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Fuzzy open Sets In fuzzy Tri Topological Space

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ABSTRACT
The main aim of this paper is to studyfuzzy open sets and fuzzy trib-open sets in fuzzy tri topological spaces along with their several propertiesand characterization .We studyfuzzy tri continuous, fuzzy tri separation, fuzzy trib-continuous, fuzzy tri-b separation and obtain few of their basic properties.

KEYWORDS: Fuzzy Tritopology, fuzzy tri open sets, fuzzy tri continuousfunction,fuzzy tri separation, fuzzy tri b-open sets,fuzzy tri b-continuousfunction,fuzzy tri-b separation.
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1. Introduction

In 1996, D. Andrijvic [1] introduced b-open sets. In 1961 J .C. Kelly [6] introduced the concept of bitopological space. Al-Hawary& A. Al-omari [3] defined the b-open set in bitopological space .Abo Khadra and Nasef [2] discussed b-open set in bitopological spaces. Martin M. Kovar[8] studied tri topological space.

S. Palaniammal [9] studied tri topological space.123 open set in tri topological spaces was studied byN.F. Hameed & Mohammed Yahya Abid[7].P. Priyadharsini and A. Parvathi [10] introduced tri-b open sets in tri topological spaces. We [11] studied properties of tri-b open sets in tri topological spaces.

fuzzy topological spaces introduced by Change C.L. [4]. Kandil A. [5] introduced fuzzy bitopological spaces in 1991. Palaniammal S.[9] introduced fuzzy tri topological space.

The present paper introducefuzzy tri open sets and fuzzy tri b-open sets in fuzzy tri topological space and studied their fundamental properties in fuzzy tri topological space.

2. Preliminaries

Definition 2.1[9]:"Let X be a nonempty set and T_1, T_2 and T_3 are three topologies on X .The set X together with three topologies is called a tri topological space and is denoted by (X, T_1, T_2, T_3)".

Definition 2.2[9]: "Let X be a non-empty set. Let F_1, F_2, F_3 be fuzzy topologies on X . Then (X, F_1, F_2, F_3) is called fuzzy tri topological space".

Definition 2.3[1]: "A subset A of a topological space (X, τ) is called a b-open sets if $A \subseteq cl(\text{int}(A)) \cup \text{int}(cl(A))$ and b-closed if $cl(\text{int}(A)) \cup \text{int}(cl(A)) \subseteq A$".

3. Fuzzy Tri-Separation Axioms in Fuzzy Tri Topological Space

Definition 3.1: Let $(X, \sigma_1, \sigma_2, \sigma_3)$ be a fuzzy tri topological space Then a subset B of X is called fuzzytri open set ($Ftri$) if $B \prec F_1 \cup F_2 \cup F_3$.

Note 3.2: Fuzzy tri-interior(resp. Fuzzy tri -closure) of any subset B is denoted by $Ftri\,\text{int}(B)$ ($Ftricl(B)$).

Definition 3.3: A fuzzy tri topological space $(X, \sigma_1, \sigma_2, \sigma_3)$ is called $F\,tri - T_0$ space iff to given pair of different points $\chi_{\mu_1}, \chi_{\mu_2}$ in χ_μ , $\exists$ a fuzzy tri- open set consisting one point but not the other.

Example 3.4: Let $X=\{1,2,3\}$ be a non-empty fuzzy set, consider three fuzzy topologies on X are $\tau_1=\{\tilde{1}_X,\tilde{0}_X\}$, $\tau_2=\{\tilde{1}_X,\tilde{0}_X,\chi_{\{1\}}\}$, $\tau_3=\{\tilde{1}_X,\tilde{0}_X,\chi_{\{2,3\}}\}$

Fuzzy tri open sets of fuzzy tri topological spaces are union of all fuzzy tri topologies.

Then fuzzy tri open sets of $X=\{\tilde{1}_X,\tilde{0}_X,\chi_{\{1\}},\chi_{\{2,3\}}\}$

So $(X,\sigma_1,\sigma_2,\sigma_3)$ is $F\,tri-T_0$ space.

