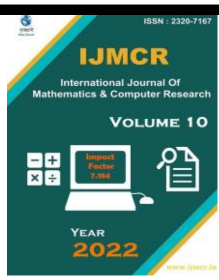
International Journal of Mathematics and Computer Research

ISSN: 2320-7167

Volume 10 Issue 12 December 2022, Page no. – 2986-2994

Index Copernicus ICV: 57.55, Impact Factor: 7.362

DOI: 10.47191/ijmcr/v10i12.01




# E-Banhatti Sombor Indices

**V.R.Kulli**

Department of Mathematics,Gulbarga University, Gulbarga 585106, India

**ARTICLE INFO**

Published Online: 03 December 2022

Corresponding Author: **V.R.Kulli**

**ABSTRACT**

In this paper, we introduce the E-Banhatti Sombor index and the modified E-Banhatti Sombor index and their corresponding exponentials of a graph. Also we compute these newly defined E-Banhatti Sombor indices and their corresponding exponentials for wheel graphs, friendship graphs, tetrameric 1,3-adamantane and honeycomb networks. Furthermore, we establish some properties of the E-Banhatti Sombor index.

**KEYWORDS:** E-Banhatti Somber index, modified E-Banhatti Sombor index, graph, tetrameric 1,3-adamantane, honeycomb network.

## I. INTRODUCTION

Let $G$ be a simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex $u$ is the number of vertices adjacent to $u$. The edge $e$ connecting the vertices $u$ and $v$ is denoted by $uv$. If $e=uv$ is an edge of $G$, then the vertex $u$ and edge $e$ are incident as are $v$ and $e$. Let $d_G(e)$ denote the degree of an edge $e$ in $G$, which is defined by $d_G(e) = d_G(u) + d_G(v) - 2$ with $e=uv$. We refer [1] for undefined term and notation.

A molecular graph is a simple graph, representing the carbon atom skeleton of an organic molecule of the hydrocarbon. Therefore the vertices of a molecular graph represent the carbon atoms and its edges the carbon-carbon bonds. Chemical Graph Theory is a branch of Mathematical Chemistry which has an important effect on the development of Chemical Sciences. Several graph indices [2] have found some applications in Chemistry, especially in QSPR/QSAR research [3, 4].

In [5], Kulli defined the Banhatti degree of a vertex $u$ of a graph $G$ as

$$B(u) = \frac{d_G(e)}{n - d_G(u)},$$

where $n$ is the number of vertices of $G$ and the vertex $u$ and edge $e$ are incident in $G$.

The first and second E-Banhatti indices were introduced by Kulli in [5] and they are defined as

$$EB_1(G) = \sum_{uv \in E(G)} [B(u) + B(v)],$$

$$EB_2(G) = \sum_{uv \in E(G)} B(u)B(v).$$

In [6], the FE-Banhatti index of a graph G is defined as

$$FEB(G) = \sum_{uv \in E(G)} \left[B(u)^2 + B(v)^2\right].$$

Recently, some E-Banhatti indices were studied, for example, in [7, 8].

We propose the E-Banhatti Sombor index of a graph G and defined it as

$$EBS(G) = \sum_{uv \in E(G)} \sqrt{B(u)^2 + B(v)^2}.$$

Considering the E-Banhatti Sombor index, we define the E-Banhatti Sombor exponential of a graph $G$ as

$$EBS(G, x) = \sum_{uv \in E(G)} x^{\sqrt{B(u)^2 + B(v)^2}}.$$

We propose the modified E-Banhatti Sombor index of a graph G and defined it as

$${}^{m}EBS(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{B(u)^2 + B(v)^2}}.$$

Considering the modified E-Banhatti Sombor index, we define the modified E-Banhatti Sombor exponential of a graph $G$ as

$${}^{m}EBS(G, x) = \sum_{uv \in E(G)} x^{\frac{1}{\sqrt{B(u)^2 + B(v)^2}}}.$$

Several types of topological or graph indices such as distance based, degree based indices have been introduced and studied in Chemical Graph Theory. Among graph indices Wiener, Zagreb, Gourava, Revan, Reverse, Banhatti indices are studied well. In [5], Kulli defined Banhatti degree of a vertex in a graph and introduced the first and second E-Banhatti indices of a graph. A study of E-Banhatti indices in Chemical Graph Theory is a new direction in Graph Index Theory.

In Chemical Graph Theory, several graph indices were introduced and studied such as the Wiener index [ 9, 10, 11, 12, 13], the Zagreb indices [14, 15, 16, 17, 18], the Revan indices [19, 20, 21, 22], the reverse indices [23, 24, 25], the Gourava indices [26, 27, 28, 29] and the Banhatti indices [30, 31, 32, 33, 34, 35].

In this paper, we compute the E-Banhatti Sombor and modified E-Banhatti Sombor indices for some standard graphs, wheel graphs, friendship graphs, tetrameric 1,3-adamantane and honeycomb networka.

## II. RESULTS FOR SOME STANDARD GRAPHS

**Proposition 1.** If $G$ is an $r$-regular graph with $n$ vertices and $r \geq 2$, then

$$EBS(G) = \frac{\sqrt{2}nr(r-1)}{(n-r)}.$$

**Proof:** Let $G$ be an $r$-regular graph with $n$ vertices and $r \geq 2$. Then $G$ has $\frac{nr}{2}$ edges. For any edge $uv=e$ in $G$, $d_G(e)= d_G(u)+ d_G(u) -2=2r-2$.

