance completion time.

A(t) = Cumulative distribution function of preventive maintenance.

B(t) = Cumulative distribution function of preventive maintenance completion time.

$\boxed{s}$ = Symbol for Laplace -stieltjes transform

$\boxed{c}$ = Symbol for Laplace-convolution

$M_o/M_g/M_r$ = Main unit under operation / good and non –operative mode / repair state

$W_o/W_g/W_r$ = Associative unit under operation / good and non –operative mode / repair state

$S_o/S_g/S_r$ = Associative unit under operation / good and non –operative mode / repair state

P.M = System under preventive maintenance

S.D = System under shutdown

Up states -

$S_0 = (M_g, W_g, S_g)$; $S_1 = (M_o, W_o, S_o)$; $S_2 = (M_o, W_r, S_o)$; $S_0 = (M_o, W_o, S_r)$

Down states -

$S_4 = (M_r, W_g, S_g)$; $S_5 = (S.D.)$; $S_6 = (P.M.)$

3. **Transition Probability : -** Using markovian regenerative process , Simple probabilistic considerations yields the following non zero transition probabilities -

1 $p_{01} = \int_0^\infty \lambda e^{-\lambda t}\, dt =$; 2. $p_{12} = \int_0^\infty \beta e^{-Xt}\, \overline{A(t)}\, dt = \frac{\beta}{X}[1 - a^*(x)]$

3. $p_{13} = \int_0^\infty \gamma\, e^{-Xt}\, \overline{A(t)}\, dt = \frac{\gamma}{X}[1 - a^*(x)]$; 4. $p_{14} = \int_0^\infty \alpha\, e^{-\alpha t}\, \overline{A(t)}\, dt = \frac{\alpha}{X}[1 - a^*(x)]$

5. $p_{16} = \int_0^\infty e^{-Xt} a(t)\, dt = a^*(x)$; 6. $p_{21} = \int_0^\infty e^{-Zt} g_2(t)\, dt = g^*{}_2(Z)$

7. $p_{25} = \int_0^\infty Z\, e^{-Zt}\, \overline{G_2}(t)\, dt = [1 - g^*{}_2(Z)]$; 8. $p_{31} = \int_0^\infty e^{-Yt} g_3(t)\, dt = g^*{}_3(Y)$

9. $p_{35} = \int_0^\infty Y\, e^{-Yt}\, \overline{G_3}(t)\, dt = [1 - g^*{}_3(Y)]$; 10. $p_{40} = \int_0^\infty g_1(t) dt$

11. $p_{50} = \int_0^\infty \varphi\, e^{-\varphi t} dt$; 12. $p_{60} = \int_0^\infty b(t)\, dt$ Eq. 1- 12

where $X = \alpha + \beta + \gamma$; $Y = \alpha + \beta$; $Z = \alpha + \gamma$

5. Mean Sojourn Time & Mean Time to System Failure:

Let μ_i in the state S_i be defined as time that system continuous to be in state S_i before transiting to any other states. If T denotes the Sojourn time in state S_i , then

$$\mu_i = E(t) = \int_0^\infty \emptyset(T < t) dt$$

Using above relation we can obtain the following equation-

$\mu_0 = \int_0^\infty \lambda\, e^{-\lambda t}\, dt = \frac{1}{\lambda}$; $\mu_2 = \int_0^\infty e^{*Zt}\, \overline{G_2(t)} dt = \frac{1}{Z}[1 - g_2{}^*(Z)]$

$\mu_3 = \int_0^\infty e^{*Yt}\, \overline{G_3(t)} dt = \frac{1}{Y}[1 - g_3{}^*(Y)]$; $\mu_4 = \int_0^\infty \overline{G_1(t)} dt = 1$; $\mu_5 = \int_0^\infty e^{-\Psi t}\, dt = \frac{1}{\Psi}$

