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Prime Labelling for Some Bipartiate Related Graphs

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ABSTRACT
A graph G = (V,E) with 'n' vertices is said to have a prime labeling if its vertices are labelled with distinct positive integers not exceeding n such that for each pair of adjacent vertices are relatively prime. A graph G which admits prime labeling is called a prime graph. In this paper, we investigate prime labeling for some bipartiate and cycle related graphs. We also discuss the prime labeling of some graph operation namely joint sum and path joining of bipartiate and cycle graphs.

KEYWORDS: Prime labelling, joint sum, path union.

INTRODUCTION

Here we considered the graphs which are finite, simple and undirected graphs. A graph G is G = [V (G), E (G)] where, V(G) denotes the vertices set and E (G) denotes the edge set. The terminology and notations in graph theory we follow Harary [1]. A complete survey of graph labelling is referred from J.A. Gallian [2]. Graph labelling where the vertices are assigned real values satisfying some conditions.

Definition1.1

Let G = (V, E) be a graph with p vertices. A bijection f: V (G) {1,2,3…p} is said to be as prime labelling if for each edge e = uv the labels assigned to u and v are relatively prime. A graph which admits prime labelling is called prime graph.

Definition 1.2

K_1 with 'n' pendent edges incident with V (K_1) is called a Star Graph and is denoted by K_1, n.

Definition 1.3

Path $P_n = v_1v_2v_3$ ………..v_n has 'n' vertices and 'n-1' edges.

Definition 1.4

Cycle $C_n = v_1v_2v_3$ ………..v_nv_1 has 'n' vertices and 'n' edges.

Definition 1.5

Let G_1 and G_2 be the two copies of fixed graph, connect a vertex of first copy to a vertex of second copy with a new edge the new graph obtained is called joint sum of G_n.

Definition 1.6

The fan graph F_n is defined as K_1+P_n, P_n is a path of n vertices.

Theorem 2.1

Vertices joined by an edge of $K_{1,\,n}$ graph (n is odd) admits prime labelling (vertices v_1 and v_{n-1}).

Proof:

Let G be $K_{1,\,n}$ graph, the vertices of V (K_1) = u and v_i, $1 \le i \le n$ be the 'n' vertices adjacent to u. Now join by an edge between the v_1 to v_{n-1} and V (G) = n+1.

Define a function f: V (G) {1,2, (n+1)} by

f (u) = 1;

f (vi) = i; $1 \le i \le n$.

As defined by definition of prime labelling,

gcd{ f(u), f (vi) } = 1, $1 \le i \le n$.

gcd{ f(v_1), f (v_{n-1}) } = 1.

Thus, G admits prime labeling. ∴ G is a prime labelling graph.

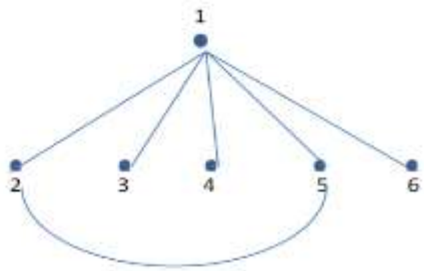

Figure 1.Vertices v_1 and v_4 joined by an edge of $K_{1,\,5}$ graph.

Theorem 2.2

Vertices joined by an edge of $K_{1,\,n}$ graph (n is even) admits prime labelling. (vertices v_1 and v_n).

Proof:

Let G be $K_{1,\,n}$ graph, the vertices of V (K_1) = u and v_i, $1 \le i \le n$ be the 'n' vertices adjacent to u. Now join by an edge between the vertices v_1 to v_n. Then V(G) = n+1.

Define a function f: V (G) $\rightarrow$ {1, 2 ………. (n+1) } by
f(u) = 1;
f(vi) = i ; $1 \leq i \leq n$. As defined by definition of prime labelling,
gcd{ f(u), f (vi) } = 1, $1 \leq i \leq n$
gcd{ f(v_1), f (v_n) } = 1.
Thus, G admits prime labeling. ∴ G is a prime labelling graph.

