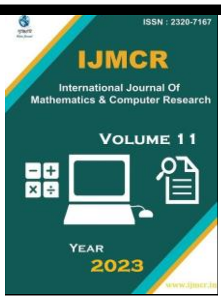
International Journal of Mathematics and Computer Research

ISSN: 2320-7167

Volume 11 Issue 04 April 2023, Page no. – 3355-3361

Index Copernicus ICV: 57.55, Impact Factor: 7.362

DOI: 10.47191/ijmcr/v11i4.07



Finite Difference Analysis of Boundary Layer Flow of Nanofluids Past A Vertical Plane with Heat Transfer

M M. WAFULA[1], J.P. KIPTUM[2]

[1]School of Science and Technology, United States International University- Africa, Kenya

[2]School of Mathematical Sciences, Pan African Univeristy of Basic Sciencs, Kenya

ARTICLE INFO	ABSTRACT
Published Online: 26 April 2023 Corresponding Author: **M M. Wafula**	An unsteady boundary layer of Nanofluids flowing past a vertical plane has numerically been investigated. The boundary layer governing equations were non-dimensionalised and non-dimensional equations obtained. The nondimensional parameters which include Grashof number, Magnetic parameter and Eckert number was obtained. The equations obtained were solved numerically by finite difference method. The velocity and temperature profiles are found to be affected by these dimensionless parameters. The present investigation provides useful information to researchers Since the results obtained is applied in various sectors which includes magnetic nanomaterial processing and automotive fields such as Nanofluid coolant.

KEYWORDS: Nanofluids; Finite difference; b0undary layer; Heat transfer.

1. INTRODUCTION

Cooling and heating of fluid has a lot of application in various sectors such as manufacturing industries, transportation and power plants. The usual normal heat transfer fluids such as water, ethylene glycol and engine oil posses slightly lower heat transfer properties compared to Metallic elements which demonstrate a higher thermal conductivity. Conductivity in fluids can be enhanced by blending it with metallic Nano particles, the resulting fluid is referred to as Nanofluids. Choi[7] developed a this kind of fluid and reported that compared to base fluids, they have higher thermal conductivities. The nanoparticle may be mettalic particles, mettallic oxides, carbon Nano tubes or Carbides which are suspended or blended within a base fluid . Experimental studies done by [11] and [6], among others reported that for effective heat transfer, the Nanofluids should contain minimum amount of nanoparticles of about 5 % . Most reseachers [11] have also reported that by use of Nanofluids, the heat transfer coefficient improved by 45 % and the thermal conductivity coefficient has ranges from 15 %-40 % over base fluid The nanofluid model proposed by [7] was used by several authors such as [15] and [14] among others Boundary layer theory is applied in various fields such as; Hydrodynamics, Aerodynamics, wind engineering , Ocean engineering and in the field of transportation. In his book [4], Blasius explained that in boundary layer, friction plays an essential part. Several researches were then made on boundary layer flow. Rohn et al [16] studied Unsteady mixed convection boundary-layer flow with suction and temperature slip effects near the stagnation point on a vertical permeable surface embedded in a porous medium. Vajravelu [17] investigated Unsteady convective boundary layer flow of a viscous fluid at a vertical surface with variable fluid properties Aziz and Khan[3] studied Natural convective boundary layer flow of a nanofluid past a convectively heated vertical plate. Fazlina and Anuar [2] studied Mixed Convection Boundary Layer Flow towards a Vertical Plate with a Convective Surface Boundary Condition. Khan et al [13] analysed MHD boundary layer flow of a nanofluid containing gyrotactic microorganisms past a vertical plate with Navier slip. Many researchers have also used a finite difference method to solve flow problems. El-Naby et al [9]analysed the Finite difference solution of radiation effects on MHD unsteady free-convection flow over vertical plate with variable surface temperature. Wafula and Kinyanjui [18] Investigated mixed convection of unsteady Nanofluid flow past a vertical plane with entropy generation. Fornberg [10] studied Finite difference formulas in the complex plane. Abbas et al [1] investigated Finite element analysis of nanofluid flow and heat transfer in a square cavity with two circular obstacles at different positions in the presence of magnetic field. Different from the previous approach,the present paper considers finite difference analysis of boundary layer flow of nanofluids past a vertical plane with heat transfer.