Definition 3.5: A fuzzy tri topological space X is calledfuzzy tri-T_1space iff for any given pair of different fuzzy points $\chi_{\{x_1\}}$, $\chi_{\{y_1\}}$ of X there be two fuzzy tri-open sets χ_{λ_1}, χ_{δ_1} such that $\chi_{\{x_1\}}\leq\chi_{\lambda_1}$, $\chi_{\{y_1\}}\succ\chi_{\lambda_1}$ and $\chi_{\{y_1\}}\leq\chi_{\delta_1}$, $\chi_{\{x_1\}}\succ\chi_{\delta_1}$.

Theorem 3.6: Everyfuzzy tri-T_1 space is a fuzzy tri-T_0 space.

Definition 3.7:fuzzy tri topological space X is called fuzzy tri-T_2 space iff for $\chi_{\{x_1\}}$, $\chi_{\{y_1\}}\leq\tilde{1}_X$, $\chi_{\{x_1\}}\neq\chi_{\{y_1\}}$ there be two disjoint fuzzy tri- open sets χ_λ, χ_δ in X such that $\chi_{\{x_1\}}\leq\chi_\lambda$, $\chi_{\{y_1\}}\leq\chi_\delta$.

Theorem 3.8: Every fuzzy tri-T_2 space is fuzzy tri-T_1 space.

Proof: Suppose X is a fuzzy tri-T_2 space and let $\chi_{\{x_1\}}$, $\chi_{\{y_1\}}$ in X with $\chi_{\{x_1\}}\neq\chi_{\{y_1\}}$,so by assumption $\exists$ two different fuzzy tri-open,say χ_λ, χ_δ such that $\chi_{\{x_1\}}\leq\chi_\lambda$, $\chi_{\{y_1\}}\leq\chi_\delta$ but $\chi_\lambda\wedge\chi_\delta=\tilde{0}_X$ hence $\chi_{\{x_1\}}\succ\chi_\delta$, $\chi_{\{y_1\}}\succ\chi_\lambda$ i.e. X isa fuzzy tri-T_1 space.

Theorem 3.9: Every fuzzy tri-b-T_3 space is afuzzy tri-T_2 space.

Proof: Suppose $(X,\sigma_1,\sigma_2,\sigma_3)$ be a fuzzy tri-T_3 space and let $\chi_{\{x_1\}}$, $\chi_{\{y_1\}}$ are two different fuzzy points of X. By definition , X is afuzzy tri--T_1 space &therefore $\chi_{\{x_1\}}$ is a fuzzy tri- closed set .furthermore $\chi_{\{y_1\}}\succ\chi_{\{x_1\}}$ .Since $(X,\sigma_1,\sigma_2,\sigma_3)$ is a fuzzy tri- regular space, $\exists$ fuzzy tri- open sets χ_λ and χ_δ such that $\chi_{\{x_1\}}\prec\chi_\lambda$, $\chi_{\{y_1\}}\prec\chi_\delta$ and $\chi_\lambda\wedge\chi_\delta=\tilde{0}_X$ .Also $\chi_{\{x_1\}}\prec\chi_\lambda\Rightarrow\chi_{\{x_1\}}\leq\chi_\lambda$ Thus $\chi_{\{x_1\}}$, $\chi_{\{y_1\}}$ belong respectively to differentfuzzy tri- open sets χ_λ and χ_δ . According $(X,\sigma_1,\sigma_2,\sigma_3)$ is a $Ftri-T_2$ space.

4. Fuzzy Tri-g Open Sets in Tri Topological Space

Definition 4.1: A fuzzy subset χ_{λ_1} of a fuzzy tri topological space (X,ρ_1,ρ_2,ρ_3) is called fuzzy tri-g-closed if $Ftri-cl(\chi_{\lambda_1})\prec\chi_{\delta_1}$ whenever $\chi_{\lambda_1}\prec\chi_{\delta_1}$ and χ_{δ_1} is fuzzy tri open set.