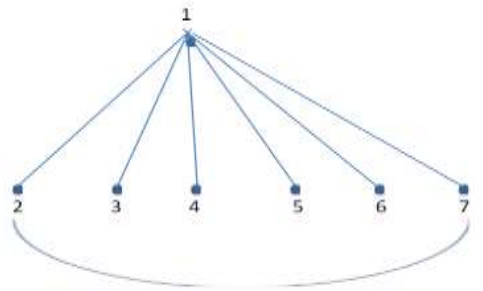

Figure 2. Vertices v_1 and v_6 joined by an edge of $k_{1,6}$ graph.

Theorem 2.3
Vertices joined by an edge between two copies of $K_{1,n}$ graph admits prime labelling (Vertices v_1 to v_1').

Proof:
Let G be two copies of $K_{1,n}$ graph. The vertices of V (K_1) = u and v_i, $1 \leq i \leq n$ be the 'n' vertices adjacent to u is the first copy of $K_{1,n}$ graph. The vertices of V (K_1) = u' and v'_i, $1 \leq i \leq n$ be the 'n' vertices adjacent to u' is the second copy of $K_{1,n}$ graph. Now join by an edge between the vertices v_1 to v'_1 .Then V (G) = 2 (n+1).
Define a function f: V(G) $\rightarrow$ {1, 2, ………. 2 (n+1) } by
f(u) = 1;
f(v_i) = 2i ; $1 \leq i \leq n$.
f(u') = 2;
f (v'_i) = 2i+1 ; $1 \leq i \leq n$.
As defined by definition of prime labelling,
gcd{ f(u), f (v_i) } = 1, $1 \leq i \leq n$.
gcd{ f(u'), f (v'_i) } = 1, $1 \leq i \leq n$.
gcd{ f(v_1), f (v'_1) } = 1,
Thus, G admits prime labeling. ∴ G is a prime labelling graph.

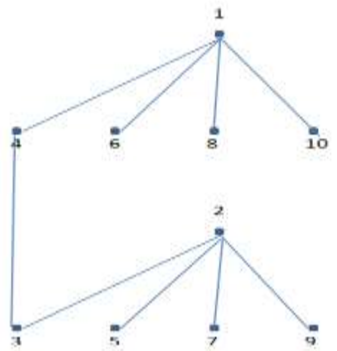

Figure 3. Vertices v_1 and v_1' joined by an edge of two copies of $k_{1,4}$.

Theorem 2.4
Vertices joined by an edge between two copies of $K_{1,n}$ graph admits prime labelling graph
(Vertices v_1 and v_i',$1 \leq i \leq n$)
Proof:
Let G be two copies of $K_{1,n}$ graph. The vertices of V (K_1) = u and v_i, $1 \leq i \leq n$ be the 'n' vertices adjacent to u is the first copy of $K_{1,n}$ graph. The vertices of V (K_1) = u' and v_i', $1 \leq i \leq n$ be the 'n' vertices adjacent to u' is the second copy of $K_{1,n}$ graph. Now join by an edge between the vertex v_1 to v'_i , $1 \leq i \leq n$. Then V (G) = 2 (n+1).
Define a function f: V (G) $\rightarrow$ {1, 2, 2 (n+1) } by
f(u) = 1;
f(v_i) = 2+2i ; $1 \leq i \leq n$.
f(u') = 2;
f ($v_{i'}$) = 2i+1 ; $1 \leq i \leq n$.
As defined by definition of prime labelling,
gcd{ f(u), f (v_i) } = 1, $1 \leq i \leq n$.
gcd{ f(u'), f (v_i') } = 1, $1 \leq i \leq n$.
gcd{ f(v_1), f (v'_i) } = 1, $1 \leq i \leq n$.
Thus, G admits prime labeling. ∴ G is a prime labelling graph.