From definition we have

$$EBS(G) = \sum_{uv\in E(G)} \sqrt{B(u)^2 + B(v)^2}$$

$$= \frac{nr}{2}\sqrt{\left(\frac{2r-2}{n-r}\right)^2 + \left(\frac{2r-2}{n-r}\right)^2} = \frac{\sqrt{2}nr(r-1)}{(n-r)}.$$

**Corollary 1.1.** Let $C_n$ be a cycle with $n \geq 3$ vertices. Then

$$EBS(C_n) = \frac{2\sqrt{2}n}{(n-2)}.$$

**Corollary 1.2.** Let $K_n$ be a complete graph with $n \geq 3$ vertices. Then

$$EBS(K_n) = \sqrt{2}n(n-1)(n-2).$$

**Proposition 2.** Let $P_n$ be a path with $n \geq 3$ vertices. Then

$$EBS(P_n) = 2\sqrt{\left(\frac{1}{n-1}\right)^2 + \left(\frac{2}{n-2}\right)^2}$$

$$+(n-3)\sqrt{\left(\frac{2}{n-2}\right)^2 + \left(\frac{2}{n-2}\right)^2}$$

$$= \frac{2\sqrt{5n^2-12n+8}}{(n-1)(n-2)} + \frac{2\sqrt{2}(n-3)}{(n-2)}.$$

**Proposition 3.** Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$ *and* $n \geq 2$. Then

$$EBS(K_{m,n}) = \sqrt{(m^2+n^2)(m+n-2)^2}.$$

**Proof:** Let $K_{m,n}$ be a complete bipartite $m$ $n$ graph with $m + n$ vertices and $mn$ *edges* such that $|V_1|= m$ , $|V_2|= n$, $V(K_{r,s}) = V_1 \cup V_2$ for $1 \leq m \leq n$, and $n \geq 2$. Every vertex of $V_1$ is incident with $n$ edges and every vertex of $V_2$ is incident with $m$ edges. Then $d_G(e)= d_G(u)+ d_G(v) -2= m + n -2$.

$$EBS(K_{m,n}) = \sum_{uv\in E(G)} \sqrt{B(u)^2 + B(v)^2}$$

$$= mn\sqrt{\left(\frac{m+n-2}{m+n-n}\right)^2 + \left(\frac{m+n-2}{m+n-m}\right)^2}$$

$$= \sqrt{(m^2+n^2)(m+n-2)^2}.$$

**Corollary 3.1.** Let $K_{n,n}$ be a complete bipartite graph with $n \geq 2$. Then

$$EBS(K_{n,n}) = 2\sqrt{2}n(n-1).$$

**Corollary 3.2.** Let $K_{1,n}$ be a star with $n \geq 2$. Then

$$EBS(K_{1,n}) = (n-1)\sqrt{(1+n^2)}.$$

**Proposition 4.** If $G$ is an $r$-regular graph with $n$ vertices and $r \geq 2$, then

$$^{m}EBS(G) = \frac{nr(n-r)}{4\sqrt{2}(r-1)}.$$

**Proof:** Let $G$ be an $r$-regular graph with $n$ vertices and $r \geq 2$. Then $G$ has $\frac{nr}{2}$ edges. For any edge $uv=e$ in $G$, $d_G(e)= d_G(u)+ d_G(u) -2=2r-2$.

From definition we have

$$^{m}EBS(G) = \sum_{uv\in E(G)} \left[B(u)^2 + B(v)^2\right]^{-\frac{1}{2}}$$

$$= \frac{nr}{2}\left[\left(\frac{2r-2}{n-r}\right)^2 + \left(\frac{2r-2}{n-r}\right)^2\right]^{-\frac{1}{2}} = \frac{nr(n-r)}{4\sqrt{2}(r-1)}.$$

**Corollary 4.1.** Let $C_n$ be a cycle with $n \geq 3$ vertices. Then

$$^{m}EBS(C_n) = \frac{n(n-2)}{2\sqrt{2}}.$$

**Corollary 4.2.** Let $K_n$ be a complete graph with $n \geq 3$ vertices. Then

$$^{m}EBS(K_n) = \frac{n(n-1)}{4\sqrt{2}(n-2)}.$$

**Proposition 5.** Let $P_n$ be a path with $n \geq 3$ vertices. Then

$$^{m}EBS(P_n) = 2\left[\left(\frac{1}{n-1}\right)^2 + \left(\frac{2}{n-2}\right)^2\right]^{-\frac{1}{2}}$$

$$+(n-3)\left[\left(\frac{2}{n-2}\right)^2 + \left(\frac{2}{n-2}\right)^2\right]^{-\frac{1}{2}}$$

$$= \frac{2(n-1)(n-2)}{\sqrt{5n^2-12n+8}} + \frac{(n-3)(n-2)}{2\sqrt{2}}.$$

**Proposition 6.** Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$ *and* $n \geq 2$. Then

$$^{m}EBS(K_{m,n}) = \frac{m^2n^2}{\sqrt{(m^2+n^2)(m+n-2)^2}}.$$

**Proof:** Let $K_{m,n}$ be a complete bipartite $m$ $n$ graph with $m + n$ vertices and $mn$ *edges* such that $|V_1| = m$, $|V_2| = n$, $V(K_{r,s}) = V_1 \cup V_2$ for $1 \leq m \leq n$, and $n \geq 2$. Every vertex of $V_1$ is incident with $n$ edges and every vertex of $V_2$ is incident with $m$ edges. Then $d_G(e) = d_G(u) + d_G(v) - 2 = m + n - 2$.

