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Modified Domination Sombor Index and its Exponential of a Graph

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ARTICLE INFO	ABSTRACT
Published Online: 12 August 2023 Corresponding Author: **V. R. Kulli**	In this paper, we introduce the modified domination Sombor index and its corresponding exponential of a graph. Also we introduce the domination Sombor exponential of a graph. We compute the modified domination Sombor index and its corresponding exponential for some standard graphs, French windmill graphs, friendship graphs and book graphs. Furthermore, we establish some properties of the domination Sombor index.

KEYWORDS: modified domination Sombor index, modified domination Sombor exponential, domination Sombor exponential, graph.

I. INTRODUCTION

Let G be a simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. We refer [1, 2] for undefined term and notation.

A molecular graph is a simple graph, representing the carbon atom skeleton of an organic molecule of the hydrocarbon. Therefore the vertices of a molecular graph represent the carbon atoms and its edges the carbon-carbon bonds. Chemical Graph Theory is a branch of Mathematical Chemistry which has an important effect on the development of Chemical Sciences. Several graph indices have found some applications in Chemistry, especially in QSPR/QSAR research [3, 4].

The domination degree $d_d(u)$ of a vertex u [5] in a graph G is defined as the number of minimal dominating sets of G which contains u.

The modified first domination Zagreb index [5] of a graph is defined as

$$DM_1^*(G) = \sum_{uv \in E(G)} \left(d_d(u) + d_d(v)\right).$$

Ref. [5] was soon followed by a series of publications [6, 7, 8, 9, 10, 11, 12, 13].

The modified forgotten domination index [5] of a graph is defined as

$$DF^*(G) = \sum_{uv \in E(G)} \left(d_d^2(u) + d_d^2(v)\right).$$

Recently, the so-called domination Sombor index was put forward, defined as [14]

$$DSO(G) = \sum_{uv \in E(G)} \sqrt{d_d(u)^2 + d_d(v)^2}.$$

Considering the domination Sombor index, we define the domination Sombor exponential of a graph G as

$$DSO(G,x) = \sum_{uv \in E(G)} x^{\sqrt{d_d(u)^2 + d_d(v)^2}}.$$

Recently, some Sombor indices were studied, for example, in [15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30].

We propose the modified domination Sombor index of a graph G and defined it as

$$^m DSO(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_d(u)^2 + d_d(v)^2}}.$$

Considering the modified domination Sombor index, we define the modified domination Sombor exponential of a graph G as

$$^m DSO(G,x) = \sum_{uv \in E(G)} x^{\frac{1}{\sqrt{d_d(u)^2 + d_d(v)^2}}}.$$

Recently, some domination indices were studied, for example, in [31, 32, 33, 34, 35, 36].

In this paper, we compute the modified domination Sombor index for some standard graphs, French windmill graphs, friendship graphs. Also we compute the domination Sombor exponential and modified domination Sombor exponential for some standard graphs, French windmill graphs, friendship graphs and book graphs. Furthermore, we establish some properties of the domination Sombor index.

II. RESULTS FOR SOME STANDARD GRAPHS

Proposition 1. If K_n is a complete graph with *n* vertices, then $^{m}DSO(K_n)=\frac{n(n-2)}{2\sqrt{2}}$.

Proof: If K_n is a complete graph, then $d_d(u)$ =1.

From definition, we have

$$^{m}DSO(G)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_d(u)^2+d_d(v)^2}}$$

$$=\frac{n(n-1)}{2}\frac{1}{\sqrt{1^2+1^2}}=\frac{n(n-1)}{2\sqrt{2}}.$$

Proposition 2. If S_{n+1} is a star graph with $d_d(u)$ =1, then

$$^{m}DSO(S_{n+1})=\frac{n}{\sqrt{2}}.$$

Proposition 3. If $S_{p+1,q+1}$ is a double star graph with $d_d(u)$ =2, then

$$^{m}DSO(S_{p+1,q+1})=\frac{p+q+1}{2\sqrt{2}}.$$

Proposition 4. Let $K_{m,n}$ be a complete bipartite graph with $2 \leq m \leq n$. Then

$$^{m}DSO(K_{m,n})=\frac{mn}{\sqrt{m^2+n^2+2(m+n+1)}}.$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with *m* + *n* vertices and *mn edges* such that $|V_1|= m$, $|V_2|= n$, $V(K_{r,s}) = V_1$ □ V_2 for $1 \leq m \leq n$, and $n \geq 2$. Every vertex of V_1 is incident with *n* edges and every vertex of V_2 is incident with *m* edges. Then

$d_d(u)$ =*m*+1,

=*n*+1, for all $u \in V(K_{m,n})$.