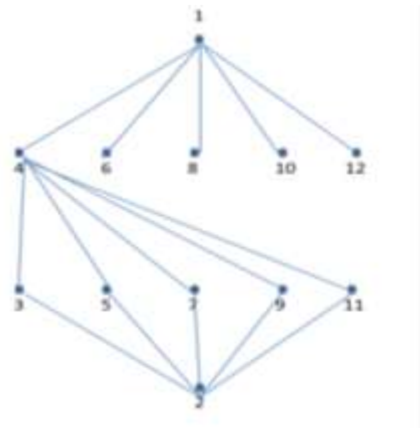

Figure 4. Vertices v_1 and v_i', ($1 \leq i \leq 5$) joined by edges of two copies of $k_{1,5}$ graph.

Theorem 2.5
Form a cycle C_m (m even) at the vertex v_1 of $K_{1,n}$ admits prime labelling graph.

Proof:
Let G be $K_{1,n}$ graph. The vertices of V (K_1) = u and v_i, $1 \leq i \leq n$ be the 'n' vertices adjacent to u. Now form a cycle C_m (m even) at v_1 of $K_{1,n}$ graph. Let V (C_m)= $v'_1v'_2$ …… $v'_mv'_1$ and the vertex $v_1 = v_1$'. Then V (G) = m + n.
Define a function f:V(G) $\rightarrow$ {1, 2 ………. (m+n) } by
f(u) = 1;
f(v_1) = f (v_1') = 2
f(v_i') = 1+i ; $2 \leq i \leq m$. (m even)
f(v_i) = m+i, $2 \leq i \leq n$
As defined by definition of prime labelling,
gcd{ f(u), f (v_i) } = 1, $1 \leq i \leq n$.
gcd{ f (v_i'), (v'_{i+1} } = 1, $1 \leq i \leq m-1$.
gcd{ f(v'_m), f (v'_1) } = 1,

Thus, G admits prime labeling. ∴ G is a prime labelling graph.

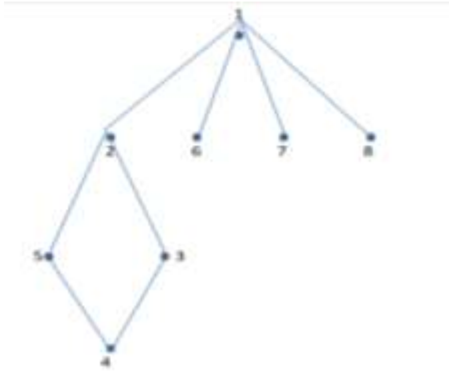

Figure 5 Forming cycle c_4 at v_1 of $k_{1\cdot4}$ graph.

Theorem 2.6

Form a cycle C_m at vertex K_1 of $K_{1,n}$ admits prime labelling graph.

Proof:

Let G be a $K_{1,n}$ graph. The vertices of V (K_1) = u and v_i, $1 \leq i \leq n$ be the 'n' vertices adjacent to u. Now form a cycle C_m at k_1 of $K_{1,n}$. Let V (C_m) = $v'_1v'_2$ ……… $v'_mv'_1$ and the vertex V (K_1) = u = v_1'. Then, V (G) = m+n.

Define a function f: V (G) {1, 2 (m+n) }by

$f(u) = 1 = f(v'_1)$

$f(v'_i) = i;\ 2 \leq i \leq m$

$f(v_i) = m+i,\ 2 \leq i \leq n$

As defined by definition of prime labelling,

gcd{ f(u), f (v_i) } = 1, $1 \leq i \leq n$.

gcd{ $f(v_i')$, $(v'_{i+1}$ } = 1, $1 \leq i \leq m-1$.

gcd{ $f(v'_m)$, f (v'_1) } = 1,

Thus, G admits prime labeling. ∴ G is a prime labelling graph.

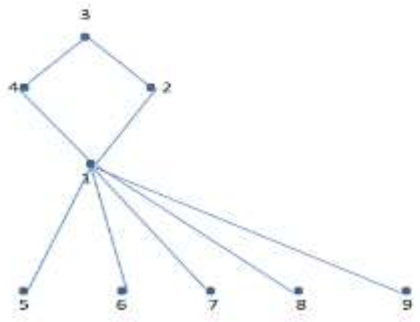

Figure 6. Formingacycle c_4 at k_1 of $k_{1,5}$ graph.