Remarks 4.2:

(i) The complement of fuzzy tri-g closed is fuzzy tri-g open.

(ii) Fuzzy tri-g-closure of χ_δ is denoted by $Ftrigcl(\chi_\delta)$ and fuzzy tri-g-interiorof χ_δ is denoted by $Ftrig\,\mathrm{int}(\chi_\delta)$.

5. Fuzzy Tri-b Open Sets in Fuzzy Tri Topological Space

Definition 5.1: Let (X,ρ_1,ρ_2,ρ_3) is a Fuzzy tri topological space. Then $B\subset X$ is called Fuzzytri-b open set if $B\subset Ftri-cl(Ftri-\mathrm{int}\,B)\cup Ftri-\mathrm{int}(Ftri-clB)$.

Not 5.2: Fuzzy tri-interior(resp. Fuzzy tri -closure) of any subset B is denoted by $Ftri-b\text{int}(B)$ ($Ftri-bcl(B)$) and $Ftri-bclA$ is the intersection of all Fuzzy tri –b closed sets containing A.

Theorem5.3: Let $(X,\sigma_1,\sigma_2,\sigma_3)$ be afuzzy tri topological space and $\chi_\mu \prec X$, then

(a) $Ftri-bcl\chi_\mu = Fscl\chi_\mu \wedge Fpcl\chi_\mu$.

(b) $Ftri-b\text{int}\,\chi_\mu = F\sin t\chi_\mu \vee Fp\text{int}\,\chi_\mu$.

Theorem 5.4: If χ_λ is a subset of a fuzzy tri topological space (X,τ_1,τ_2,τ_3), then

$Ftri-bint\ \left(Ftri-bcl\ \chi_\lambda\right) = \text{Ftri-}bcl\left(Ftri-b\text{int}\,\chi_\lambda\right)$.

Proof: Let (X,τ_1,τ_2,τ_3) be a fuzzy tri topological space.

Now, $Ftri-bint\ \left(Ftri-bcl\ (\chi_\lambda)\right) = \text{Fsint(Ftri-bcl}\chi_\lambda) \vee Fp\text{int}\left(Ftri-bcl\chi_\lambda\right)$.

$= Ftri-bcl\left(\text{Fsint}\ \chi_\lambda\right) \vee Fp\text{int}\left(Ftri-bcl\chi_\mu\right)$--------------- (i)

& $Ftri-bcl\left(\text{Ftri-bint}\ \chi_\lambda\right) = Ftri-bcl(\text{F}\sin t\chi_\mu \vee Fp\text{int}\,\chi_\mu)$

$= Ftri-bcl(\text{F}\sin t\chi_\mu) \vee Ftri-bcl(\text{F}\,p\,\text{int}\,\chi_\mu)$

$= Fscl(\text{F}\sin t\chi_\mu) \vee Fp\,\text{int}(\text{F}\,pcl\chi_\mu)$------------------------------ (ii)

Hence from (i) & (ii),

$Ftri-bint\ \left(Ftri-bcl\ \chi_\lambda\right) = \text{Ftri-}bcl\left(Ftri-b\text{int}\,\chi_\lambda\right)$.

Corollary 5.5:A subset χ_λ in a fuzzy tri topological space $(X,\sigma_1,\sigma_2,\sigma_3)$ is fuzzy tri-b open if and only if it contains fuzzy tri pre-open set but not its fuzzy tri pre-closure.

6. Fuzzy Tri-b Continuous Function in Fuzzy Tri Topological Space

Dentition6.1: $f:(X,\tau_1,\tau_2,\tau_3) \to (\text{Y},\rho_1,\rho_2,\rho_3)$ iscalledfuzzy tri-b closed (resp. fuzzy tri-b open) if for every fuzzy tri-b closed (resp. fuzzy tri-b open) subset χ_λ of X, $f(\chi_\lambda)$ is fuzzy tri b-closed (resp. fuzzy tri b-open) in Y.

