



## An Important Conclusion for Fermat’s Last Theorem

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ARTICLE INFO	ABSTRACT
Published Online: 02 August 2022	Fermat’s Last Theorem is perhaps the only most famous mathematical problem of all times. Although finally proved, but the Theorem never stopped being a challenge mainly because the first proof didn’t used mathematics known in Fermat’s era.
Corresponding Author: <b>Alkis Mazaris</b>	In the present work we arrive at a very important conclusion for the Theorem. If this conclusion is taken into account, the formulation of the Theorem should be different.
<b>KEYWORDS:</b> Fermat, Fermat’s Last Theorem, diophantine’s equations	

### INTRODUCTION

Studying the Theorem using only mathematics that was known at the time of Fermat, we reached a very important conclusion:

The wording of the theorem should be different. The work employs the wellknown technique of the proof by contradiction and is structured in 2 parts, leading to the final result.

We accept that  $a^v + b^v = c^v$  holds and in first part we arrive in a conclusion using the idea of the definition of the zero-sequence of numbers. In the second part using the result of part 1 in  $\left(\frac{a}{c}\right)^v + \left(\frac{b}{c}\right)^v = 1$  formula that is equivalent to  $a^v + b^v = c^v$  if  $c$  is not zero, we arrive at a contradiction.

### A NOVEL APPROACH USING CLASSIC METHOD

Fermat’s last theorem states that:

If  $a, b, c$  is non-zero natural number, there is no natural number  $v > 2$  such that  $a^v + b^v = c^v$ .

We will assume that there are  $a, b, c$ , non-zero natural numbers such that:

$$a^v + b^v = c^v \tag{1}$$

Also let’s assume that

$$a \geq b \tag{2}$$

It is very easy to prove that:  $c > a \geq$  (Appendix A)

Dividing all members of the relation (1) by  $c^v$  we will have:

$$\left(\frac{a}{c}\right)^v + \left(\frac{b}{c}\right)^v = 1 \tag{3}$$

### Part 1

Let us now consider the sequence of numbers  $d_n = \left(\frac{a}{c}\right)^n$  like this:

$a, c$  constant non-zero natural numbers  $c > a$  and  $\eta$  is any non-zero natural number. It is very easy to prove that; this sequence is a zero-sequence of numbers (Appendix B). By the definition of the zero-sequence (Appendix C) it holds that: for every  $\varepsilon > 0$  there is  $n_0 \in \mathbb{N}^*$ :

$$\text{For every } n > n_0 \text{ holds: } \left(\frac{a}{c}\right)^n < \varepsilon$$

So, for  $\varepsilon = \frac{1}{2}$  there is  $n_0 \in \mathbb{N}^*$  (we will calculate  $n_0$ ):

For every  $n > n_0$  holds:

$$\left(\frac{a}{c}\right)^v < \frac{1}{2} \tag{4}$$

Calculation of  $n_0$

$$\left(\frac{a}{c}\right)^v < \frac{1}{2}$$

$$\ln \left(\frac{a}{c}\right)^v < \ln \frac{1}{2}$$

$$v \ln \frac{a}{c} < \ln \frac{1}{2}$$

$$n > \frac{\ln \frac{1}{2}}{\ln \frac{a}{c}} \quad \left(\frac{a}{c} < 1\right)$$

$$\text{Finally: } n > \frac{\ln 2}{\ln \frac{a}{c}} \tag{5}$$

$$\text{So: } n_0 = \left\lceil \frac{\ln 2}{\ln \frac{a}{c}} \right\rceil \text{ (integral or integer part of } \frac{\ln 2}{\ln \frac{a}{c}})$$

We reached to the following:

CONCLUSION A: When  $n > n_0 = \left\lceil \frac{\ln 2}{\ln \frac{a}{c}} \right\rceil$  then

$$\left(\frac{a}{c}\right)^n < \frac{1}{2}, \quad c > a$$

### Part 2

Now let’s get back to the relation (3)

$$\left(\frac{a}{c}\right)^v + \left(\frac{b}{c}\right)^v = 1$$

We study the quantity  $\left(\frac{a}{c}\right)^v, c > a, c, a, v \in N^*$

According to CONCLUSION A:

When  $v > v_0 = \left\lfloor \frac{\ln 2}{\ln \frac{c}{a}} \right\rfloor$  then  $\left(\frac{a}{c}\right)^v < \frac{1}{2}, c > a$

In this case, because  $\left(\frac{a}{c}\right)^v > \left(\frac{b}{c}\right)^v$  (it holds  $a \geq b$ )

We will have:  $\left(\frac{a}{c}\right)^v + \left(\frac{b}{c}\right)^v < \frac{1}{2} + \frac{1}{2} = 1$

It means finally:  $\left(\frac{a}{c}\right)^v + \left(\frac{b}{c}\right)^v < 1$  it is contradiction

Because we have accepted that

$$\left(\frac{a}{c}\right)^v + \left(\frac{b}{c}\right)^v = 1$$

Therefore the relation  $\left(\frac{a}{c}\right)^v + \left(\frac{b}{c}\right)^v = 1$  to be valid, it must

$$\text{be } v \leq n_0 = \left\lfloor \frac{\ln 2}{\ln \frac{c}{a}} \right\rfloor$$

(integral or integer part of a number).