2. METHODOLOGY

Consider the steady two-dimensional laminar flow of a viscous incompressible boundary layer flow over a vertical plate as shown in the Figure 2.1 below.

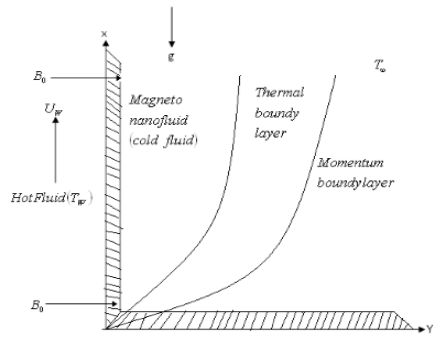


Figure 2.1. Geometry of the problem

It is assumed that the fluid properties such as viscosity, thermal conductivity and specific heat capacity are constants, the velocity of the fluid is too small compared with that of light i.e $\frac{q^2}{c^2} << 1$ and no slip condition was satisfied. The Maxwell's equations which provides the relations between the interacting electric and magnetic fields are given by;

$$\nabla \times \vec{B} = \mu_e \vec{j}$$
$$\nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\nabla \cdot \vec{j} = 0 \quad (2.1)$$

The isotropic constitutive equations are

$$\vec{B} = \mu_e \vec{H} \text{ and } \vec{D} = \varepsilon \vec{E} \quad (2.2)$$
$$\vec{j} = \sigma(\vec{E} + \vec{q} \times \vec{B}) \quad (2.3)$$

The current density in a moving conductor is given by Ohm's law is valid under the assumptions that the Hall effect and electric displacement current can be neglected and that there are no thermoelectric voltage sources.

The Ohm's law yields

$$\vec{q} \times \vec{B} = \begin{vmatrix} i & j & k \\ u & v & 0 \\ 0 & B_0 & 0 \end{vmatrix} = uB_0\vec{k} \quad (2.4)$$

The Lorentz force $\vec{j} \times \vec{B}$ is given by;

$$\vec{j} \times \vec{B} = \begin{vmatrix} i & j & k \\ 0 & 0 & \sigma u B_0 \\ 0 & B_0 & 0 \end{vmatrix} = -\sigma u B_0^2 \vec{i} \quad (2.5)$$

Heat generated due to electrical resistance of the fluid to the flow of induced electric current is;

$$\frac{j^2}{\sigma} = \sigma u^2 B_0^2 \quad (2.6)$$

The flow field and the electromagnetic field are to be determined by solving the fundamental equations under appropriate boundary conditions for the velocity field and electromagnetic field.

Using boundary layer approximation and the assumptions above, the equation of continuity, equation of conservation of momentum and energy equation governing the nanofluid fluid flow in the presence of a transverse magnetic field can be expressed as

(2.7) $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

(2.8) $$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho_{nf}}\frac{\partial p}{\partial x}+\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial x^2}\right)+g\beta_{nf}(T-T_\infty)-\frac{\sigma_{nf}B_0^2 u}{\rho_{nf}}$$

(2.9) $$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\frac{2\mu_{nf}}{(\rho C_p)_{nf}}\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]+\frac{\mu_{nf}}{(\rho C_p)_{nf}}\left[\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right]+\frac{\sigma_{nf}B_0^2u^2}{(\rho C_p)_{nf}}$$

Both the velocity u and the temperature T of the fluid depend only on the transversal coordinate y, the continuity equation is satisfied identically, the transversal component of the hydrodynamic pressure gradient $\frac{\partial p}{\partial y}$ is vanishing, the planar one $\frac{\partial p}{\partial x}$ is a given constant. Under this assumptions, the continuity, momentum and energy equations becomes