$$^{m}EBS(K_{m,n}) = \sum_{uv \in E(G)} \left[B(u)^2 + B(v)^2\right]^{-\frac{1}{2}}$$

$$= mn\left[\left(\frac{m+n-2}{m+n-n}\right)^2 + \left(\frac{m+n-2}{m+n-m}\right)^2\right]^{-\frac{1}{2}}$$

$$= \frac{m^2n^2}{\sqrt{(m^2+n^2)(m+n-2)^2}}.$$

**Corollary 6.1.** Let $K_{n,n}$ be a complete bipartite graph with $n \geq 2$. Then

$$^{m}EBS(K_{n,n}) = \frac{n^3}{2\sqrt{2}(n-1)}.$$

**Corollary 6.2.** Let $K_{1,n}$ be a star with $n \geq 2$. Then

$$^{m}EBS(K_{1,n}) = \frac{n^2}{(n-1)\sqrt{(1+n^2)}}.$$

## III. RESULTS FOR WHEEL GRAPHS

A wheel graph $W_n$ is the join of $C_n$ and $K_1$. Then $W_n$ has $n+1$ vertices and $2n$ edges. A graph $W_n$ is shown in Figure 1.

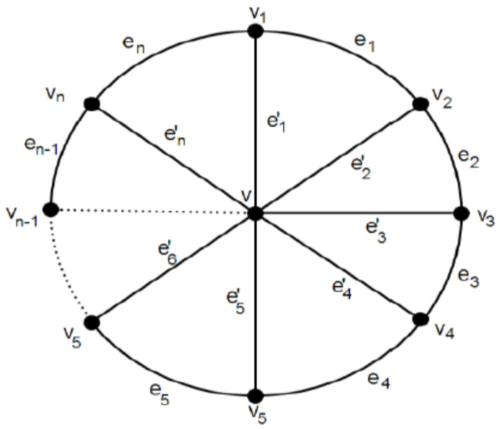



**Figure 1. Wheel graph $W_n$**

In $W_n$, there are two types of edges as follows:

$E_1 = \{uv \in E(W_n) \mid d(u) = d(v) = 3\}$, $|E_1| = n$.

$E_2 = \{uv \in E(W_n) \mid d(u) = 3, d(v) = n\}$, $|E_2| = n$.

Therefore, in $W_n$, there are two types of Banhatti edges based on Banhatti degrees of end vertices of each edge follow:

$BE_1 = \{uv \in E(W_n) \mid B(u) = B(v) = \frac{4}{(n-2)}\}$, $|BE_1| = n$.

$BE_2 = \{uv \in E(W_n) \mid B(u) = \frac{n+1}{n-2}, B(v) = n+1\}$, $|BE_2| = n$.

We compute the E-Banhatti Sombor index of the wheel graph $W_n$.

**Theorem 1.** Let $W_n$ be a wheel graph with $n + 1$ vertices and $2n$ edges. Then

$$EBS(W_n) = \frac{n}{n-2}\left(4\sqrt{2} + (n+1)\sqrt{n^2-4n+5}\right).$$

**Proof:** From definition and by cardinalities of the Banhatti edge partition of $W_n$, we obtain

$$EBS(W_n) = \sum_{uv \in E(W_n)} \sqrt{B(u)^2 + B(v)^2}$$

$$= n\sqrt{\left(\frac{4}{n-2}\right)^2 + \left(\frac{4}{n-2}\right)^2}$$

$$+n\sqrt{\left(\frac{n+1}{n-2}\right)^2+\left(n+1\right)^2}$$

$$=\frac{n}{n-2}\left(4\sqrt{2}+(n+1)\sqrt{(n^2-4n+5}\right).$$

We compute the modified E-Banhatti Sombor index of the wheel graph $W_n$.

**Theorem 2.** Let $W_n$ be a wheel graph with $n+1$ vertices and $2n$ edges. Then

$${}^{m}EBS\left(W_n\right)=n\left(n-2\right)\left[\frac{1}{4\sqrt{2}}+\frac{1}{(n+1)\sqrt{n^2-4n+5}}\right].$$

**Proof:** From definition and by cardinalities of the Banhatti edge partition of $W_n$, we have

$${}^{m}EBS\left(W_n\right)=\sum_{uv\in E(W_n)}\frac{1}{\sqrt{B(u)^2+B(v)^2}}$$

$$=n\left[\left(\frac{4}{n-2}\right)^2+\left(\frac{4}{n-2}\right)^2\right]^{-\frac{1}{2}}$$

$$+n\left[\left(\frac{n+1}{n-2}\right)^2+\left(n+1\right)^2\right]^{-\frac{1}{2}}$$

$$=n\left(n-2\right)\left[\frac{1}{4\sqrt{2}}+\frac{1}{(n+1)\sqrt{n^2-4n+5}}\right].$$

By using definitions and by cardinalities of the Banhatti edge partition of a wheel graph $W_n$, we obtain the E- Banhattil Sombor and modified SE-Banhatti Sombor exponentials of $W_n$.