Theorem 2.7

Form a cycle C_m with vertex v_1 of F_n admits prime labelling graph.

Proof:

Let G be F_n graph. The vertices of F_n is $v_1v_2v_3$……….$v_nv_{n+1}v_1$, where v_1 is adjacent to v_2v_3….v_{n+1} and v_{i+1} is adjacent to v_{i+2} $(1 \leq i \leq n-1)$. Now form a cycle C_m at v_1 of F_n graph. Let V (C_m) = $v'_1v'_2$ …… $v'_mv'_1$ and the vertex $v_1 = v_1'$. Then, V (G) = m + n.

Define a function f: V (G) {1, 2 (m+n) } by

$f(v_1) = f(v_1') = 1$

$f(v'_i) = 1+i\ ;\ 2 \leq i \leq m$.

$f(v_i) = m+i,\ 1 \leq i \leq n$ .As defined by definition of prime labelling,

gcd{ $f(v'_i)$, $(v'_{i+1}$ } = 1, $1 \leq i \leq m-1$.

gcd{ $f(v'_m)$, $f(v'_1)$ } = 1.

gcd{ $f(v_1)$, $f(v_{i+1})$ } = 1, $1 \leq i \leq n$.

gcd{ $f(v_i)$, $f(v_{i+1})$ } = 1, $2 \leq i \leq n$.

Thus, G admits prime labeling. ∴ G is a prime labelling graph

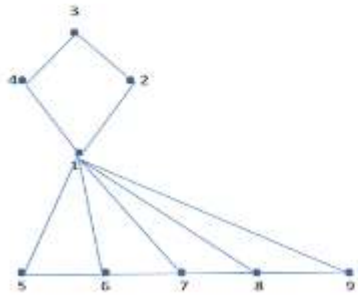

Figure 7. forming a cycle c_4 at v_1 of F_5 graph.

REFERENCES

1. F. Harary. Graph Theory, Addison Wesley, Reading Mass 1972.
2. J.A. Gallian. A dynamic survey of graph labeling. Electronic J. combinatorics 17 (2014).
3. S.K. Vaidya and K.K. Kanani. Prime labeling for some cycle related graphs. Journal of Mathematics Research. Vol.2, No. 2010. 98-103.
4. S. Meena and K. Vaithilingam. Prime labeling for some helm related graphs. International Journal of Innovative Research in Science, Engineering and Technology, 2(4)(2013).
5. S. Ashok kumar and S. Maragathavalli. Prime labeling of special graphs. IOSR, Journal of Mathematics (IOSR-JM), Vol.11 (2015), 01-05.
6. S.K. Vaidya and Udaya M. Prajapati. Some new results on prime graph. Open Journal of Discrete Mathematics. (2012), 99-104.
7. S. Meena, A. Ezhil. Total prime labeling of some graphs, International journal of research In Advent Technology Research, 7(1) (2019), 1–9.
8. S. Meena, A. Ezhil. Total prime labeling of some cycle and path related graphs, Journal of Emerging Technologies and Innovative Research, 6(4) (2019), 685–693.
9. S. Meena, A. Ezhil. Total prime labeling of some subdivision graphs, AIP conference proceedings, 2177 (2019), 020044-1 – 020044-10.
10. S. Meena, A. Ezhil. Total prime labeling of barycentric subdivision of cycle graphs, Journal of critical reviews, ISSN-2394-5125, Vol 7, ISSUE 05,2020.

11. S. Meena, A. Ezhil. Total prime of labeling of certain types of graphs, Advances in Mathematics: Scientific Journal 9 (2020), no.11, 9685–9689 ISSN: 1857-8365 (printed); 1857-8438 (electronic) https://doi.org/10.37418/amsj.9.11.75 Spec. Iss. on ICRTAMS-2020.
12. [12] S. Meena, A. Ezhil. Total prime of labeling of some special graphs, Advances in Mathematics: Scientific Journal 9 (2020), no.11, 9647–9651 ISSN: 1857-8365 (printed); 1857-8438 (electronic) https://doi.org/10.37418/amsj.9.11.70 Spec. Iss. on ICRTAMS-2020.