(2.10) $$\frac{\partial u}{\partial x}=0$$

(2.11) $$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\left(\frac{\partial^2 u}{\partial y^2}\right)+g\beta_{nf}(T-T_f)-\frac{\sigma_{nf}B_0^2 u}{\rho_{nf}}$$

(2.12) $$\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}=\frac{k_{nf}}{(\rho C_p)_{nf}}\frac{\partial^2 T}{\partial y^2}+\frac{\mu_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\sigma_{nf}B_0^2u^2}{(\rho C_p)_{nf}}$$

With initial boundary conditions

$u(y,0) = 0$ and $T(y,0) = 0$: at $t = 0$ $u(y,t) = u_\infty$ and $T(y,t) = T_\infty$ at $t > 0$ Replacing the variables with the following non-dimensionless parameters

$$v^*=\frac{v}{U}\quad u^*=\frac{u}{U}\quad T^*=\frac{T-T_\infty}{T-T_f}\quad B^*=\frac{B}{B_0}\quad t^*=\frac{tU^2}{\nu}$$

$$M^2=\frac{B_0^2}{U^2}\frac{\mu_{nf}\sigma_{nf}}{\rho_{nf}}\quad G_{rt}=\frac{\nu g\beta_{nf}(T-T_f)}{U_0^3}\quad \mathrm{Pr}=\frac{\mu C_p}{k_f}\quad \mathrm{Ec}=\frac{U^2}{C_p(T-T_\infty)}$$

Equations (2.11) and (2.12) become

(2.13) $$\frac{\partial u*}{\partial t^*}+v^*\frac{\partial u^*}{\partial y^*}=\left(\frac{\partial^2 u^*}{\partial y^{*2}}\right)+G_{rT}\phi_3-M^2u^*\phi_1$$

(2.14) $$\frac{\partial T^*}{\partial t^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{\phi_2}{Pr}\frac{\partial^2 T^*}{\partial y^{*2}}+Ec\phi_4\left(\frac{\partial u^*}{\partial y^*}\right)^2+Ec\phi_5M^2(u^*)^2$$

where

; $$\phi_1=\frac{(1-\phi)+\phi\frac{\sigma_s}{\sigma_f}}{(1-\phi)+\phi\frac{\rho_s}{\rho_f}};\quad \phi_2=\frac{\frac{k_s-2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)}}{(1-\phi)+\phi\frac{\rho Cp_s}{\rho Cp_f}};\quad \phi_3=\frac{(1-\phi)^{-2.5}}{(1-\phi)+\phi\frac{p_s}{\rho_f}}$$

$$\phi_4=\frac{(1-\phi)^{-2.5}}{(1-\phi)+\phi\frac{\rho Cp_s}{\rho Cp_f}};\quad \phi_5=\frac{(1-\phi)+\phi\frac{\sigma_s}{\sigma_f}}{(1-\phi)+\phi\frac{\rho Cp_s}{\rho Cp_f}}$$

The above parameters were estimated with the existing reactions for the two-phase mixture given by [5] where nanofluid viscosity is given by

$$\mu_{nf}=\frac{\mu_f}{(1-\phi)^{2.5}}\qquad (2.15)$$

The effective density, heat capacitance, electrical conductivity and effective thermal conductivity of the nanofluid is [19], [12]

$$\sigma_{nf} = (1-\phi)\sigma_f + \phi\sigma_s$$

$$\frac{k_n f}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)}, \qquad \alpha_{nf} = \frac{k_{nf}}{(\rho c_p)_{nf}}$$

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s \qquad (2.16)$$

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s \qquad (2.17)$$

(2.18)

(2.19)

Thermophysical properties of the nanofluid are given in Table 1.