$\mu_4 = \int_0^\infty \overline{B(t)} dt = 1$ Eq. 13- 18

Time to system failure can be regarded as the first passage time to the failed state. To obtain it we regarded to down state as absorbing states. Using argument as for the regenerative process, we obtain the following recursive relation for $\pi_i(t)$ as follows:

$$\mu_i(t) = \int_0^t \pi_i(t-u) dQ_{ij}(u) = \tilde{Q}_{ij}(t) \boxed{S} \pi_i(t)$$

$$\pi_0(t) = \tilde{Q}_{01}(t) \boxed{S} \pi_1(t)$$

$$\pi_1(t) = \tilde{Q}_{12}(t) \boxed{S} \pi_2(t) + \tilde{Q}_{13}(t) \boxed{S} \pi_3(t) + \tilde{Q}_{14}(t) + \tilde{Q}_{16}(t)$$

$$\pi_2(t) = \tilde{Q}_{21}(t) \boxed{S} \pi_1(t) + \tilde{Q}_{25}(t)$$

$$\pi_3(t) = \tilde{Q}_{31}(t) \boxed{S} \pi_1(t) + \tilde{Q}_{35}(t)$$

Eq. 19- 22

In matrix Form

$$\begin{bmatrix} 1 & -\tilde{Q}_{01} & 0 & 0 \\ 0 & 1 & -\tilde{Q}_{12} & -\tilde{Q}_{13} \\ 0 & -\tilde{Q}_{21} & 1 & 0 \\ 0 & -\tilde{Q}_{31} & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{\pi}_0 \\ \tilde{\pi}_1 \\ \tilde{\pi}_2 \\ \tilde{\pi}_3 \end{bmatrix} = \begin{bmatrix} 0 \\ \tilde{Q}_{14} + \tilde{Q}_{16} \\ \tilde{Q}_{25} \\ \tilde{Q}_{35} \end{bmatrix}$$

Then, we calculate the value of $D_1(s)$, $N_1(s)$, $D_1(0)$ and $N_1(0)$ as follows :

$$D_1(s) = 1 - \tilde{Q}_{12}\tilde{Q}_{21} - \tilde{Q}_{13}\tilde{Q}_{31}$$

$$D_1(0) = 1 - p_{12}\, p_{21} - p_{13}\, p_{31}$$

Eq. 23

$$N_1(s) = \tilde{Q}_{01}[\tilde{Q}_{14} + \tilde{Q}_{16} + \tilde{Q}_{12}\tilde{Q}_{25} + \tilde{Q}_{13}\tilde{Q}_{35}]$$

$$N_1(0) = p_{14} + p_{16} + p_{12}p_{25} + p_{13}p_{35}$$

Eq. 24

To Calculate the MTSF , we use the following formula :

$$MTSF = \frac{D_1^{'}(0) - N_1^{'}(0)}{D_1(0)}$$

$$MTSF = \frac{\mu_0(P_{14} + P_{16} + P_{12}P_{25} + P_{13}P_{35}) + \mu_1 + \mu_2 P_{12} + \mu_3 P_{13}}{1 - p_{12}\, p_{21} - p_{13}\, p_{31}}$$

Eq. 25

6. Availability Analysis:

Let $M_i(t)$ denote the probability that the system is up initially in regenerative state S_i at epoch t without passing through any other regenerative state. It might return to itself through one or more non regenerative states so that either it continues to remain in regenerative state without visiting any regenerative state including itself by probability arguments.

We observe that the entry to any of the state S_0, S_1, S_2 *and* S_3 is a regenerative point. $A_i(t)$ is defined as the probability that the system is up in state S_0, S_1, S_2 *and* S_3 at epoch it.

To obtain it consider all possible consequences.

1. Probability that the system initially up is S_0 is up at epoch t without transiting to any other regenerative state in E which is $M_0(t)$.

2. Probability that the system transits to S_i in E during (u,u+du) and then starting from S_0 it is up at epoch t which is

$$A_0 = \int_0^t q_{0i}(t) A_i(t-u)du = q_{0i} \boxed{c} A_i(t)$$

Thus we have

$$A_0(t) = M_0(t) + q_{01}(t) \boxed{c} A_1(t)$$

$$A_1(t) = M_1(t) + q_{12}(t) \boxed{c} A_2(t) + q_{13}(t) \boxed{c} A_3(t) + q_{14}(t) \boxed{c} A_4(t) + q_{16}(t) \boxed{c} A_5(t)$$

