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Solving Two Topical Problems

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ABSTRACT
The Goldbach-Euler binary problem is formulated as follows: Any even number, starting from 4, can be represented as the sum of two primes. The ternary Goldbach problem is formulated as follows: Every odd number greater than 7 can be represented as the sum of three odd primes, which was finally solved in 2013. In 1995, Olivier Ramare proved that any even number is the sum of no more than 6 primes.[9] The second problem is about the infinity of twin primes. The author carries out the proof by the methods of elementary number theory.

KEYWORDS: solving, problems, in, number, theory

1. INTRODUCTION, LITERATURE REVIEW AND SCOPE OF WORK

In 1742, the Prussian mathematician Christian Goldbach sent a letter to Leonard Euler, in which he made the following conjecture: Every odd number greater than 5 can be represented as the sum of three prime numbers [1; 2; 3; 4]. Euler became interested in the problem and put forward a stronger conjecture: Every even number greater than two can be represented as the sum of two prime numbers. The first statement is called the ternary Goldbach problem, the second the binary Goldbach problem (or Euler problem). From the validity of the ternary Goldbach conjecture (proved in 2013 by year) it follows that any even number is a sum of at most 4 numbers [6]. As of July 2008, Goldbach's binary conjecture has been tested for all even numbers not exceeding 1.2 × × 1018 [2]. The binary Goldbach conjecture can be reformulated as statement about the unsolvability of a Diophantine equation of the 4th degree some special kind [5; 6]. The question of whether there are infinitely many twin primes has been one of the most open questions in theory numbers for many years. This is the content of the twin prime conjecture, which states that there are infinitely many primes p such that that p+2 is also prime. In 1849 de Polignac advanced more the general conjecture that for every natural number k there exists infinitely many primes p such that p + 2k is also simple. [7] The case k = 1 of the de.

2. CONTENT

Note: under prime numbers, so as not to repeat, further implied odd prime numbers.The numbers under p are not critical.

Under the value N, N_1 is an uninterrupted series of integers from the starting value to infinity.

Below the value of K is a fixed integer.

Task 1. Binary Goldbach-Euler problem

Lemma1. The difference of the sum of two primes and a prime is any odd number, starting from 9.

Based on the solved Goldbach-Euler ternary problem:

$$p_1+p_2+p_3+2=p_4+p_5+p_6 \quad (01)$$

After transferring primes, we have:

$$p_1+p_2-p_6+2=p_4+p_5-p_3 \quad (02)$$

From what follows:

$$p_1+p_2-p_3=2N_1+1 \quad (03)$$

where $N \geq 4$

Thus, any odd number starting from 9 has at least two representation. One is the sum of three prime numbers, the other is the difference of the sum of two primes and one prime number.

Lemma2. Any even number starting from 12 is representable as a sum four odd prime numbers.

1. For the first even number 12 = 3+3+3+3.

We allow justice for the previous N> 5:

$$p_1 + p_2 + p_3 + p_4 = 2N \quad (04)$$

We will add to both parts on 1

$$p_1 + p_2 + p_3 + p_4 + 1 = 2N + 1 \quad (05)$$

where on the right the odd number also agrees

$$p_1 + p_2 + p_3 + p_4 + 1 = p_5 + p_6 + p_7 \quad (06)$$

Having added to both parts still on 1

$$p_1 + p_2 + p_3 + p_4 + 2 = p_5 + p_6 + p_7 + 1 \quad (07)$$

We will unite $p_6 + p_7 + 1$ again we have some odd number,

which according to finally solved ternary Goldbach problemwe replace with the sum of three simple and as a result we receive:

$$p_1 + p_2 + p_3 + p_4 + 2 = p_5 + p_6 + p_7 + p_8 \quad (08)$$

at the left the following even number is relative , and on the right the sum four prime numbers.

$$p_1 + p_2 + p_3 + p_4 = 2N \quad (09)$$

Thus obvious performance of an inductive mathematical method.

As was to be shown.

Corollary1: Possible value of one of the four primes odd equals in the sum of 2N from 3 to 2N-9 inclusive.

This follows from the finally solved Goldbach's ternary problem.

So the sum of four primes is:

$$p_1 + p_2 + p_3 + p_4 = 2N_1 + 1 + p_4 = 2\ N \quad (10)$$

where $2N_1 + 1$ the sum of three primes, $N_1 \geq 4 \quad N \geq 6$

Theorem1. The sum of four primes is equal to the sum of two primes and any even

numbers starting with 12.

According to Lemma1:

$$p_1 + p_2 + p_3 = p_5 + p_6 - p_4 = 2N_1 + 1 \quad (11)$$

$$p_1 + p_2 + p_3 + p_4 = p_5 + p_6 = 2N_1 + p_4 = 2N \quad (12)$$

watch corollary1 (10).