Definition 6.2: Let (X,τ_1,τ_2,τ_3) and $(\text{Y},\rho_1,\rho_2,\rho_3)$ are two fuzzy tri-topological spaces. A fuzzy function $f:\text{I}^X \to I^Y$ is called fuzzy tri-b open map (fuzzy tri- b closed map) if $f(\chi_\lambda)$ is fuzzy tri-b open(fuzzy tri-b closed) in Y for every fuzzy tri-b open set(fuzzy tri-b closed) χ_λ in X.

Definition 6.3: Consider two fuzzy tri topological spaces (X,τ_1,τ_2,τ_3), $(Y,\sigma_1,\sigma_2,\sigma_3)$. A fuzzy function $f:I^X \to I^Y$ is called a fuzzy tri-b continuous function if χ_λ is fuzzy tri-b open in X, for every tri-b open set χ_λ in Y.

Theorem 6.4: A function $f:(X,\tau_1,\tau_2,\tau_3) \to (Y,\sigma_1,\sigma_2,\sigma_3)$ is fuzzy tri-b continuous iff the inverse image of every fuzzy tri-b open set in Y is fuzzy tri-b open in X.

Theorem 6.5: Let (X,τ_1,τ_2,τ_3) and $(Y,\sigma_1,\sigma_2,\sigma_3)$ be two fuzzy tri topological spaces. Then, $f:I^X \to I^Y$ is fuzzy tri-b continuous function if and only if $f^{-1}(\chi_\lambda)$ is fuzzy tri-b closed in X whenever χ_λ is fuzzy tri-b closed in Y.

Theorem 6.6: Let (X,τ_1,τ_2,τ_3) and $(Y,\sigma_1,\sigma_2,\sigma_3)$ be two fuzzy tri topological spaces. Then $f:I^X \to I^Y$ is fuzzy tri-b continuous function if and only if $f(Ftri-bcl(\chi_\lambda)) \prec Ftri-bcl(f(\chi_\lambda))\ \forall\chi_\lambda \prec \tilde{1}_X$

Proof: Suppose $f: I^X \to I^Y$ is a fuzzy tri-b continuous function. Since $Ftri-bcl[f(\chi_\lambda)]$ is fuzzy tri closed in Y .So using theorem (3.4) $f^{-1}[F\text{ tri}-bcl(f(\chi_\lambda))]$ is fuzzy tri closed in X ,

$Ftri-bcl(f^{-1}(F\text{ tri}-bclf(\chi_\lambda))) = f^{-1}(F\text{ tri}-bclf(\chi_\lambda))$ ----(1).

Now : $f(\chi_\lambda) \prec Ftri-bcl(f(\chi_\lambda)),\ \chi_\lambda \prec f^{-1}(f(\chi_\lambda)) \prec f^{-1}(F\text{ tri}-bclf(\chi_\lambda))$.

Then $Ftri-bcl(\chi_\lambda) \prec Ftri-bcl(f^{-1}(F\text{ tri}-bclf(\chi_\lambda))) = f^{-1}(F\text{ tri}-bclf(\chi_\lambda))$ by (1).

Then $f(F\text{ tri}-bcl(\chi_\lambda)) \prec Ftri-bcl(f(\chi_\lambda))$.

Conversely, Let $f(F\text{ tri}-bcl(\chi_\lambda)) \prec Ftri-bcl(f(\chi_\lambda))\ \forall \chi_\lambda \prec \tilde{1}_X$.

Let χ_δ be fuzzy tri closed set in Y, So that $Ftri-bcl(\chi_\delta) = \chi_\delta$. Now $f^{-1}(\chi_\delta) \prec \tilde{1}_X$, by supposition,

$f(F\text{ tri}-bcl(f^{-1}(\chi_\delta))) \prec Ftri-bcl(f(f^{-1}(\chi_\delta)) \prec Ftri-bcl(\chi_\delta) = \chi_\delta$.

