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Radio Gd-Distance Number of Some Basic Graph

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ABSTRACT

A Radio Gd-distance labeling of a connected graph G is an injective map f from the vertex set V(G) to ℕ such that for two distinct vertices u and v of G, $dGd(u,v)+|f(u)-f(v)|\geq 1+diamGd(G)$, where $dGd(u,v)$ denotes the Gd-distance between u and v and $diamGd(G)$ denotes the Gd-diameter of G. The Radio Gd-distance number of f, (f) is the maximum label assigned to any vertex of G. The Radio Gd-distance number of G, $rn(G)$ is the minimum value f of G. In this paper we find the radio Gd-distance number of some basic graph.

KEYWORDS: Gd-distance, Radio Gd-distance, Radio Gd-distance number.

I. INTRODUCTION

By a graph $G = (V(G), E(G))$ we mean a finite undirected graph without loops or multiple edges. Let $V(G)$ and $E(G)$ denotes the vertex set and edge set of G. The order and size of G are denoted by p and q respectively.

The Gd-distance was introduced by V. Maheswari and M. Joice Mabel. If u and v are vertices of a connected graph G, Gd-length of a u-v path is defined as $d^{Gd}(u,v) = d(u,v) + \deg(u) + \deg(v)$. The Gd-radius, denoted by $r^{Gd}(G) = min\{e^{Gd}(v): v \in V(G)\}$. Similarly the Gd-diameter $d^{Gd}(G) = max\{e^{Gd}(v): v \in V(G)\}$. We observe that for any two vertices u and v of G we have d(u,v) $\leq$ $d^{Gd}(u,v)$. The equality holds if and only if u, v are identical. If G is any connected graph, then the d^{Gd} distance is a metric on the set of vertices of G. We can check easily that for any non-trivial connected graph, $r^{Gd}(G) \leq d^{Gd}(G) \leq 2r^{Gd}(G)$.The lower bound is clear From the definition and the upper bound follows from the triangular inequality.

In this paper, we introduced the concept of radio Gd-distance labeling of a graph G. Radio Gd-distance labeling is a function f from V(G) to ℕ satisfying the condition $d^{Gd}(u,v) + |f(u) - f(v)| \geq 1 + diam^{Gd}(G)$, where $diam^{Gd}(G)$ is the Gd-distance diameter of G. A Gd-distance radio labeling number of G is the maximum label assigned to any vertex of G. It is denoted by $rn^{Gd}(G)$

Radio labeling can be regarded as an extension of distance-two labeling which is motivated by the channel assignment problem introduced by W. K. Hale [6]. G. Chartrand et al.[2] introduced the concept of radio labeling of graph. Also G. Chartrand et al.[3] gave the upper bound for the radio number of path. The exact value for the radio number of path and cycle was given by Liu and Zhu [10]. However G. Chartrand et al.[2] obtained different values for them. They found the lower and upper bound for the radio number of cycle. Liu [9] gave the lower bound for the radio number of Tree. The exact value for the radio number of Hypercube was given by R. Khennoufa and O. Togni [8]. In [4] C. Fernandez et al. found the radio number for complete graph, Star graph, Complete Bipartite graph, Wheel graph and Gear graph. In this paper, we fined the radio Gd-distance labeling of some basic graphs.

II. MAIN RESULTS

Theorem 2.1

The radio Gd-distance number of the complete graph, $rn^{Gd}(K_n) = n \; \forall \; n$

Proof.

Let, $V(K_n) = \{v_1, v_2, \dots, v_n\}$ be the vertex set

Then, $d^{Gd}(v_i, v_j) = 2n - 1 \; for \; 1 \leq i, j \leq n$

It is obvious that the $diam^{Gd}(K_n) = 2n - 1$.