Further from Lemma2, namely (8) applying the method by contradiction we

have:

$$p_5 + p_6 \neq p_1 + p_2 + p_3 + p_4 \quad (13)$$

$$p_5 + p_6 - p_4 \neq p_1 + p_2 + p_3 \quad (14)$$

Which is not admissible by Lemma 1. This implies:

$$p_1 + p_2 + p_3 + p_4 = p_5 + p_6 = 2\ N \quad (15)$$

where N is a continuous series of integers and even numbers 2 N , starting from 12.

Up to 12 we show arithmetically.

Corollary 2: The difference of two odd prime numbers is any even number.

From what follows:

$$p_1 + p_2 = p_3 + p_4 + 2 \quad (16)$$

$$p_1 - p_3 = p_4 - p_2 + 2 \quad (17)$$

and we assert:

$$p_1 - p_2 = 2\ N \quad (18)$$

where N=1,2... ∞

Corollary 3. If the sum of four simple odd , then the sum two-even number from 6 to 2N-6 inclusive.

What follows from the solved Goldbach-Euler conjecture.

Task 2.

Twin primes are infinite

Theorem2. Starting from 14, even numbers are the sum of two odd primes not

less than two different representations.

$$p_1 + p_2 + p_3 + p_4 = p_1 + p_5 = 2N \quad (01)$$

$$p_2 + p_3 + p_4 = p_5 \qquad (02)$$

Assume by analogy with (02):

$$p_1 + p_3 + p_4 = p_6 \qquad (03)$$

add up (02)+(03):

$$p_5 + p_6 = p_1 + p_2 + 2(p_3 + p_4) \qquad (04)$$

according to theorem 1:

$$2(p_3 + p_4) = p_7 + p_8 = 4K \qquad (05)$$

where 4K is a fixed even number.

and

$$p_5 + p_6 = p_1 + p_2 + p_7 + p_8 \qquad (06)$$

and further :

$$p_5 + p_6 - p_7 = p_1 + p_2 + 4K - p_7 \qquad (07)$$

$$p_1 + p_2 + 2p_3 + 2p_4 = p_1 + p_2 + 4K \qquad (08)$$

and finally:

$$p_3 + p_4 = 2K \qquad (09)$$

corresponds to Theorem 2, which confirms Assumption (03).
(03),(04) - inequality in case (09) is not equal to the corresponding certain even number 2K with respect to 2N. However redistribution by replacing simple p_1, p_2, p_3, p_4 we find an even 2K for $p_1 \neq p_2$, which means two representations by the sum of two prime for even 2N.

Let's say $p_1 = p_2$, then $2p_5 = 2N$. Introducing an even through the sum of four simple ones:

$$p_5 + p_1 + p_3 + p_4 = 2p_5 \qquad (10)$$

$$p_5 = p_1 + p_3 + p_4 \qquad (11)$$

$$p_6 = p_5 + p_3 + p_4 \qquad (12)$$

Thus we have $p_5 \neq p_6$ и $p_1 \neq p_2$.

The second representation would be absent if there were even numbers that cannot be represented as the sum of two prime numbers.

From this follows:

$$p_5 + p_1 = p_6 + p_2 = 2N \qquad (13)$$

where N= 7, 8, 9,10.., ∞

As a result, even numbers starting with 16 are the sum of two prime numbers, at least than two presentations. Up to 16 we determine arithmetically -6,8,12 in one presentations. Hence the values of N.

Corollary 4: The number of twins is infinite.

Corollary 4 is a special case of the above theorem

Let p_1, p_2 a pair of twins. Then according to (13) p_5, p_6 inevitably next set of twins. Next, instead of p_1, p_2 , we substitute in (13) p_5, p_6 we have the next pair, etc. So the process is endless and there is no finite pair of twins!

Corollary 5: A prime number starting at 5 is the arithmetic mean of two

simple .

According to Theorem 2:

$$p_1 + p_2 = 2p_3 \qquad (14)$$

where is one representation of an even number, indicating different values in (02) and

(03).

And the second representation of an even number:

$$p_3 + p_3 = 2p_3 \qquad (15)$$

and (11) confirms the infinity of primes!

3. CONCLUSION

the solution of these problems opens up opportunities for solving a number of

problems in number theory.

REFERENCES

1. Terence Tao Google+- Busy day in analytic number theory; – Harald. Helfgott has…
2. Major arcs for Goldbach`s, H.A. Helfgott // arxiv 1305.2897
3. Goldbach Variations// SciAm blogs, Evelyn Lamb, May 15, 2013.
4. Two Proofs Spark a Prime Week for Number Theory // Science 24 May, 2013.
5. Vol. 340 no. 6135 p. 913. Doi:10.1126/Science.340.6135.913
6. Weisstein, Eric W. Goldbach Conjecture(English.)on the site Wolfram MathWorld.
7. Yuri Matiyasevich. Hilbert's Tenth Problem: What was done and what is to be done.
8. de Polignac A. Recherches nouvelles sur les nombres premiers. 1849 .