Therefore, $Ftri-bcl(f^{-1}(\chi_\delta) \prec f^{-1}(\chi_\delta)$ .But $f^{-1}(\chi_\delta) \prec Ftri-bcl(f^{-1}(\chi_\delta)$ always.

Hence $Ftri-bcl(f^{-1}(\chi_\delta) = f^{-1}(\chi_\delta)$ and so $f^{-1}(\chi_\delta)$ is fuzzy tri closed in X .

Hence by theorem (6.5) f is fuzzy tri continuous function.

Theorem 6.7: Let (X,τ_1,τ_2,τ_3) and $(Y,\sigma_1,\sigma_2,\sigma_3)$ be two fuzzy tri topological spaces. Then $f: I^X \to I^Y$ is fuzzytri continuous function if and only if $f^{-1}(F\text{ tri}-b\text{int}(\chi_\lambda)) \prec Ftri-b\text{int}(f^{-1}(\chi_\lambda)) \quad \forall \chi_\lambda \prec \tilde{1}_Y$.

Proof: Let $f: I^X \to I^Y$ be a fuzzy tri continuous function. Since $Ftri-b\text{int}(\chi_\lambda)$ is fuzzy tri-b open in Y, then by theorem (3.3) $f^{-1}(F\text{ tri}-b\text{int}(\chi_\lambda))$ is fuzzy tri open in X and therefore

$F\text{ tri}-b\text{int}(f^{-1}(F\text{ tri}-b\text{int}(\chi_\lambda))) = f^{-1}(F\text{ tri}-b\text{int}(\chi_\lambda))$.........(1)

Now $F\text{ tri}-b\text{int}(\chi_\lambda) \prec \chi_\lambda$, then

$f^{-1}(F\text{ tri}-b\text{int}(\chi_\lambda)) \prec f^{-1}(\chi_\lambda)$ Then

$F\text{ tri}-b\text{int}(f^{-1}(F\text{ tri}-b\text{int}(\chi_\lambda))) \prec Ftri-b\text{int}(f^{-1}(\chi_\lambda))$ By (1)

Conversely: Let the condition hold and let χ_δ be any fuzzy tri open set in Y so that

$F\text{ tri}-b\text{int}(\chi_\delta) = \chi_\delta$.

By hypothesis

$f^{-1}(\text{tri}-\text{int}(\chi_\delta)) \prec tri-\text{int}\, f^{-1}(\chi_\delta)$

Since $f^{-1}(\text{tri}-\text{int}(\chi_\delta)) = f^{-1}(\chi_\delta)$ then

$f^{-1}(\chi_\delta) \prec tri-\text{int}(f^{-1}(\chi_\delta))$

But $\text{int}(f^{-1}(\chi_\delta)) \prec f^{-1}(\chi_\delta)$ always

So, $\text{tri}-\text{int}(f^{-1}(\chi_\delta)) = f^{-1}(\chi_\delta)$.

Therefore $f^{-1}(\chi_\delta)$ is fuzzy tri open in X . Consequently by theorem (6.4) f is fuzzy tri continuous function.

7. Fuzzy Tri-gb Open Sets in Tri Topological Space

Definition 7.1: A fuzzy subset χ_λ of a fuzzy tri topological space $(X,\sigma_1,\sigma_2,\sigma_3)$ is said to be fuzzy tri-gb-closed if $Ftri-bcl(\chi_\lambda) \prec \chi_\delta$ whenever $\chi_\lambda \prec \chi_\delta$ and χ_δ is fuzzy tri open set.

Remarks 7.2:

(i) The complement of fuzzy tri-gb closed is fuzzy tri-gb open.