The radio Gd-distance condition is $d^{Gd}(u,v) + |f(u) - f(v)| \geq 1 + diam^{Gd}(G) = 2n$

Now, fix $f(v_1) = 1$

$$d^{Gd}(v_{1,}\, v_2) + |f(v_1) - f(v_2)| \geq 2n - 1 + |1 - f(v_2)| \geq 2n$$

$|1 - f(v_2)| \geq 1$, which implies $f(v_2) = 2$

$\therefore f(v_i) = i,\ 1 \leq i \leq n$

Hence, $rn^{Gd}(K_n) = n, \forall\, n$

Theorem 2.2

The radio Gd-distance number of a star graph, $rn^{Gd}(K_{1,n}) = n^2 - 2n + 2,\ \text{n} \geq 3$

Proof.

Let $\text{V}(K_{1,n}) = \{v_{0,}\, v_{1,} v_2, \ldots, v_n\}$ be the vertex set, where v_0 be the central vertex and

$\text{E}(K_{1,n}) = \{v_0 v_i\, ; 1 \leq i \leq n\}$ be the edge set

Then, $d^{Gd}(v_0, v_i) = n + 2; 1 \leq i \leq \text{n},\ d^{Gd}(v_i, v_j) = 4;\ 1 \leq i, j \leq n;\ i \neq j$

So, $diam^{Gd}(K_{1,n}) = n + 2$

Without loss of generality,

$f(v_1) < f(v_0) < f(v_2) < \cdots < f(v_n)$

We shall check the radio Gd-distance condition

$$d^{Gd}(u, v) + |f(u) - f(v)| \geq 1 + diam^{Gd}(G) = n + 3$$

Fix $f(v_1) = 1$, for (v_1, v_0)

$$d^{Gd}(v_1, v_0) + |f(v_1) - f(v_0)| \geq \text{n} + 2 + |1 - f(v_0)| \geq n + 3$$

$|1 - f(v_0)| \geq 1$, which implies $f(v_0) = 2$

For (v_1, v_2)

$$d^{Gd}(v_1, v_2) + |f(v_1) - f(v_2)| \geq 4 + |1 - f(v_2)| \geq n + 3$$

$|1 - f(v_2)| \geq n - 1$, which implies $f(v_2) = n$

For (v_2, v_3)

$$d^{Gd}(v_2, v_3) + |f(v_2) - f(v_3)| \geq 4 + |n - f(v_3)| \geq n + 3$$

$|n - f(v_3)| \geq n - 1$, which implies $f(v_3) = 2n - 1$

$\therefore f(v_i) = n(i - 1) - i + 2,\ 1 \leq i \leq n$

Hence, $rn^{Gd}(K_{1,n}) = n^2 - 2n + 2,\ n \geq 3$

Theorem 2.3

The radio Gd-distance number of a path $rn^{Gd}(P_n) \leq n^2 - 5n + 8,\ n \geq 4$

Proof.

Let $\text{V}(P_n) = \{v_{1,}\, v_{2,}, \ldots, v_n\}$ be the vertex set and $\text{E}(P_n) = \{v_i v_{i+1}\, ; 1 \leq i \leq n - 1\}$ be the edge set

Then, $d^{Gd}(v_{1,}\, v_n) = d^{Gd}(v_{2,}\, v_n) = n + 1$,

$d^{Gd}(v_{1,}\, v_2) = d^{Gd}(v_{n-1,}\, v_n) = 4$,

$d^{Gd}(v_{i,}\, v_{i+1}) = 5;\ 2 \leq i \leq n - 2$

It is clear that $diam^{Gd}(P_n) = n + 1$

Without loss of generality

$$f(v_1) < f(v_n) < f(v_2) < \cdots < f(v_{n-1})$$

We shall check the radio Gd-distane condition

$$d^{Gd}(u, v) + |f(u) - f(v)| \geq 1 + diam^{Gd}(G) = n + 2$$

Fix $f(v_1) = 1$ for $(v_{1,}\, v_n)$

$$d^{Gd}(v_1, v_n) + |f(v_1) - f(v_n)| \geq \text{n} + 1 + |1 - f(v_n)| \geq n + 2$$