**Theorem 3.** The E-Banhatti Sombor exponential of $W_n$ is given by

$$EBS\left(W_n,x\right)=nx^{\frac{4\sqrt{2}}{n-2}}+nx^{\frac{(n+1)\sqrt{n^2-4n+5}}{n-2}}.$$

**Theorem 4.** The modified E-Banhatti Sombor exponential of $W_n$ is given by

$${}^{m}EBS\left(W_n,x\right)=nx^{\frac{n-2}{4\sqrt{2}}}+nx^{\frac{n-2}{(n+1)\sqrt{n^2-4n+5}}}.$$

## IV. RESULTS FOR FRIENDSHIP GRAPHS

A friendship graph $F_n$, $n \geq 2$, is a graph that can be constructed by joining $n$ copies of $C_3$ with a common vertex. A graph $F_4$ is presented in Figure 2.

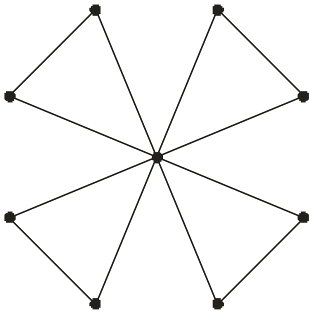


**Figure 2. Friendship graph $F_4$**

Let $F_n$ be a friendship graph with $2n+1$ vertices and $3n$ edges. By calculation, we obtain that there are two types of edges as follows:

$E_1=\left\{uv\in E\left(F_n\right)\mid d_{F_n}(u)=d_{F_n}(v)=2\right\}$, $|E_1| = n$.

$E_2=\left\{uv\in E\left(F_n\right)\mid d_{F_n}(u)=2, d_{F_n}(v)=2n\right\}$, $|E_2| = 2n$.

Therefore, in $F_n$, there are two types of Banhatti edges based on Banhatti degrees of end vertices of each edge follow:

$BE_1 = \{uv \in E(F_n) \mid B(u) = B(v) = \frac{2}{2n-1}\}$, $|BE_1| = n$.

$BE_2 = \{uv \in E(F_n) \mid B(u) = \frac{2n}{2n-1}, B(v) = 2n\}$, $|BE_2| = n$.

We compute the E-Banhatti Sombor index of the friendship graph $F_n$.

**Theorem 5.** Let $F_n$ be a friendship graph with $n+1$ vertices and $2n$ edges. Then

$$EBS\left(F_n\right)=\frac{n}{n-2}\left(4\sqrt{2}+(n+1)\sqrt{n^2-4n+5}\right).$$

**Proof:** From definition and by cardinalities of the Banhatti edge partition of $F_n$, we obtain

$$EBS\left(F_n\right)=\sum_{uv\in E(F_n)}\sqrt{B(u)^2+B(v)^2}$$

$$=n\sqrt{\left(\frac{4}{n-2}\right)^2+\left(\frac{4}{n-2}\right)^2}$$

$$+n\sqrt{\left(\frac{n+1}{n-2}\right)^2+\left(n+1\right)^2}$$

$$=\frac{n}{n-2}\left(4\sqrt{2}+(n+1)\sqrt{(n^2-4n+5}\right).$$

We compute the modified E-Banhatti Sombor index of the friendship graph $F_n$.

**Theorem 6.** Let $F_n$ be a friendship graph with $n+1$ vertices and $2n$ edges. Then

$$ {}^{m}EBS(F_n) = n(n-2)\left[\frac{1}{4\sqrt{2}} + \frac{1}{(n+1)\sqrt{n^2-4n+5}}\right]. $$

**Proof:** From definition and by cardinalities of the Banhatti edge partition of $F_n$, we have

$$ {}^{m}EBS(F_n) = \sum_{uv\in E(F_n)} \frac{1}{\sqrt{B(u)^2 + B(v)^2}} $$

$$ = n\left[\left(\frac{4}{n-2}\right)^2 + \left(\frac{4}{n-2}\right)^2\right]^{-\frac{1}{2}} $$

$$ + n\left[\left(\frac{n+1}{n-2}\right)^2 + (n+1)^2\right]^{-\frac{1}{2}} $$

$$ = n(n-2)\left[\frac{1}{4\sqrt{2}} + \frac{1}{(n+1)\sqrt{n^2-4n+5}}\right]. $$

By using definitions and by cardinalities of the Banhatti edge partition of a friendship graph $F_n$, we obtain the E- Banhatti Sombor and modified E-Banhatti Sombor exponentials of $F_n$.

**Theorem 7.** The E-Banhatti Sombor exponential of $F_n$ is given by

$$ EBS(F_n, x) = nx^{\frac{4\sqrt{2}}{n-2}} + nx^{\frac{(n+1)\sqrt{n^2-4n+5}}{n-2}}. $$

**Theorem 8.** The modified E-Banhatti Sombor exponential of $F_n$ is given by

$$ {}^{m}EBS(F_n, x) = nx^{\frac{n-2}{4\sqrt{2}}} + nx^{\frac{n-2}{(n+1)\sqrt{n^2-4n+5}}}. $$

## V.RESULTS FOR TETRAMERIC 1,3-ADAMANTANE

In Chemistry, diamondoids are variants of the carbon cage known as adamantane ($C_{10}$, $H_{16}$), the smallest unit cage structure of the diamond crystal lattice. We focus on the molecular graph structure of the family of tetrameric 1,3-adamantane, denoted by $TA[n]$. Let $G$ be the graph of tetrameric 1,3-adamantane $TA[n]$. The graph of tetrameric 1,3-adamantane $TA[4]$ is shown in Figure 3.