(ii) Fuzzy tri-gb-closure of χ_δ is denoted by $Ftri-gb-cl(\chi_\delta)$ and fuzzy tri-gb-interior of χ_δ denoted by $Ftri-gb\,\mathrm{int}(\chi_\delta)$ is c

Theorem 7.3: Fuzzy tri closed subset χ_λ of a fuzzy tri topological space $(X,\sigma_1,\sigma_2,\sigma_3)$ is fuzzy tri-b closed.

Proof: Suppose $\chi_\lambda \prec \chi_\delta$ is a fuzzy tri closed set, since $\mathrm{Ftri\,int}\,\chi_\lambda \prec Ftricl(\mathrm{int}\,\chi_\lambda)$, hence $\mathrm{Ftri\,int}(\mathrm{int}\,\chi_\lambda) \prec Ftricl(\mathrm{int}(\mathrm{int}\,\chi_\lambda))$ hence $Ftri\,\mathrm{int}\,\chi_\lambda \prec \chi_\lambda$ for any subset χ_λ ,hence $\mathrm{Ftri\,int}(\mathrm{int}\,\chi_\lambda) \prec Ftricl(\mathrm{int}(\mathrm{int}\,\chi_\lambda))$ and $Ftri\,\mathrm{int}\,\chi_\lambda \prec Ftri\,\mathrm{int}(\mathrm{F}tricl(Ftri\,\mathrm{int}\,\chi_\lambda)) \vee \mathrm{F}tricl(\mathrm{F}tri\,\mathrm{int}(\mathrm{Ftriint}\,\chi_\lambda))$ hence $Ftri\,\mathrm{int}\,\chi_\lambda$ is fuzzy tri-b open set,hence χ_λ is fuzzy tri-b open set.

Theorem 7.4:fuzzy tri-b closed subset χ_λ of a fuzzy tri topological space $(X,\sigma_1,\sigma_2,\sigma_3)$ is a fuzzy tri-gbclosed.

8. Fuzzy Tri-b Separation AxiomsinFuzzy Tri Topological Space

Definition 8.1: A fuzzy tri topological space $(X,\sigma_1,\sigma_2,\sigma_3)$ is called $Ftri-b-T_0$ space iff to given pair of different points $\chi_{\lambda_1},\chi_{\lambda_2}$ in χ_λ , $\exists$ a fuzzy tri-b open set consisting one point but not the other

Example 8.2: Let $X=\{1,2,3\}$ be a non-empty fuzzy set**,**consider three fuzzy topologies on X $\tau_1=\{\tilde{1}_X,\tilde{0}_X\}$, $\tau_2=\{\tilde{1}_X,\tilde{0}_X,\chi_{\{1\}}\}$, $\tau_3=\{\tilde{1}_X,\tilde{0}_X,\chi_{\{2,3\}}\}$

Fuzzy tri open sets of fuzzy tri topological spaces are union of all fuzzy tri topologies.

Then fuzzy tri open sets of $X=\{\tilde{1}_X,\tilde{0}_X,\chi_{\{1\}},\chi_{\{2,3\}}\}$

Fuzzy tri-b open set of X is $Ftri-BO(X)=\{\tilde{1}_X,\tilde{0}_X,\chi_{\{1,2\}},\chi_{\{3\}},\chi_{\{1,3\}},\chi_{\{2,3\}}\}$.

So (X,τ_1,τ_2,τ_3) is $Ftri-b-T_0$ space.

Theorem 8.3: If $\chi_{\{x_1\}}$ is a fuzzy tri-b-open for some $\chi_{\{x_1\}} \le \chi_\lambda$ then $\chi_{\{x_1\}} \le Ftri-b-cl(\chi_{\{y_1\}})$, for all $\chi_{\{y_1\}} \ne \chi_{\{x_1\}}$.