From definition, we have

$$^{m}DSO(K_{m,n})=\sum_{uv\in E(K_{m,n})}\frac{1}{\sqrt{d_d(u)^2+d_d(v)^2}}$$

$$=\frac{mn}{\sqrt{(m+1)^2+(n+1)^2}}=\frac{mn}{\sqrt{m^2+n^2+2(m+n+1)}}.$$

In the following proposition, by using definitions, we obtain the modified domination Sombor exponential of K_n, S_{n+1}, $S_{p+1,q+1}$ and $K_{m,n}$.

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Proposition 5. The modified domination Sombor exponential of K_n, S_{n+1}, $S_{p+1,q+1}$ and $K_{m,n}$ are given by

(i) $$^{m}DSO(K_n,x)=\sum_{uv\in E(G)}x^{\frac{1}{\sqrt{d_d(u)^2+d_d(v)^2}}}$$

$$=\frac{n(n-1)}{2}x^{\frac{1}{\sqrt{1^2+1^2}}}=\frac{n(n-1)}{2}x^{\frac{1}{\sqrt{2}}}.$$

(ii) $$^{m}DSO(S_{n+1},x)=nx^{\frac{1}{\sqrt{2}}}.$$

(iii) $$^{m}DSO(S_{p+1,q+1},x)=(p+q+1)x^{\frac{1}{2}}.$$

(iv) $$^{m}DSO(K_{m,n},x)=mnx^{\frac{1}{\sqrt{m^2+n^2+2(m+n+1)}}}.$$

In the following proposition, by using definitions, we obtain the domination Sombor exponential of K_n, S_{n+1}, $S_{p+1,q+1}$ and $K_{m,n}$.

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Proposition 6. The domination Sombor exponential of K_n, S_{n+1}, $S_{p+1,q+1}$ and $K_{m,n}$ are given by

(i) $$DSO(K_n,x)=\sum_{uv\in E(G)}x^{\sqrt{d_d(u)^2+d_d(v)^2}}$$

$$=\frac{n(n-1)}{2}x^{\sqrt{1^2+1^2}}=\frac{n(n-1)}{2}x^{\sqrt{2}}.$$

(ii) $$DSO(S_{n+1},x)=nx^{\sqrt{2}}.$$

(iii) $$DSO(S_{p+1,q+1},x)=(p+q+1)x^{2\sqrt{2}}.$$

(iv) $$DSO(K_{m,n},x)=mnx^{\sqrt{m^2+n^2+2(m+n+1)}}.$$

III. RESULTS FOR FRENCH WINDMILL GRAPHS

The French windmill graph F_n^m is the graph obtained by taking *m* □ 3 copies of K_n, *n* □ 3 with a vertex in common [18]. The graph F_n^m is presented in Figure 4. The French windmill graph F_3^m is called a friendship graph.

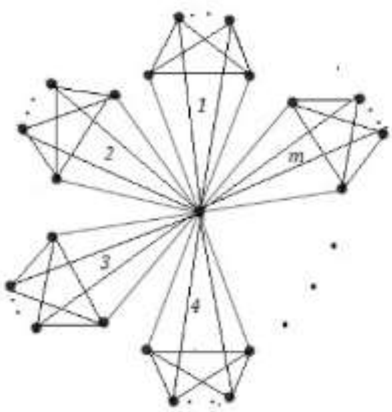

Figure 4. French windmill graph F_n^m

Let *F* be a French windmill graph F_n^m . Then

$d_d(u)$ =1, if *u is in center*

$= (n-1)^{m-1}$, otherwise.

Theorem 1. Let F be a French windmill graph F_n^m. Then

$$ {}^m DSO(F) = \frac{m(n-1)}{\sqrt{1+(n-1)^{(m-1)2}}} + \frac{[(mn(n-1)/2) - m(n-1)]}{(n-1)^{(m-1)}\sqrt{2}}. $$

Proof: In F, there are two sets of edges. Let E_1 be the set of all edges which are incident with the center vertex and E_2 be the set of all edges of the complete graph. Then

$$ {}^m DSO(F) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_d(u)^2 + d_d(v)^2}} $$

$$ = \sum_{uv\in E_1(G)} \frac{1}{\sqrt{d_d(u)^2 + d_d(v)^2}} + \sum_{uv\in E_2(G)} \frac{1}{\sqrt{d_d(u)^2 + d_d(v)^2}} $$

$$ = \frac{m(n-1)}{\sqrt{1^2+(n-1)^{(m-1)2}}} + \frac{[(mn(n-1)/2) - m(n-1)]}{\sqrt{(n-1)^{(m-1)2}+(n-1)^{(m-1)2}}} $$

$$ = \frac{m(n-1)}{\sqrt{1+(n-1)^{(m-1)2}}} + \frac{[(mn(n-1)/2) - m(n-1)]}{(n-1)^{(m-1)}\sqrt{2}}. $$