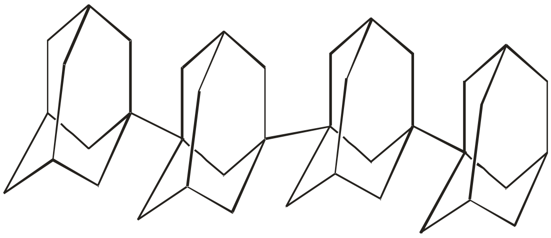


**Figure 3**

By calculation, $G$ has $10n$ vertices and $13n - 1$ edges. Also by calculation, we obtain three edge partitions of $G$ based on the degrees of the end vertices of each edge as follows:

$E_1=\{uv$□$E(G) \mid d_G(u)=2, d_G(v)=3\}$, $|E_1| = 6n + 6$.
$E_2=\{uv$□$E(G) \mid d_G(u)=2, d_G(v)=4\}$, $|E_2| = 6n - 6$.
$E_3=\{uv$□$E(G) \mid d_G(u)=d_G(v) = 4\}$, $|E_3| = n - 1$.

Therefore, in $TA[n]$, there are three types of edges based on the temperature of end vertices of each edge as follow:

$BE_1 = \{uv$ □ $E(G) \mid B(u) = \frac{3}{10n-2},\ B(v) = \frac{3}{10n-3}\}$, $|BE_1| = 6n+6$.

$BE_2 = \{uv$ □ $E(G) \mid B(u) = \frac{4}{10n-2}, B(v) = \frac{4}{10n-4}\}$, $|BE_2| = 6n-6$.

$BE_3 = \{uv$ □ $E(G) \mid B(u) = \frac{6}{10n-4}, B(v) = \frac{6}{10n-4}\}$, $|BE_3| = n-1$.

We compute the E-Banhatti Sombor index of $TA[n]$.

**Theorem 9.** Let $G$ be the graph of a tetrameric 1,3-adamantane $TA[n]$ with $10n$ vertices and $13n-1$ edges. Then

$$ EBS(TA[n]) = \frac{3(6n+6)\sqrt{(10n-2)^2+(10n-3)^2}}{(10n-2)(10n-3)} + \frac{2(6n-6)\sqrt{(5n-1)^2+(5n-2)^2}}{(5n-1)(5n-2)} + \frac{3\sqrt{2}(n-1)}{(5n-2)}. $$

**Proof:** From definition and by cardinalities of the Banhatti edge partition of $TA[n]$, we obtain

$$ EBS(TA[n]) = \sum_{uv\in E(TA[n])} \sqrt{B(u)^2 + B(v)^2} $$

$$ = (6n+6)\sqrt{\left(\frac{3}{10n-2}\right)^2 + \left(\frac{3}{10n-3}\right)^2} $$

$$ + (6n-6)\sqrt{\left(\frac{4}{10n-2}\right)^2 + \left(\frac{4}{10n-4}\right)^2} $$

$$ + (n-1)\sqrt{\left(\frac{6}{10n-4}\right)^2 + \left(\frac{6}{10n-4}\right)^2}. $$

After simplification, we get the desired result.

We compute the modified E-Banhatti Sombor index of $TA[n]$.

**Theorem 10.** Let $G$ be the graph of a tetrameric 1,3-adamantane $TA[n]$ with $10n$ vertices and $13n-1$ edges. Then

$$^{m}EBS(TA[n]) = \frac{(6n+6)(10n-2)(10n-3)}{3\sqrt{(10n-2)^2+(10n-3)^2}} + \frac{(6n-6)(5n-1)(5n-2)}{2\sqrt{(5n-1)^2+(5n-2)^2}} + \frac{(n-1)(5n-2)}{3\sqrt{2}}.$$

**Proof:** From definition and by cardinalities of the Banhatti edge partition of $TA[n]$, we have

$$^{m}EBS(TA[n]) = \sum_{uv\in E(TA[n])} \frac{1}{\sqrt{B(u)^2+B(v)^2}}$$

$$= (6n+6)\left[\left(\frac{3}{10n-2}\right)^2 + \left(\frac{3}{10n-3}\right)^2\right]^{-\frac{1}{2}}$$

$$+(6n-6)\left[\left(\frac{4}{10n-2}\right)^2 + \left(\frac{4}{10n-4}\right)^2\right]^{-\frac{1}{2}}$$

$$+(n-1)\left[\left(\frac{4}{10n-2}\right)^2 + \left(\frac{4}{10n-4}\right)^2\right]^{-\frac{1}{2}}$$

gives the desired result after simplification.

By using definitions and by cardinalities of the Banhatti edge partition of a friendship graph $TA[n]$, we obtain the E- Banhatti Sombor and modified E-Banhatti Sombor exponentials of $TA[n]$ .