$$A_2(t) = M_2(t) + q_{21}(t) \boxed{c} A_1(t) + q_{25}(t) \boxed{c} A_5(t)$$

$$A_3(t) = M_3(t) + q_{31}(t) \boxed{c} A_1(t) + q_{35}(t) \boxed{c} A_5(t)$$

$$A_4(t) = q_{40}(t) \boxed{c} A_0(t)$$

$$A_5(t) = q_{50}(t) \boxed{c} A_0(t)$$

$$A_6(t) = q_{60}(t) \boxed{c} A_0(t)$$

Eq. 25- 32

Taking in the matrix form of above equation

$$\begin{bmatrix} 1 & -q_{01}^* & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -q_{12}^* & -q_{13}^* & -q_{14}^* & 0 & -q_{16}^* \\ 0 & -q_{21}^* & 1 & 0 & 0 & -q_{25}^* & 0 \\ 0 & -q_{31}^* & 0 & 1 & 0 & -q_{35}^* & 0 \\ -q_{40}^* & 0 & 0 & 0 & 1 & 0 & 0 \\ -q_{50}^* & 0 & 0 & 0 & 0 & 1 & 0 \\ -q_{60}^* & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_0^* \\ A_1^* \\ A_2^* \\ A_3^* \\ A_4^* \\ A_5^* \\ A_6^* \end{bmatrix} = \begin{bmatrix} M_0^* \\ M_1^* \\ M_2^* \\ M_3^* \\ M_4^* \\ M_5^* \\ M_6^* \end{bmatrix}$$

Then we calculate the values of $D_2(s)$, $N_2(s)$, $D_2(0)$ and $N_2(0)$

$$D_2(s) = (1 - q_{12}^{*} q_{25}^{*} - q_{13}^{*} q_{35}^{*}) - q_{01}^{*}[q_{14}^{*} q_{40}^{*} - q_{50}^{*}(q_{12}^{*} q_{25}^{*} + q_{13}^{*} q_{35}^{*}) - q_{16}^{*} q_{60}^{*}]$$

$$D_2(0) = (1 - p_{12}p_{25} - p_{13}p_{35}) - [p_{14} + (p_{12}p_{25} + p_{13}p_{35}) + p_{16}] \qquad Eq33$$

$$N_2(s) = M_0^{*}(1 - q_{12}^{*} q_{21}^{*} - q_{13}^{*} q_{31}^{*}) + M_1^{*} q_{01}^{*} + M_2^{*} q_{01}^{*} q_{12}^{*} + M_3^{*} q_{01}^{*} q_{13}^{*}$$

$$N_2(0) = \mu_0(1 - p_{12}p_{21} - p_{13}p_{31}) + \mu_1 + \mu_2 p_{12} + \mu_3 p_{13} \qquad Eq34$$

To obtain the value of

$$D_2'(0) = (\mu_o + \mu_5)(1 - p_{12}p_{21} - p_{13}p_{31}) + \mu_1 + \mu_2 p_{12} + \mu_3 p_{13} \qquad \text{Eq. }35$$

Thus the steady state Availability of the system

$$A_0(\infty) = \lim_{s \to 0} A_0^{*}(s) = \frac{N_2(0)}{D_2'(0)}$$

$$A_0(\infty) = \frac{\mu_0(1 - p_{12}p_{21} - p_{13}p_{31}) + \mu_1 + \mu_2 p_{12} + \mu_3 p_{13}}{(\mu_o + \mu_5)(1 - p_{12}p_{21} - p_{13}p_{31}) + \mu_1 + \mu_2 p_{12} + \mu_3 p_{13}} \qquad \text{Eq. }36$$

7. Busy Period Analysis :

(a) Busy period repairman for performing Normal repair :

Let $W_i(t)$ denote the probability that the repairman is busy initially with repair in regenerative state S_4 and remains busy at epoch t without transiting to any other state or returning to itself through one or more regenerative state. By probabilistic argument, we have

$$W_i(t) = \overline{G_i}(t)$$

Developing Similarly relationship as in availability for normal repair, we have

$$B_0'(t) = q_{01}(t) \boxed{\text{C}} B_1^1(t)$$

$$B_1^1(t) = q_{12}(t) \boxed{\text{C}} B_2^1(t) + q_{13}(t) \boxed{\text{C}} B_3^1(t) + q_{14}(t) \boxed{\text{C}} B_4^1(t) + q_{16}(t) \boxed{\text{C}} B_6^1(t)$$

$$B_2^1(t) = W_2(t) + q_{21}(t) \boxed{\text{C}} B_1^1(t) + q_{25}(t) \boxed{\text{C}} B_5^1(t)$$

$$B_3^1(t) = W_3(t) + q_{31}(t) \boxed{\text{C}} B_1^1(t) + q_{35}(t) \boxed{\text{C}} B_5^1(t)$$

$$B_4^1(t) = W_4(t) + q_{40}(t) \boxed{\text{C}} B_0^1(t)$$

$$B_5'(t) = q_{50}(t) \boxed{\text{C}} B_0^1(t)$$

$$B_6'(t) = q_{60}(t) \boxed{\text{C}} B_0^1(t) \qquad \text{Eq. }37\text{ - }43$$