Physical properties	Water/base fluid	Cu(Copper)	Al2O3(Alumina)	TiO2(Titanium Oxide)
$\rho(kgm^{-3})$	997.1	8933	3970	4250
$C_p(J/KgK)$	4179	385	765	686.2
k(WmK)	0.613	401	40	8.9538
ϕ	0.0	0.05	0.15	0.2
$\sigma(S/m)$	5.5×10^{-6}	59.6×10^{6}	35×10^{6}	2.6×10^{6}

Table 1. Thermophysical Properties of water and nanoparticles[[?], [?]]

3. NUMERICAL SOLUTION

The nonlinear differential equations are nonlinear hence solved by use of numerical method. In this work, finite difference method (FDM) is used to dicretize the physical domain into finite grids. By use of forward time central space (FTCS), the differential equations are replaced by finite difference equations and given as follows

$$U_{j,k+1} = \Delta t\left[-V_0\frac{U_{j+1,k}-U_{j-1,k}}{2\Delta y} + \frac{U_{j+1,k}-2U_{j,k}+U_{j-1,k}}{(\Delta y)^2} + \phi_3 G_{rT} - U_{j,k}M^2\phi_1\right] + U_{j,k} \quad (3.1)$$

$$T_{j,k+1} = \Delta t\left[-V_0\frac{T_{j+1,k}-T_{j-1,k}}{2\Delta y} + \frac{T_{j+1,k}-2T_{j,k}+T_{j-1,k}}{(\Delta y)^2} + \phi_4 Ec(\frac{T_{j+1,k}-T_{j-1,k}}{2\Delta y})^2 + EcM^2(U_{j,k})^2\right] + T_{j,k} \quad (3.2)$$

The above equations are then solved by use of MATLAB and results given in graphical format.

4. RESULTS AND DISCUSSION

The effect of parameters on velocity and temperature profiles are presented in graphs and discussed.

Figure 4.1 presents the effect of varying Hartman number on velocity profile. It is observed that as Hartman number increases, the velocity decreases. This is due to the increasing influence of the magnetic field, which generates Lorentz forces that can counteract the fluid's natural tendency to flow thus decreasing the velocity.

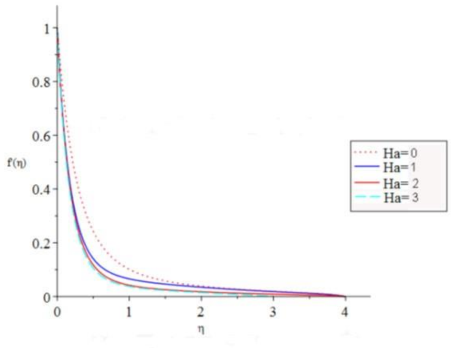


Figure 4.1. Velocity profile for different Ha when ϕ = 0.1

Figure 4.2 Shows velocity profile for different volume fraction parameter. It is observed that, increase in volume fraction parameter increases the velocity of the fluid. This can be explained by the fact that an increase in the volume fraction parameter of nanoparticles in a fluid can lead to an increase in the fluid velocity to overcome the increased resistance to flow caused by the presence of the nanoparticles.

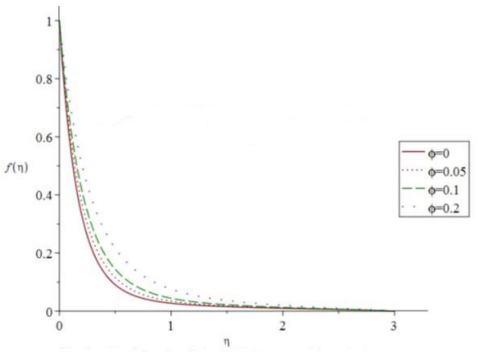


Figure 4.2. Velocity profile for different volume fraction parameter when *Ha* = *λ* = 1

Figure 4.3 displays velocity profiles for different values of Eckert numbers Ec. It is observed that, increase in Ec leads to the increase in the velocity of the fluid.

As the Eckert number increases, the ratio of kinetic energy to thermal energy becomes larger, resulting in a higher velocity.