Proof: Let $\chi_{\{x_1\}}$ be a fuzzy tri-b-open for some $\chi_{\{x_1\}} \le \chi_\lambda$,then $\chi_\lambda - \chi_{\{x_1\}}$ is fuzzy tri-. If $\chi_{\{x_1\}} \le Ftri-b(\chi_{\{y_1\}})$, for some $\chi_{\{y_1\}} \ne \chi_{\{x_1\}}$, then $\chi_{\{y_1\}},\chi_{\{x_1\}}$ are in fuzzy tri-gb-closed sets containing $\chi_{\{y_1\}}$, so $\chi_{\{x_1\}} \le \chi_\lambda - \chi_{\{x_1\}}$ which is denial, hence $\chi_{\{x_1\}} \le Ftri-b(\chi_{\{y_1\}})$

Theorem 8.4: In fuzzy tri topological space $(X,\sigma_1,\sigma_2,\sigma_3)$, any fuzzy distinct points have distinct fuzzy tri-b-closure.

Proof: Let $\chi_{\{y_1\}},\chi_{\{x_1\}} \le \tilde{1}_X$ with $\chi_{\{y_1\}} \ne \chi_{\{x_1\}}$ and let $\chi_\lambda = \chi_{\{x_1\}^c}$ hence $Ftri-b-cl(\chi_\lambda)=\chi_\lambda$ or χ_δ .Now if then $Ftri-b-cl(\chi_\lambda)=\chi_\lambda$,then χ_λ is fuzzy tri-b-closed so $\tilde{1}_X - \chi_\delta = \chi_{\{x_1\}}$ is fuzzy tri-b open & not containing $\chi_{\{y_1\}}$. So by theorem(8.3) $\chi_{\{x_1\}} \succ Ftri-b-cl(\chi_{\{y_1\}})$ & $\chi_{\{y_1\}} \le Ftri-b-cl(\chi_{\{y_1\}})$ which implies that $Ftri-b-cl(\chi_{\{y_1\}})$ and $Ftri-b-cl(\chi_{\{x_1\}})$ are distinct. If $Ftri-b-cl(\chi_\lambda)=\tilde{1}_X$ then χ_δ is fuzzy tri-b open,hence $\chi_{\{x_1\}}$ is fuzzy tri-b closed,which mean that $Ftri-b-cl(\chi_{\{x_1\}})=\chi_{\{x_1\}}$ which is not equal to $Ftri-b-cl(\chi_{\{y_1\}})$.

Theorem 8.5: In any fuzzy tri topological space $(X,\sigma_1,\sigma_2,\sigma_3)$, if differentfuzzy points have different fuzzy tri-b closure then χ_λ is $Ftri-b-T_0$ space.

Proof: Let $\chi_{\{x_1\}}$, $\chi_{\{y_1\}} \leq \chi_\lambda$ with $\chi_{\{x_1\}} = \chi_{\{y_1\}}$,with $Ftri-b-cl(\chi_{\{y_1\}})$ is not equal to $Ftri-b-cl(\chi_{\{x_1\}})$,hence there exists $\chi_{\{z\}} \leq \chi_\lambda$ such that $\chi_{\{z\}} \leq Ftri-b-cl(\chi_{\{x_1\}})$ but $\chi_{\{z\}} \succ Ftri-b-cl(\chi_{\{y_1\}})$ or $\chi_{\{z\}} \leq Ftri-b-cl(\chi_{\{x_1\}})$ but now without loss of generality,let $\chi_{\{z\}} \leq Ftri-b-cl(\chi_{\{x_1\}})$

But $\chi_{\{z\}} \succ Ftri-b-cl(\chi_{\{x_1\}})$, If $\chi_{\{x\}} \leq Ftri-b-cl(\chi_{\{y_1\}})$, then $Ftri-b-cl(\chi_{\{x_1\}})$ is contained in $Ftri-b-cl(\chi_{\{y_1\}})$, hence $\chi_{\{z\}} \leq Ftri-b-cl(\chi_{\{y_1\}})$, which is a opposition,this mean that $\chi_{\{x\}} \succ Ftri-b-cl(\chi_{\{y_1\}})$ hence $\chi_{\{z\}} \succ Ftri-b-cl(\chi_{\{y_1\}})$ $\chi_{\{x_1\}} \leq Ftri-c1-b(\chi_{\{y_1\}^c})$,hence X is fuzzy tri-b -T_0 space.