In Matrix form, we have :

$$\begin{bmatrix} 1 & -q_{01}^* & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -q_{12}^* & -q_{13}^* & -q_{14}^* & 0 & -q_{16}^* \\ 0 & -q_{21}^* & 1 & 0 & 0 & -q_{25}^* & 0 \\ 0 & -q_{31}^* & 0 & 1 & 0 & -q_{35}^* & 0 \\ -q_{40}^* & 0 & 0 & 0 & 1 & 0 & 0 \\ -q_{50}^* & 0 & 0 & 0 & 0 & 1 & 0 \\ -q_{60}^* & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} B_0^{*1} \\ B_1^{*1} \\ B_2^{*1} \\ B_3^{*1} \\ B_4^{*1} \\ B_5^{*1} \\ B_6^{*1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ W_2^* \\ W_3^* \\ W_4^* \\ 0 \\ 0 \end{bmatrix}$$

$$N_3(s) = W_2^*(q_{01}^* q_{12}^*) + W_3^*(q_{01}^* q_{13}^*) + W_4^*(q_{01}^* q_{14}^*)$$

$$N_3(0) = \mu_2 p_{12} + \mu_3 p_{13} + \mu_4 p_{14} \qquad \text{Eq. 44}$$

To find the steady state the fraction of time for which the repairman of busy with repair , we first calculate

$$W_2^* = \mu_2 \quad ; \quad W_3^* = \mu_3 \quad ; \quad W_4^* = \mu_4 \qquad \text{Eq. 44-47}$$

Therefore in long run the fraction of fine for the repairman in busy with the normal repair is given by-

$$B_0^1(\infty) = \lim_{s \to 0} B_0^{*1}(t) = \frac{N_3(0)}{D_2'(0)}$$

$$B_0^1(\infty) = \frac{\mu_2 p_{12} + \mu_3 p_{13} + \mu_4 p_{14}}{(\mu_o + \mu_5)(1 - p_{12}p_{21} - p_{13}p_{31}) + \mu_1 + \mu_2 p_{12} + \mu_3 p_{13}} \qquad \text{Eq. 48}$$

(b) Busy period repairman performing for Shutdown repair

Developing similar relationships as in availability for shutdown repair, we have

$$B_0^2(t) = q_{01}(t) \boxed{c} B_1^2(t)$$

$$B_1^2(t) = q_{12}(t) \boxed{c} B_2^2(t) + q_{13}(t) \boxed{c} B_3^2(t) + q_{14}(t) \boxed{c} B_4^2(t) + q_{16}(t) \boxed{c} B_6^2(t)$$

$$B_2^2(t) = q_{21}(t) \boxed{c} B_1^2(t) + q_{25}(t) \boxed{c} B_5^2(t)$$

$$B_3^2(t) = q_{31}(t) \boxed{c} B_1^2(t) + q_{35}(t) \boxed{c} B_5^2(t)$$

$$B_4^2(t) = q_{40}(t) \boxed{c} B_0^2(t)$$

$$B_5^2(t) = W_5 + q_{50}(t) \boxed{c} B_0^2(t)$$

$$B_6^2(t) = q_{60}(t) \boxed{c} B_0^2(t) \qquad \text{Eq. 49 - 55}$$

Taking Matrix form

$$\begin{bmatrix} 1 & -q_{01}^{*} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -q_{12}^{*} & -q_{13}^{*} & -q_{14}^{*} & 0 & -q_{16}^{*} \\ 0 & -q_{21}^{*} & 1 & 0 & 0 & -q_{25}^{*} & 0 \\ 0 & -q_{31}^{*} & 0 & 1 & 0 & -q_{35}^{*} & 0 \\ -q_{40}^{*} & 0 & 0 & 0 & 1 & 0 & 0 \\ -q_{50}^{*} & 0 & 0 & 0 & 0 & 1 & 0 \\ -q_{60}^{*} & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} B_0^{*2} \\ B_1^{*2} \\ B_2^{*2} \\ B_3^{*2} \\ B_4^{*2} \\ B_5^{*2} \\ B_6^{*2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ W_5^{*} \\ 0 \end{bmatrix}$$