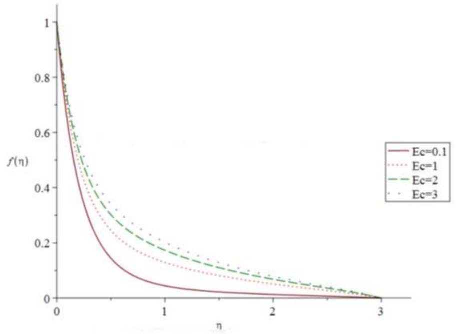


Figure 4.3. Velocity profile for different Ec when ϕ = 0.1 and *Ha* = *λ* = 1

From figure 4.4, it is observed that there is a direct proportion between change in volume fraction parameter and temperature. The increase in the volume fraction parameter can lead to an increase in the rate of heat transfer through the fluid, which in turn can cause an increase in the temperature profile.

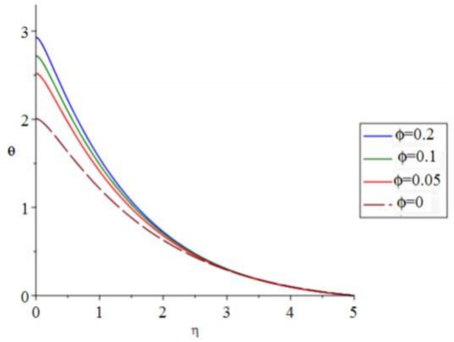


Figure 4.4. Temperature profile for different Volume fraction parameter when

Ha = *λ* = 1

Figure 4.5 shows the effect of and Hartmann number (Ha) on temperature profile. It is observed that the fluid temperature increases as the magnetic field strengthens since extra work is executed by the fluid in overcoming the drag-force for nanofluid, leading to an increase in both temperature and thermal boundary layer thickness. This accompanying work is then dissipated as thermal energy which acts to heat the fluid and elevates temperature.

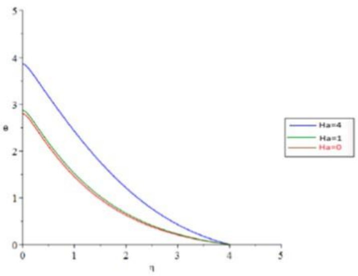


Figure 4.5. Temperature profile for different Hartman number when ϕ = 0.1

Increase in Eckert number (Ec) leads to an increase in both temperature and thermal boundary layer thickness. This is because an increase in the Eckert number may lead to an increase in the heat transfer rate, which can result in a more significant temperature gradient.

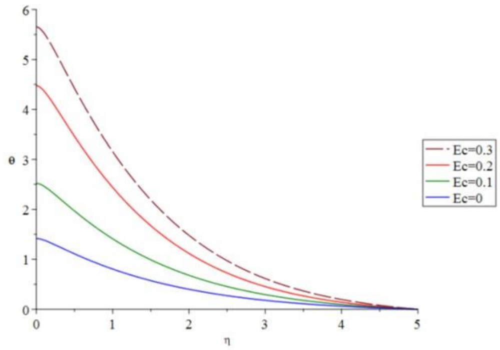


Figure 4.6. Temperature profile for different Ec number when ϕ = 0.1

5. CONCLUSION

In this work, finite difference of boundary layer flow of nanofluids past a vertical plane with heat transfer was analsed. The influences of the different types of non dimensional parameters on flow field was examined. the following conclusions were made:

It was observed that the velocity of nanofluid decreases as the strength of magnetic field increases and the temperature increases with increase of magnetic parameter. Also, the velocity and temperature of nanofluid increases with increase of volume fraction parameter. It was also observed that Increase of Eckert number leads to the increase of both velocity and temperature. This research is important since it can applied in automotive fields and Nanofluid coolants.