Corollary 1.1. Let F_3^m be a friendship graph. Then

$$ {}^m DSO(F_3^m) = \frac{2m}{\sqrt{1+2^{(m-1)2}}} + \frac{m(n-2)}{2^{(m-1)}\sqrt{2}}. $$

In the following theorem, by using definition, we obtain the modified domination Sombor exponential of a French windmill graph F_n^m.

Theorem 2. The modified domination Sombor exponential of F_n^m is given by

$$ {}^m DSO(F_n^m, x) = m(n-1)\, x^{\frac{1}{\sqrt{1+(n-1)^{(m-1)2}}}} + [(mn(n-1)/2) - m(n-1)]\, x^{\frac{1}{(n-1)^{(m-1)}\sqrt{2}}} $$

In the following theorem, by using definition, we obtain the domination Sombor exponential of a French windmill graph F_n^m.

Theorem 3. The domination Sombor exponential of F_n^m is given by

$$ DSO(F_n^m, x) = m(n-1)\, x^{\sqrt{1+(n-1)^{(m-1)2}}} + [(mn(n-1)/2) - m(n-1)]\, x^{(n-1)^{(m-1)}\sqrt{2}}. $$

In the following theorem, by using definition, we obtain the modified domination Sombor index of a friendship graph F_3^m.

Theorem 4. The modified domination Sombor index of F_3^m is given by

$$ {}^m DSO(F_3^m) = \frac{2m}{\sqrt{1+2^{(m-1)2}}} + \frac{m}{2^{(m-1)}\sqrt{2}}. $$

In the following theorem, by using definition, we obtain the modified domination Sombor exponential of a friendship graph F_3^m.

Theorem 5. The modified domination Sombor exponential of F_3^m is given by

$$ {}^m DSO(F_3^m, x) = 2mx^{\frac{1}{\sqrt{1+2^{(m-1)2}}}} + mx^{\frac{1}{2^{(m-1)}\sqrt{2}}}. $$

In the following theorem, by using definition, we obtain the domination Sombor exponential of obtain a friendship graph F_3^m.

Theorem 6. The domination Sombor exponential of F_3^m is given by

$$ DSO(F_3^m, x) = 2mx^{\sqrt{1+2^{(m-1)2}}} + mx^{2^{(m-1)}\sqrt{2}}. $$

IV. RESULTS FOR B_n

The book graph B_n, $n\geq 3$, is a cartesian product of star S_{n+1} and path P_2.

For B_n, $n\geq 3$, we have

$d_d(u) = 3$, if u is the center vertex,

$= 2^{n-1}+1$, otherwise.

Theorem 7. If B_n, $n\geq 3$, is a book graph, then

$$ {}^m DSO(B_n) = \sqrt{6} + 2n\sqrt{4+2^{n-1}} + n\sqrt{2(2^{n-1}+1)}. $$

Proof: In B_n, there are three types of edges as follow:

$E_1 = \{uv \in E(B_n) \mid d_d(u)=d_d(v)=3\}$, $|E_1| = 1$

$E_2 = \{uv \in E(B_n) \mid d_d(u) = 3, d_d(v)= 2^{n-1}+1\}$, $|E_2| = 2n$.

$E_3 = \{uv \in E(B_n) \mid d_d(u) = d_d(v)= 2^{n-1}+1\}$, $|E_3| = n$.

By definition, we have

$$ {}^m DSON(B_n) = \sum_{uv\in E(G)} \frac{1}{\sqrt{d_d(u)^2 + d_d(v)^2}} $$

$$=1\frac{1}{\sqrt{3^2+3^2}}+2n\frac{1}{\sqrt{3^2+\left(2^{n-1}+1\right)^2}}$$

$$+n\frac{1}{\sqrt{\left(2^{n-1}+1\right)^2+\left(2^{n-1}+1\right)^2}}$$

$$=\frac{1}{\sqrt{18}}+\frac{2n}{\sqrt{10+2^{2(n-1)}+2^n}}+\frac{n}{\left(2^{n-1}+1\right)\sqrt{2}}.$$

In the following theorem, by using definition, we obtain the modified domination Sombor exponential of B_n.