But the relations:

$$\left(\frac{a}{c}\right)^v + \left(\frac{b}{c}\right)^v = 1 \quad \text{and} \quad a^v + b^v = c^v \quad \text{are equivalent}$$

because  $c \neq 0$ , so we reached to CONCLUSION B.

CONCLUSION B:

The relation  $a^v + b^v = c^v$  it could be right only if

$$v \leq n_0 = \left\lfloor \frac{\ln 2}{\ln \frac{c}{a}} \right\rfloor \text{ (integral or integer part)}$$

## RESULTS

In Part 1 we reached the CONCLUSION A

In Part 2 we used CONCLUSION A in the relation

$$\left(\frac{a}{c}\right)^v + \left(\frac{b}{c}\right)^v = 1 \quad \text{to reach}$$

CONCLUSION B

## DISCUSSION and CONCLUSION

1. According to CONCLUSION B the formulation of FLT should have been: if  $a, b, c$  are non-zero natural numbers, there is no natural

number  $v > 2, v \leq \left\lfloor \frac{\ln 2}{\ln \frac{c}{a}} \right\rfloor$  such that:  $a^v + b^v = c^v$

It is a very important conclusion

Suppose looking for number  $v$  such that:

$$4^v + 5^v = 6^v, \quad v > 2 \quad \text{We can}$$

looking for only  $v$  such that:

$$v \leq \left\lfloor \frac{\ln 2}{\ln \frac{6}{5}} \right\rfloor = \left\lfloor \frac{\ln 2}{\ln 1,2} \right\rfloor = [3,8017 \dots] = 3$$

and not for any natural number  $v$ .

2. Pythagorean theorem:

$$\text{Ex. } 3^2 + 4^2 = 5^2$$

$$\text{It must be } 2 \leq \left\lfloor \frac{\ln 2}{\ln \frac{6}{5}} \right\rfloor = \left\lfloor \frac{\ln 2}{\ln 1,25} \right\rfloor = [3,1062 \dots] = 3$$

whichever applies

3. Simple addition of natural numbers:

$$\text{Ex. } 4^1 + 2^1 = 6^1$$

$$\text{It must be } 1 \leq \left\lfloor \frac{\ln 2}{\ln \frac{6}{5}} \right\rfloor = \left\lfloor \frac{\ln 2}{\ln 1,5} \right\rfloor = [1,7095 \dots] = 1$$

whichever applies

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## Conflict of interest

The author declares that there is no conflict of interest

## APPENDIX A

If it holds  $a^v + b^v = c^v, a, b, c, v \in N^*$  and  $a \geq b$  then  $c > a \geq b$  Proof:

The minimum price for  $b = 1$  So:  $a^v + 1^v = c^v$

$$a^v + 1 = c^v$$

It means that  $c^v > a^v$

$$\text{So: } c > a$$

$$\text{And } c > a \geq b$$

## APPENDIX B

Because of Appendix A,  $c > a$ . Consider  $\varepsilon$  to be an arbitrary small

number such that  $\left(\frac{a}{c}\right)^v < \varepsilon$ .

$$\text{Then } \ln \left(\frac{a}{c}\right)^v < \ln \varepsilon \text{ and finally } v > \frac{\ln \varepsilon}{\ln \frac{a}{c}}, (c > a)$$

If  $k_1$  is the lower-value natural number that satisfy:

$k_1 > \frac{\ln \varepsilon}{\ln \frac{a}{c}}$  then for every natural number  $v > k_1$  it holds

$$\left(\frac{a}{c}\right)^v < \varepsilon$$

## APPENDIX C

Definition: Let’s assume a sequence of number like this:

$$d_n = \left(\frac{a}{c}\right)^n, c > a, c, a \in N^*$$

$n \in N^*$  any non-zero natural number the  $d_n$  sequence is a zero-sequence of numbers when for every  $\varepsilon > 0$  there is natural number  $n_0$  non zero:

$|d_n| < \varepsilon$  for every natural non-zero  $n$ : ( $|d_n|$ : absolute value of modulus)

$$n > n_0$$

when the number of sequence are positive number  $|d_n| = d_n$ .

This happened in case we study.

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