REFERENCES

1. Abbas, S., Wang, X., Khan, W., Hobiny, A., and Iqbal, K. Finite element analysis of nanofluid flow and heat transfer in a square cavity with two circular obstacles at different positions in the presence of magnetic field. *Journal of Energy Storage*, 51:104462, 2022.
2. Aman, F. and Ishak, A. Mixed convection boundary layer flow towards a vertical plate with a convective surface boundary condition. *Mathematical Problems in Engineering*, 2012, 2012.
3. Aziz, A. and Khan, W. Natural convective boundary layer flow of a nanofluid past a convectively heated vertical plate. *International Journal of Thermal Sciences*, 52:83–90, 2012.
4. Blasius, H. Grenzschichten in flussigkeiten mit kleiner. *Reibung. Eng. transl. NACA TM*, 1256, 1908.
5. Brinkman, H. C. A calculation of the viscous force exerted by a flowing fluid on a dense swarm of particles. *Flow, Turbulence and Combustion*, 1:27–34, 1949.
6. Das, S. K., Putra, N., Thiesen, P., and Roetzel, W. Temperature dependence of thermal conductivity enhancement for nanofluids. *J. Heat Transfer*, 125(4):567–574, 2003.
7. Das, S. K., Choi, S. U., Yu, W., and Pradeep, T. *Nanofluids: science and technology*. John Wiley & Sons, 2007.
8. Eastman, J. A., Choi, S., Li, S., Yu, W., and Thompson, L. Anomalously increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles. *Applied physics letters*, 78(6):718–720, 2001.
9. El-Naby, A., Elbarbary, E. M., and Abdelazem, N. Y. Finite difference solution of radiation effects on mhd unsteady free-convection flow over vertical

plate with variable surface temperature. *Journal of Applied Mathematics*, 2003(2):65–86, 2003.

10. Fornberg, B. Finite difference formulas in the complex plane. *Numerical Algorithms*, 90(3): 1305–1326, 2022.
11. Ilias, M. R., Ismail, N. S., AbRaji, N. H., Rawi, N. A., and Shafie, S. Unsteady aligned mhd boundary layer flow and heat transfer of a magnetic nanofluids past an inclined plate. *International Journal of Mechanical Engineering and Robotics Research*, 9(2):197–206, 2020.
12. Kakaç, S. and Pramuanjaroenkij, A. Review of convective heat transfer enhancement with nanofluids. *International journal of heat and mass transfer*, 52(13-14):3187–3196, 2009.
13. Khan, W., Makinde, O., and Khan, Z. Mhd boundary layer flow of a nanofluid containing gyrotactic microorganisms past a vertical plate with navier slip. *International journal of heat and mass transfer*, 74:285–291, 2014.
14. Mat Yasin, M. H., Arifin, N. M., Nazar, R., Ismail, F., and Pop, I. Mixed convection boundary layer flow embedded in a thermally stratified porous medium saturated by a nanofluid. *Advances in Mechanical Engineering*, 5:121943, 2013.
15. Mohd Rohni, A., Ahmad, S., and Pop, I. Boundary layer flow over a moving surface in a nanofluid beneath a uniform free stream. *International Journal of Numerical Methods for Heat & Fluid Flow*, 21(7):828–846, 2011.
16. Rohni, A. M., Ahmad, S., Pop, I., and Merkin, J. H. Unsteady mixed convection boundarylayer flow with suction and temperature slip effects near the stagnation point on a vertical permeable surface embedded in a porous medium. *Transport in porous media*, 92:1–14, 2012.
17. Vajravelu, K., Prasad, K., and Ng, C.-O. Unsteady convective boundary layer flow of a viscous fluid at a vertical surface with variable fluid properties. *Nonlinear Analysis: Real World Applications*, 14(1):455–464, 2013.
18. Wafula, M. and Kinyanjui, M. N. Mixed convection of unsteady nanofluids flow past a vertical plane with entropy generation. *Mathematical Theory and Modeling*, 2019.
19. Xuan, Y. and Li, Q. Heat transfer enhancement of nanofluids. *International Journal of heat and fluid flow*, 21(1):58–64, 2000.