$|1 - f(v_n)| \geq 1$, which implies $f(v_n) = 2$

For (v_1, v_2)

$$d^{Gd}(v_1, v_2) + |f(v_1) - f(v_2)| \geq 4 + |1 - f(v_2)| \geq n + 2$$

$|1 - f(v_2)| \geq n - 2$, which implies $f(v_2) = n - 1$

For (v_2, v_3)

$$d^{Gd}(v_2, v_3) + |f(v_2) - f(v_3)| \geq 5 + |n - 1 - f(v_3)| \geq n + 2$$

$|n - 1 - f(v_3)| \geq n - 3$, which implies $f(v_3) = 2n - 4$

$\therefore f(v_i) = n(i - 1) - 3i + 5,\ 2 \leq i \leq n - 1$

Hence, $rn^{Gd}(P_n) \leq n^2 - 5n + 8,\ n \geq 4$

Note. $rn^{Gd}(P_n) = 3$ if n=3

Theorem 4

The radio Gd-distance number of a subdivision of a star, $rn^{Gd}\text{S}(K_{1,n}) = 2n^2 - 5n + 3,\ n \geq 3$

Proof

Let , $\text{V}(\text{S}(K_{1,n})) = \{v_0, v_{1,}\, v_{2,}, \ldots, v_n, u_{1,} u_{2,}, \ldots, u_{n,}\}$ be the vertex set, where v_0 is the central vertex and $\text{E}(\text{S}(K_{1,n})) = \{v_0 u_i\, , v_i u_i;\ 1 \leq i \leq n\}$ be the edge set

Then, $d^{Gd}(v_{0,}\, v_i) = d^{Gd}(v_{0,}\, u_i) = n + 3; 1 \leq i \leq n$,

$d^{Gd}(v_{I,}\, v_{i+1}) = d^{Gd}(u_{i,}\, u_{i+1}) = 6; 1 \leq i \leq n$

$d^{Gd}(v_{i,}\, u_i) = 4;\ 1 \leq i \leq n$

It is clear that $diam^{Gd}(\text{S}(K_{1,n})) = n + 3$

Without loss of generality $f(u_1) < f(v_0) < f(u_2) < \cdots < f(u_n) < f(v_1) < \cdots < f(v_n)$

We shall check the radio Gd-distance condition

$$d^{Gd}(u, v) + |f(u) - f(v)| \geq 1 + diam^{Gd}(G) = n + 4$$

Fix $f(u_1) = 1$, for $(u_{1,}\, v_0)$ $1 \leq i \leq n$

$$d^{Gd}(u_1, v_0) + |f(u_1) - f(v_0)| \geq \text{n} + 3 + |1 - f(v_0)| \geq n + 4$$

$|1 - f(v_0)| \geq 1$ which implies $f(v_0) = 2$

For (u_1, u_2)

$$d^{Gd}(u_1, u_2) + |f(u_1) - f(u_2)| \geq 6 + |1 - f(u_2)| \geq n + 4$$

$|2 - f(u_2)| \geq n - 2$, which implies $f(u_2) = n - 1$

For (u_2, u_3)

$d^{Gd}(u_2,u_3)+|f(u_2)-f(u_3)|\geq 6+|n-1-f(u_3)|$
$\geq n+4$
$|n-1-f(u_3)|\geq n-2$, which implies $f(u_3)=2n-3$

$\therefore f(u_i)=n(i-1)-2i+3,\ 1\leq i\leq n$

Therefore, $f(u_n)=n^2-3n+3$
For $(u_n,v_i),\ 1\leq i\leq n$
$d^{Gd}(u_n,v_1)+|f(u_n)-f(v_1)|$
$\geq 6+|n^2-3n+3-f(v_1)|\geq n+4$

$|n^2-3n+3-f(v_1)|\geq n-2$ which implies
$f(v_1)=n^2-2n+1$
For (v_1,v_2)
$d^{Gd}(v_1,v_2)+|f(v_1)-f(v_2)|$
$\geq 6+|n^2-2n+1-f(v_2)|\geq n+4$
$|n^2-2n+1-f(v_2)|\geq n-2$, which implies
$f(v_2)=n^2-n-1$

$\therefore f(v_i)=n^2+n(i-3)-2i+3,\ 1\leq i\leq n$

Hence, $rn^{Gd}S(K_{1,n})=2n^2-5n+3,\ n\geq 4$

Note. $rn^{Gd}S(K_{1,n})=2n+1\ if\ n=2,3$

Theorem 5

The radio Gd-distance number of bistar graph, $rn^{Gd}(B_{n,n})=4n^2-3n+4,\ n\geq 2$

Proof.