So that, $N_4(s) = W_5^{*}[q_{01}^{*}(q_{12}^{*}q_{25}^{*} + q_{13}^{*}q_{35}^{*})]$

To find the steady state the fraction of time for which the repairman of busy with repair , we first calculate

$$W_5^{*} = \mu_5 \qquad \text{Eq. 56}$$

$$N_4(0) = \mu_5(p_{12}p_{25} + p_{13}p_{35}) \qquad \text{Eq. 57}$$

Therefore in long run the fraction of fine for the repairman in busy with the shutdown repair is given by

$$B_0^{2}(\infty) = \lim_{s \to 0} B_0^{*2}(t) = \lim_{s \to 0} s B_0^{*2}(s) = \frac{N_4(0)}{D_2'(0)}$$

$$B_0^{2}(\infty) = \frac{\mu_5(p_{12}p_{25} + p_{13}p_{35})}{(\mu_o + \mu_5)(1 - p_{12}p_{21} - p_{13}p_{31}) + \mu_1 + \mu_2 p_{12} + \mu_3 p_{13}} \qquad \text{Eq. 58}$$

(c) Similarly to calculate the Busy Period of repairman performing the Preventive maintenance

$$N_5(s) = W_6^{*} q_{01}^{*} q_{16}^{*}$$

To find the steady state the fraction of time for which the repairman of busy with repair , we first calculate

$$W_6^{*} = \mu_6$$

$$N_5(0) = \mu_6 p_{16}$$

Therefore, in long run ,for the fraction of time the repairman in busy with the preventive maintentance is given by

$$B_0^{\ 3}(\infty) = \lim_{s \to 0} B_0^{*3}(t) = \lim_{s \to 0} s\, B_0^{*3}(s) = \frac{N_5(0)}{D_2'(0)}$$

$$B_0^{\ 3}(\infty) = \frac{\mu_6 p_{16}}{(\mu_o + \mu_5)(1 - p_{12}p_{21} - p_{13}p_{31}) + \mu_1 + \mu_2 p_{12} + \mu_3 p_{13}}$$

7. Particular Cases:

When all repair time distributation are n-phases Erlangian distributation . i.e.,

Density Function And Survial Function

$$g_i(t) = \sum \frac{nr_i(nr_it)^{n-1} e^{-nr_it}}{n-1!} \quad ; \quad \overline{G}_i(t) = \sum_{j-0}^{n-1} \frac{(nr_it)^j e^{-nr_it}}{j!}$$

And other distributation are negative exponential

$$a(t) = \theta e^{-\theta t} \quad , \quad b(t) = \eta e^{-\eta t} \quad , \quad \overline{A}(t) = e^{-\theta t} \quad , \quad \overline{B}(t) = e^{-\eta t}$$

For n=1

$$p_{01} = 1 \; ; \; p_{40} = 1 \; ; \; p_{50} = 1 \; ; \; p_{60} = 1$$

$$p_{12} = \frac{\beta}{X + \theta} \; ; \; p_{13} = \frac{\gamma}{X + \theta} \; ; \; p_{14} = \frac{\alpha}{X + \theta} \; ; \; p_{16} = \frac{\theta}{X + \theta} \; ; \; p_{21} = \frac{r_2}{Z + r_2}$$

$$p_{25} = \frac{Z}{Z + r_2} \; ; \; p_{31} = \frac{r_3}{Y + r_3} \; ; \; p_{35} = \frac{Y}{Y + r_3}$$

We can see that

$p_{12} + p_{13} + p_{14} + p_{16} = 1 \; ; \; p_{21} + p_{25} = 1 \; ; \; p_{31} + p_{35} = 1$ and

$$\mu_0 = \frac{1}{\lambda} \;;\; \mu_1 = \frac{1}{X+\theta} \;;\; \mu_2 = \frac{1}{Z+r_2} \;;\; \mu_3 = \frac{1}{Y+r_3}$$

$$\mu_4 = \frac{1}{r_1} \;;\; \mu_5 = \frac{1}{\psi} \;;\; \mu_6 = \frac{1}{\eta}$$