Definition 8.6: A fuzzy tri topological space X is calledfuzzy tri-b-T_1space iff for any given pair of differentfuzzy points $\chi_{\{x_1\}}$, $\chi_{\{y_1\}}$ of X $\exists$ two fuzzy tri-b-open sets χ_{λ_1}, χ_{δ_1} such that $\chi_{\{x_1\}} \leq \chi_{\lambda_1}$, $\chi_{\{y_1\}} \succ \chi_{\lambda_1}$ and $\chi_{\{y_1\}} \leq \chi_{\delta_1}$, $\chi_{\{x_1\}} \succ \chi_{\delta_1}$.

Theorem 8.7: Everyfuzzy tri-T_1 space is a fuzzy tri-b-T_0 space.

Definition 8.8: A fuzzy tri topological space X is calledfuzzy tri-b-T_2 space iff for $\chi_{\{x_1\}}$, $\chi_{\{y_1\}} \leq \tilde{1}_X$, $\chi_{\{x_1\}} \neq \chi_{\{y_1\}}$ $\exists$ two disjoint fuzzy tri-b open sets χ_λ, χ_δ in X such that $\chi_{\{x_1\}} \leq \chi_\lambda$, $\chi_{\{y_1\}} \leq \chi_\delta$.

Theorem 8.9: Every fuzzy tri-b-T_2 space is fuzzy tri-b-T_1 space.

Proof: Let X is a fuzzy tri-b T_2 space and let $\chi_{\{x_1\}}$, $\chi_{\{y_1\}}$ in X with $\chi_{\{x_1\}} \neq \chi_{\{y_1\}}$,so by theory $\exists$ two disjoint fuzzy tri-b open,say χ_λ, χ_δ such that $\chi_{\{x_1\}} \leq \chi_\lambda$, $\chi_{\{y_1\}} \leq \chi_\delta$ but $\chi_\lambda \wedge \chi_\delta = \tilde{0}_X$ hence $\chi_{\{x_1\}} \succ \chi_\delta$, $\chi_{\{y_1\}} \succ \chi_\lambda$ i.e. X isa fuzzy tri-b-T_1 space.

Theorem 8.10: Every fuzzy tri-b-T_3 space is afuzzy tri-b-T_2 space.

Proof: Let $(X,\sigma_1,\sigma_2,\sigma_3)$ be a fuzzy tri-b T_3 space and let $\chi_{\{x_1\}}$, $\chi_{\{y_1\}}$ are two differentfuzzy points of X. According to definition , X is afuzzy tri-b-T_1 space and so $\chi_{\{x_1\}}$ is a fuzzy tri- closed set .Also $\chi_{\{y_1\}} \succ \chi_{\{x_1\}}$ .Since (X,τ_1,τ_2,τ_3) is a fuzzy tri-b regular space, $\exists$ fuzzy tri-b open sets χ_λ and χ_δ such that $\chi_{\{x_1\}} \prec \chi_\lambda$, $\chi_{\{y_1\}} \prec \chi_\delta$ and $\chi_\lambda \wedge \chi_\delta = \tilde{0}_X$ .Also $\chi_{\{x_1\}} \prec \chi_\lambda \Rightarrow \chi_{\{x_1\}} \leq \chi_\lambda$ Thus $\chi_{\{x_1\}}$, $\chi_{\{y_1\}}$ belongs to disjointfuzzy tri-b open sets χ_λ and χ_δ. Hence $(X,\sigma_1,\sigma_2,\sigma_3)$ is a $Ftri-b-T_2$ space.

Conclusion

In thispaper , fuzzy tri continuity, fuzzy tri separation, fuzzy tri –b separationand fuzzy tri-b continuity in fuzzy tri topological spaces wereintroduced and studied .

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