Let, $V(B_{n,n})=\{v_{1,}\ v_{2,},\dots,v_n,x_{1,}\ x_{2,}u_1,u_{2,},\dots,u_{n,}\}$ be the vertex set and $E(B_{n,n})=\{x_1u_i\ ,x_2v_{i,}x_1x_{2,};1\leq i\leq n\}$ be the edge set

Then, $d^{Gd}(x_{1,}u_i)=d^{Gd}(x_2,v_i)=n+3;\ 1\leq i\leq n,$
$d^{Gd}(x_{1,}x_2)=2n+3,\ d^{Gd}(u_i,v_j)=5;1\leq i,j\leq n,$
$i\neq j, d^{Gd}(u_i,u_j)=\ d^{Gd}(v_i,v_j)=4;\ 1\leq i,j\leq n,$
$i\neq j,\ d^{Gd}(x_{1,}v_i)=d^{Gd}(x_2,u_i)=n+4;\ \ 1\leq i\leq n$

It is clear that $diam^{Gd}(B_{n,n})=2n+3$

Without loss of generality $f(v_1)<f(u_1)<f(v_2)<f(u_2)<\cdots<f(v_n)<f(u_n)<f(x_2)<f(x_1)$

We shall check the radio Gd-distance condition
$d^{Gd}(u,v)+|f(u)-f(v)|\geq 1+diam^{Gd}(G)=2n+4$

Fix, $f(v_1)=1$, For $(v_{i,}u_i),1\leq i\leq n$
$d^{Gd}(v_{1,}u_1)+|f(v_1)-f(u_1)|\geq 5+|1-f(u_1)|$
$\geq 2n+4$
$|1-f(u_1)|\geq 2n-1$, which implies $f(u_1)=2n$

For $(u_{i,}v_{i+1}),1\leq i\leq n-1$
$d^{Gd}(u_{1,}v_2)+|f(u_1)-f(v_2)|\geq 5+|2n-f(v_2)|$
$\geq 2n+4$
$|2n-f(v_2)|\geq 2n-1$ which implies $f(v_2)=4n-1$
For (v_2,u_2)
$d^{Gd}(v_2,u_2)+|f(v_2)-f(u_2)|\geq 5+|4n-1-f(u_2)|$
$\geq 2n+4$
$|4n-1-f(u_2)|\geq 2n-1$, which implies $f(u_2)=6n-2$

$\therefore f(u_i)=n(4i-2)-2i+2,\ 1\leq i\leq n$
$f(v_i)=n(4i-4)-2i+3,\ 1\leq i\leq n$

Therefore, $f(u_n)=4n^2-4n+2$

For (u_n,x_2)
$d^{Gd}(u_{n,}x_2)+|f(u_n)-f(x_2)|$
$\geq n+3+|\ 4n^2-4n+2-f(x_2)|$
$\geq 2n+4$
$|4n^2-4n+2-f(x_2)|\geq n+1$, which implies $f(x_2)=4n^2-3n+3$

For (x_2,x_1)
$d^{Gd}(x_2,x_1)+|f(x_2)-f(x_1)|$
$\geq 2n+3+|4n^2-3n+3-f(x_1)|$
$\geq 2n+4$
$|4n^2-3n+3-f(x_1)|\geq 2n$, which implies $f(x_1)=4n^2-3n+4$

Hence, $rn^{Gd}(B_{n,n})=4n^2-3n+4,\ n\geq 2$

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