So, we Calculate ,

$$MTSF = \frac{L_1}{\lambda}\frac{[K_1+K_2]}{K_3} \;;\; Availabilty = \frac{\frac{k_3}{\lambda}+L_1+K_2}{M_1+K_3+L_1(K_2+M_2)}$$

Busy period

$$B_0^{1*}(\infty) = \frac{M_3 L_1}{M_1 K_3 + L_1(K_2+M_2)} \;;\; B_0^{2*}(\infty) = \frac{M_4}{\psi[M_1 K_3 + L_1(K_2+M_2)]}$$

Where

$$L_1 = \frac{1}{X+\theta} \;;\; L_2 = \frac{1}{Y+r_3} \;;\; L_3 = \frac{1}{Z+r_3}$$

$$K_1 = \alpha+\theta+\beta Z L_2 \;;\; K_2 = 1+\beta L_3+\gamma L_2 \;;\; K_3 = 1-\beta r_2 L_1 L_2 - \gamma r_3 L_1 L_2$$

$$M_1 = (\frac{1}{\lambda}+\frac{1}{\psi}) \;;\; M_2 = (\frac{\alpha}{r_1}+\frac{\theta}{\eta}) \;;\; M_3 = (\beta L_3+\gamma L_2+\frac{\alpha}{r_1}) \;;\; M_4 = \beta Z L_1 L_3 + \gamma Y L_1 L_2$$

8. Profit Analysis:

The profit analysis of the system can be carried out by considering the expected busy period of repairman in repair of the unit in [0,t] . Therefore,

G(t)= total revenue earned by the system in [0,t]- Expected repair cost in [0,t]

$$= C_1\mu_{up}(t) - C_2\mu_{b1} - C_3\mu_{b2}$$

Where

$$\mu_{up}(t) = \int_0^t A_0(t)dt \;;\; \mu_{b1}(t) = \int_0^t B^1{}_0(t)dt \;;\; \mu_{b2}(t) = \int_0^t B^{21}{}_0(t)dt$$