**Theorem 11.** The E-Banhatti Sombor exponential of $TA[n]$ is given by

$$EBS(TA[n],x) = (6n+6)\,x^{\frac{3\sqrt{(10n-2)^2+(10n-3)^2}}{(10n-2)(10n-3)}}$$

$$+(6n-6)\,x^{\frac{2\sqrt{(5n-1)^2+(5n-2)^2}}{(5n-1)(5n-2)}} + (n-1)\,x^{\frac{3\sqrt{2}}{(5n-2)}}.$$

**Theorem 12.** The modified E-Banhatti Sombor exponential of $TA[n]$ is given by

$$^{m}EBS(TA[n],x) = (6n+6)\,x^{\frac{(10n-2)(10n-3)}{3\sqrt{(10n-2)^2+(10n-3)^2}}}$$

$$+(6n-6)\,x^{\frac{(5n-1)(5n-2)}{2\sqrt{(5n-1)^2+(5n-2)^2}}} + (n-1)\,x^{\frac{5n-2}{3\sqrt{2}}}.$$

## VI. RESULTS FOR HONEYCOMB NETWORKS

Honeycomb networks are useful in Computer Graphics and Chemistry. A honeycomb network of dimension $n$ is denoted by $HC_n$, where $n$ is the number of hexagons between central and boundary hexagon. A 4-dimensional honeycomb network is shown in Figure 4.

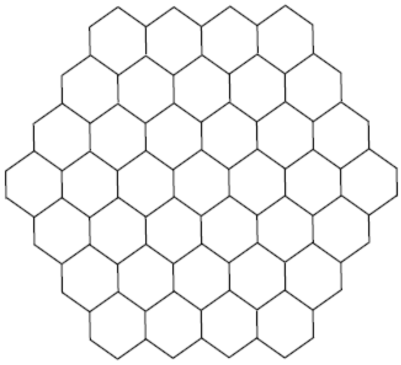


**Figure 4. A 4-dimensional honeycomb network**

Let $G$ be the graph of a honeycomb network $HC_n$. By calculation, we obtain that $G$ has $6n^2$ vertices and $9n^2–3n$ edges. In $G$, by algebraic method, there are three types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(HC_n) \mid d_G(u) = d_G(v) = 2\}$,
$|E_1| = 6$.
$E_2 = \{uv \in E(HC_n) \mid d_G(u) = 2, d_G(v) = 3\}$,
$|E_2| = 12n – 12$.
$E_3 = \{uv \in E(HC_n) \mid d_G(u) = d_G(v) = 3\}$,
$|E_3| = 9n^2 – 15n + 6$.

Therefore, in $HC_n$, there are three types of Banhatti edges based on Banhatti degrees of end vertices of each edge as follow:

$BE_1 = \{uv \in E(HC_n) \mid B(u) = B(v) = \frac{2}{6n^2-2}\}$,
$|BE_1| = 6$.

$BE_2 = \{uv \in E(HC_n) \mid B(u) = \frac{2}{6n^2-2},\ B(v) = \frac{3}{6n^2-3}\}$,
$|BE_2| = 12n – 12$.

$BE_3 = \{uv \in E(HC_n) \mid B(u) = B(v) = \frac{4}{6n^2-3}\}$,
$|BE_3| = 9n^2 – 15n + 6$.

We now compute the E-Banhatti Sombor index of a honeycomb network $HC_n$.

**Theorem 13.** Let $G$ be the graph of a honeycomb network $HC_n$ with $6n^2$ vertices and $9n^2 – 15n + 6$ edges. Then

$$EBS(HC_n)=\frac{12\sqrt{2}}{6n^2-2}+\frac{3(12n-12)\sqrt{(6n^2-2)^2+(6n^2-3)^2}}{(6n^2-2)(6n^2-3)}+\frac{4\sqrt{2}(9n^2-15n+6)}{6n^2-3}.$$

**Proof:** From definition and by cardinalities of the Banhatti edge partition of $HC_n$, we obtain

$$EBS(HC_n)=\sum_{uv\in E(HC_n)}\sqrt{B(u)^2+B(v)^2}$$

$$=6\sqrt{\left(\frac{2}{6n^2-2}\right)^2+\left(\frac{2}{6n^2-2}\right)^2}$$

$$+(12n-12)\sqrt{\left(\frac{3}{6n^2-2}\right)^2+\left(\frac{3}{6n^2-3}\right)^2}$$

$$+(9n^2-15n+6)\sqrt{\left(\frac{4}{6n^2-3}\right)^2+\left(\frac{4}{6n^2-3}\right)^2}.$$

After simplification, we get the desired result.

We now compute the modified E-Banhatti Sombor index of a honeycomb network $HC_n$.

**Theorem 14.** Let $G$ be the graph of a honeycomb network $HC_n$ with $6n^2$ vertices and $9n^2-15n+6$ edges. Then

$$^{m}EBS(HC_n)=\frac{3(6n^2-2)}{\sqrt{2}}+\frac{(4n-4)(6n^2-2)(6n^2-3)}{\sqrt{(6n^2-2)^2+(6n^2-3)^2}}+\frac{(9n^2-15n+6)(6n^2-3)}{4\sqrt{2}}.$$

**Proof:** From definition and by cardinalities of the Banhatti edge partition of $HC_n$, we have

$$^{m}EBS(HC_n)=\sum_{uv\in E(HC_n)}\frac{1}{\sqrt{B(u)^2+B(v)^2}}$$

$$=6\left[\left(\frac{2}{6n^2-2}\right)^2+\left(\frac{2}{6n^2-2}\right)^2\right]^{-\frac{1}{2}}$$

$$+(12n-12)\left[\left(\frac{3}{6n^2-2}\right)^2+\left(\frac{3}{6n^2-3}\right)^2\right]^{-\frac{1}{2}}$$

$$+(9n^2-15n+6)\left[\left(\frac{4}{6n^2-3}\right)^2+\left(\frac{4}{6n^2-3}\right)^2\right]^{-\frac{1}{2}}$$

gives the desired result after simplification.