Theorem 8. The modified domination Sombor exponential of B_n is given by

$$^{m}DSO\left(B_n,x\right)=x^{\frac{1}{\sqrt{18}}}+2nx^{\frac{1}{\sqrt{10+2^{2(n-1)}+2^n}}}+nx^{\frac{1}{\left(2^{n-1}+1\right)\sqrt{2}}}.$$

In the following theorem, by using definition, we obtain the domination Sombor exponential of B_n.

Theorem 9. The domination Sombor exponential of B_n is given by

$$^{m}DSO\left(B_n,x\right)=x^{\sqrt{18}}+2nx^{\sqrt{10+2^{2(n-1)}+2^n}}+nx^{\left(2^{n-1}+1\right)\sqrt{2}}.$$

V. RESULTS FOR GoK_p

Theorem 10. Let $H=GoK_p$, where G is a connected graph with n vertices and m edges; and K_p is a complete graph. Then

$$^{m}DSO(H)=\frac{2m+np^2+np}{2\sqrt{2}\left(p+1\right)^{(n-1)}}.$$

Proof: If $H=GoK_p$, then $d_d(u)=\left(p+1\right)^{n-1}$. In K_p, there are $\frac{p(p-1)}{2}$. edges. Thus H has $\frac{1}{2}(2m+np^2+np)$ edges. Thus

$$^{m}DSO(H)=\sum_{uv\in E(G)}\frac{1}{\sqrt{d_d(u)^2+d_d(v)^2}}$$

$$=\frac{1}{2}(2m+np^2+np)\frac{1}{\sqrt{\left(p+1\right)^{2(n-1)}+\left(p+1\right)^{2(n-1)}}}$$

$$=\frac{2m+np^2+np}{2\sqrt{2}\left(p+1\right)^{(n-1)}}.$$

In the following theorem, by using definition, we obtain the modified domination Sombor exponential of GoK_p.

Theorem 11. The modified domination Sombor exponential of GoK_p is given by

$$^{m}DSO\left(GoK_p,x\right)=\frac{1}{2}(2m+np^2+np)x^{\frac{1}{\sqrt{2}\left(p+1\right)^{(n-1)}}}.$$

In the following theorem, by using definition, we obtain the domination Sombor exponential of GoK_p.

Theorem 12. The domination Sombor exponential of GoK_p is given by

$$DSO\left(GoK_p,x\right)=\frac{1}{2}(2m+np^2+np)x^{\sqrt{2}\left(p+1\right)^{(n-1)}}.$$

VI. PROPERTIES OF DOMINATION SOMBOR INDEX

Theorem 13. Let G be a connected graph with m edges. Then

$$\frac{1}{\sqrt{2}}DM_1^*(G)\leq DSO(G)\leq DM_1^*(G).$$

Proof: For any two positive numbers a and b,

$$\frac{1}{\sqrt{2}}(a+b)\leq\sqrt{(a^2+b^2)}\leq a+b.$$

For $a=d_d(u)$ and $b=d_d(v)$, the above inequalities transform into

$$\frac{1}{\sqrt{2}}\left(d_d(u)+d_d(v)\right)\leq\sqrt{d_d(u)^2+d_d(v)^2}$$

$$\leq d_d(u)+d_d(v)$$

Now, we obtain

$$\frac{1}{\sqrt{2}}\sum_{uv\in E(G)}\left(d_d(u)+d_d(v)\right)\leq\sum_{uv\in E(G)}\sqrt{d_d(u)^2+d_d(v)^2}$$

$$\leq\sum_{uv\in E(G)}\left(d_d(u)+d_d(v)\right)$$

with the help of definitions, we arrive the desired result.

Theorem 14. Let G be a connected graph with m edges. Then

$$DSO(G)\leq\sqrt{mDF^*(G)}.$$

Proof: Using the Cauchy-Schwarz inequality, we obtain

$$\left(\sum_{uv\in E(G)}\sqrt{d_d(u)^2+d_d(v)^2}\right)^2$$

$$\leq\sum_{uv\in E(G)}1\sum_{uv\in E(G)}\left(d_d(u)^2+d_d(v)^2\right).$$

$$=mDF^*(G).$$

Thus

$$DSO(G)\leq\sqrt{mDF^*(G)}.$$

VII. CONCLUSION

In this paper, we have introduced the modified domination Sonbor index and its corresponding exponential of a graph. Also we have introduced the domination Sonbor exponential of a graph. We have computed newly defined modified domination Sombor index and exponentials for some standard graphs, French windmill graphs, friendship graphs. We have established some properties of the domination Sombor index.

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