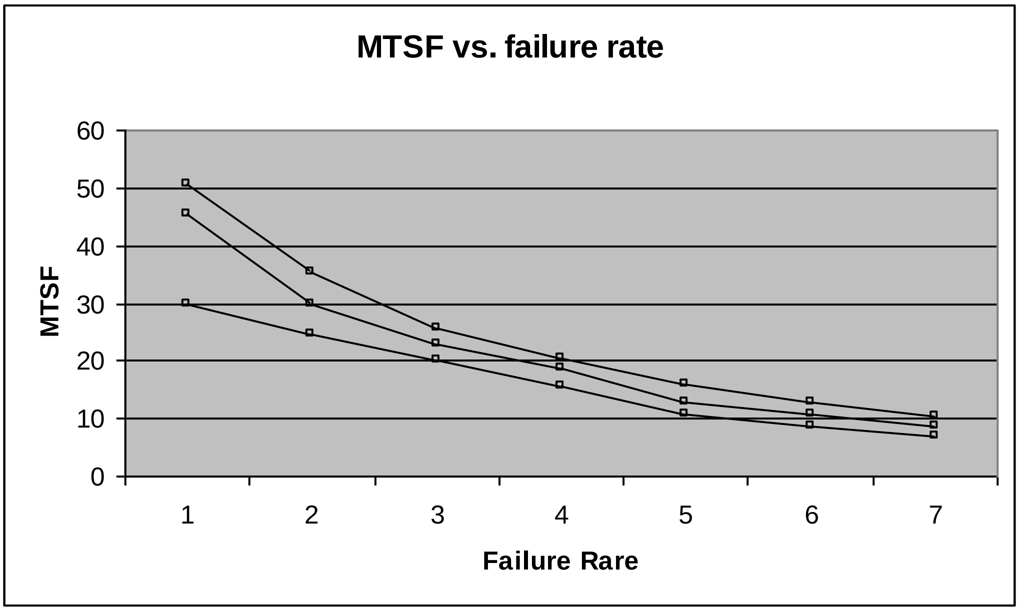


Figure 1.1

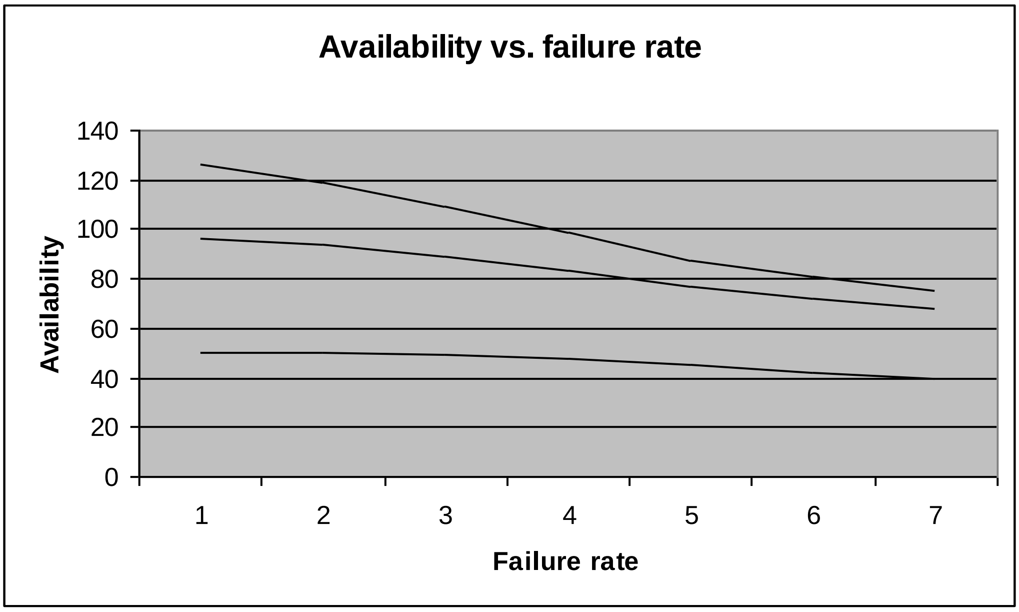


Figure 1.2

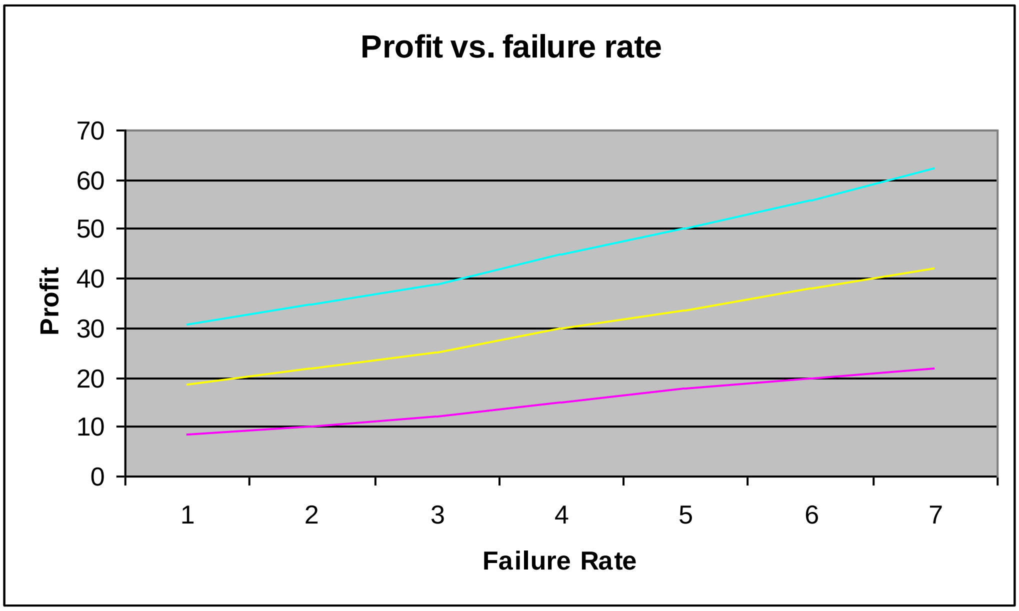


Figure 1.3

Discussion : A pioneering work in this direction involving component of two-unit system was initiated by Harris(1968). We have considered a two unit redundant system in which failure times of the component are taken to be exponential to derive mean time to system failure by using regenerative point technique for arbitrary repair time distribution.

9. Conclusion:

The mean time to system failure (MTSF) and availability of the system decreases rapidly with the increase of failure rates $\alpha \,\&\, \beta$ for fixed values of other parameters. However, it is noted that values of profit decreases with the increase in the failure rate but increases as and when repair rates $r_1 \,\&\, r_2$ increase. With preventive maintenance the reliability of the system increases considerably.

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