By using definitions and by cardinalities of the Banhatti edge partition of a friendship graph $HC_n$, we obtain the E- Banhatti Sombor and modified E-Banhatti Sombor exponentials of $HC_n$ .

**Theorem 15.** The E-Banhatti Sombor exponential of $HC_n$ is given by

$$EBS(HC_n,x)=6x^{\frac{2\sqrt{2}}{6n^2-2}}+(12n-12)x^{\frac{3\sqrt{(6n^2-2)^2+(6n^2-3)^2}}{(6n^2-2)(6n^2-3)}}+(9n^2-15n+6)x^{\frac{4\sqrt{2}}{6n^2-3}}.$$

**Theorem 16.** The modified E-Banhatti Sombor exponential of $HC_n$ is given by

$$^{m}EBS(HC_n,x)=6x^{\frac{6n^2-2}{2\sqrt{2}}}+(12n-12)x^{\frac{(6n^2-2)(6n^2-3)}{3\sqrt{(6n^2-2)^2+(6n^2-3)^2}}}+(9n^2-15n+6)x^{\frac{6n^2-3}{4\sqrt{2}}}.$$

## VII. PROPERTIES OF E-BANHATTI SOMBOR INDEX

**Theorem 17.** Let $G$ be a connected graph with $m$ edges. Then

$$\frac{1}{\sqrt{2}}EB_1(G)\le EBS(G)\le EB_1(G).$$

**Proof:** For any two positive numbers $a$ and $b$,

$$\frac{1}{\sqrt{2}}(a+b)\le\sqrt{(a^2+b^2)}\le a+b.$$

*For* $a=B(u)$ and $b=B(v)$ , the above inequalities transform into

$$\frac{1}{\sqrt{2}}\big(B(u)+B(v)\big)\le\sqrt{(B(u)^2+B(v)^2)}\le B(u)+B(v)$$

Now, we obtain

$$\frac{1}{\sqrt{2}}\sum_{uv\in E(G)}\left(B(u)+B(v)\right)\leq \sum_{uv\in E(G)}\sqrt{\left(B(u)^2+B(v)^2\right.}$$

$$\leq \sum_{uv\in E(G)}\left(B(u)+B(v)\right)$$

with the help of definitions, we arrive the desired result.

**Theorem 18.** Let $G$ be a connected graph with $m$ edges. Then

$$EBS(G)\leq \sqrt{mFEB(G)}.$$

**Proof:** Using the Cauchy-Schwarz inequality, we obtain

$$\left(\sum_{uv\in E(G)}\sqrt{B(u)^2+B(v)^2}\right)^2\leq \sum_{uv\in E(G)}1\sum_{uv\in E(G)}\left(B(u)^2+(v)^2\right).$$

$$=mFEB(G).$$

Thus $$EBS(G)\leq \sqrt{mFEB(G)}.$$

## VIII. CONCLUSION

In this paper, we have introduced the E-Banhatti Sonbor index, modified E-Banhatti Sombor index and their corresponding exponentials of a graph. We have computed these newly defined indices and exponentials for some standard graphs, wheel graphs, friendship graphs and certain nanostructures. This study is a new direction in Graph Indices.

