



An algorithm for performing traveling salesman on given N points

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Abstract

Given N points in some General metric space M , we want to connect them so that from each point we can access any other points such that the union of all connecting lines have minimal length.

Main arguments

Given N points $a_i, i = 1, 2, \dots, N$ in some metric space

M . For the above given N points we denote by S_N , a continua connecting all the

above points and such that the sum of the length of all corresponding Geodesics $L(S_N)$

will be minimal. Our aim is to find an algorithm to solve this problem in minimal cost.

In doing this randomly the cost will be N factorial ie', $N!$ which can be immense.

Furthermore the minimal continua might be achieved by adding extra vertices and then

connecting them to the existing vertices as we will show and this can not be done

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using random technics or by neural network optimization technics .Also by the term

continua or continuum, we mean compact and connected subspace of a metric space. Next, for a given positive real number x , let us set $[x]$ to be

the integer that is larger than x and is lower upper bound among the integers larger

than x . In this article we proceed with the following approach and prove that using this

approach we can get the minimal continua in less than $2[\log_2 N] + [\log_3 N]$ steps..

By Geodesic we mean lines connecting two point of minimal length.Also in this article

M can be any Euclidean metric space (the ideas can be extended to non Euclidean spaces using the arguments in [1]) where in these spaces the geodesics are

usual classic straight lines Also there might be more than one of such a minimal continua

,It is clear that a minimal continua exist and consists of points we call them Vertices

and connecting Geodesics which we denote them by edges. In the following

arguments, we need to define concept called a Tree which is a connected simple

graph without loops. Given to Trees T_1 and T_2 , we say $T_1 \geq T_2$ if

$L(T_1) \geq L(T_2)$. It is clear that the solution of the above problem is a Tree.

It is easy to show that the connecting Tree is actually a minimal continua if and if omitting an edge and replacing it by other connecting edge it will

result on larger Tree. Next it is clear that connected finite graphs are continuum.We introduce solution to this problem as in the following, our solution

is performed on the K number of steps , with $K \leq 2[\log_2 N] + [\log_3(N)]$ and is based on fact that for any two disjoints connected graphs we can find

a minimal arc connecting them which can be measured easily using quadratic programming

, since we are dealing with unions of straight lines and intervals. Now our solution is

performed in the following steps, (1) We first connect each vertex by an edge to closest vertex. As a result we get a sequence of connected simple graphs.

(2) The above graphs are considered new vertices. Now the same process as in step(1),

will be done with these new given vertices (that are actually connected finite graphs),

hence as in the above we get a new set of finite connected simple graphs that we regard them

as vertices and perform the same actions as in step(1). Continuing the above steps, after K_1 , $K_1 \leq \log_2(N)$ steps we get a connected simple graph. At this stage we try to minimize the connections between the graphs. Beginning

from the final graph. Suppose Graphs A and B are connected to graph C . Then we consider

the Geodesic α from A to the line BC and β from B to AC . Suppose without the loss of generality that $\alpha + |BC|$ is less or equal than $\beta + |AC|$, then we skip the arc AC and conversely. We proceed this process for the stages

$K_1, K_1 - 1, K_1 - 2, \dots, 1$ Finally in order to get the final solution which is a Tree

we have to get rid of the loops that are existed in this final graph. In order to do that

consider the graphs achieved in every stage and if the edge connecting two disjoint

graphs does not hit the loops then from each loop delete the largest edge and so

the resulted graph will be loop free. But if the connecting edge hits one of the loop

then the edge hit is a union of two edges, at this point we delete the largest of

the edges in the loop and hence we get a loop free graph. continuing this process

with the graphs at all the stages we will get the connecting continuum connecting all the vertices which is a Tree T . Finally since if we replace

any connecting edge by another connecting edge will result in larger connecting

Tree, by the above argument T is the desired minimal connecting Tree.

References

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