## REFERENCES

1. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
2. V.R.Kulli, Graph indices, in *Hand Book of Research on Advanced Applications of Application Graph Theory in Modern Society*, M. Pal. S. Samanta and A. Pal, (eds.) IGI Global, USA (2020) 66-91.
3. I.Gutman and O.E. Polansky, *Mathematical Concepts in Organic Chemistry*, Springer, Berlin (1986).
4. V.R.Kulli, *Multiplicative connectivity indices of nanostructures*, LAP LEMBERT Academic Publishing (2018).
5. V.R.Kulli, New direction in the theory of graph index in graphs, submitted.
6. V.R.Kulli, FE-Banhatti index, submitted.
7. V.R.Kulli, Hyper E-Banhatti indices of certain networks, submitted.
8. V.R.Kulli, Computation of E-Banhatti Nirmala indices of tetrameric 1,3-Adamantane, submitted.
9. H.Wiener, Structural determination of parattin boiling points, *Journal of the American Chemical Society*, 69(1) (1947) 17-20.
10. I.Gutman, Y.N.Yeh, S.L.Leeand Y.L.Luo, Some recent results in the theory of the Wiener number, *Indian J. Chem.* 32A (1993) 651-661.
11. P.W.Fowler, G.Caporossi and P.Hansen, Distance matrices, Wiener indices and related invariants of fullerenes, *The Journal of Physical Chemistry A*, 105(25) (2001) 6232-6242.
12. H.S.Ramane, K.P.Narayankar, S.S.Shirkol and A.B.Ganagi, Terminal wiener index of line graphs, *MATCH Commun. Math. Comput. Chem.*, 69 (2013) 775-782.
13. M.Zeryouh, M.Marraki and M.Essalih, Wiener and terminal Wiener indices of some rooted trees, *Applied Mathematical Sciences*, 8 (2014) 4995-5002
14. I. Gutman and N. Trinajstić, Graph theory and molecular orbitals. Total -electron energy of alternant hydrocarbons, *Chem. Phys. Lett.* vol. 17, pp 535-538, 1972.
15. K.C.Das, I.Gutman and B.Horoldagva, Comparing Zagreb indices and coindices of trees, *MATCH Commun. Math. Comput. Chem.*, 67 (2012) 189-198.
16. T.Doslic, B.Furtula, A.Graovac, I.Gutman, S.Moradi and Z.Yarahmadi, On vertex degree based molecular structure descriptors, *MATCH Commun. Math. Comput. Chem.*, 66 (2011) 613-626.
17. G.H.Fath-Tabar, Old and new Zagreb indices of graphs, *MATCH Commun. Math. Comput. Chem.*, 65 (2011) 79-84.
18. S.Nikolic, G.Kovacevic, A.Milicevic and N.Trinajstic, The Zagreb indices 30 years after, Croat.Chem. Acta, 76(2003) 113-124.
19. V.R. Kulli, Revan indices of oxide and honeycomb networks, *International Journal of Mathematics and its Applications*, 5(4-E) (2017) 663-667.
20. A.Q.Baig, M.Nadeem and W.Gao, Revan and hyper Revan indices of Octahedral and icosahdral networks, *Applied Mathematics and Nonlinear Sciences*, 3(1) (2018) 33-40.
21. P.Kandan, E.Chandrasekaran and M.Priyadharshini, The Revan weighted Szeged index of graphs, *Journal of Emerging Technologies and Innovative Research*, 5(9) (2018) 358-366.
22. R.Aguilar-Sanchez, I.F.Herrera-Gonzalez, J.A.Mendez-Bermudez and J.M.Sigarreta, Revan degree indices on random graphs, arXiv:2210.04749v1 [math.CO] 10 Oct 2022.

23. S.Ediz, Maximal graphs of the first reverse Zagreb beta index, *TWMS J. Appl.Eng. Math.* 8 (2018) 306-310.
24. V.R.Kulli, The reverse Zagreb and reverse hyper Zagreb indices and their polynomials of rhombus silicate networks, *Annals of Pure and Applied Mathematics*, 16(1) (2018) 47-51.
25. W.Gao, M.Younas, A.Farooq, A ur Rehman and W.Nazeer, Some Reverse degree based topological indices and polynomials of dendrimers, *Mathematics*, 6 (2018) 214.
26. V.R.Kulli, The Gourava indices and coindices of graphs, *Annals of Pure and Applied Mathematics*, 14(1) (2017).
27. M.Aruvi, M.Joseph and E.Ramganesh, The second Gourava index of some graph products, *Advances in Mathematics: Scientific Journal* 9(12) (2020) 10241-10249.
28. B.Basavanagoud and S.Policepatil, Chemical applicability of Gourava and hyper Gourava indices, *Nanosystems: Physics, Chemistry, Mathematics* 12(2) (2021) 142-150.
29. V.R.Kulli, V.Lokesha, S.Jain and M.Manjunath, The Gourava index of four operations on graphs, *International J. Math. Combin.* 4 (2018) 65-76.
30. V.R.Kulli, On *K* Banhatti indices of graphs, *Journal of Computer and Mathematical Sciences,* 7 (2016) 213-218.
31. V.R.Kulli, B.Chaluvaraju and H.S.Boregowda, Some bounds on connectivity Banhatti index of graphs, *Palestine Journal of Mathematics,* 3(1) (2015) 11-19.
32. Y.C.Kwun, J.Ahmad, W.Nazeer, W.Khalid and S.M.Kang, Topological invariants of molecular graph of grapheme, *JP Journal of Geometry and Topology*, 21(2) (2018) 95-103.
33. I.Gutman, V.R.Kulli, B.Chaluvaraju and H.S.Boregowda, On Banhatti and Zagreb indices, *Journal of the international Mathematical Virtual Institute*, 7 (2017) 53-67.
34. W.Wang, M.Naeem, A.Rauf, A.Riasat, A.Aslam and K.A.Yannick, On analysis of Banhatti indices for hyaluronic acid curcumin and hydroxychloroquine, *Journal of Chemistry,* 2021 Article ID 7468857 10 pages.
35. D.Zhao, M.A.Zahid, R.Irfan. M.Arshad, A.Fahad, Z.Ahmad and L.Li, Banhatti, Revan and hyper indices of silicon carbide Si2C3-III[*n*, *m*], *Open Chemistry*, 19 (2021) 646-652.


**V. Results for H-Naphtalenic Nanotubes**

In this section we consider a family of *H*-Naphtalenic nanotubes. This nanotube is a trivalent decoration having a sequence of $C_6$, $C_6$, $C_4$, $C_6$, $C_6$, $C_4$, … in the first row and a sequence of $C_6$, $C_8$, $C_6$, $C_8$, … in other row. This nanotube is denoted by *NHPX*[*m*, *n*], where *m* is the number of pair of hexagons in first row and *n*is the number of alternative hexagons in a column as shown in Figure 4.

| | | |
|---|---|---|
| $6n + 6$ | $6n - 6$ | $n - 1$ |