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Ricci Solitons on α-Para Kenmotsu Manifolds with Semi Symmetric Metric Connection

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ABSTRACT
In this paper we introduce notion of Ricci solitons in α-para Kenmotsu manifold with semi - symmetric metric connection. We have found the relations between curvature tensor, Ricci tensors and scalar curvature of α-para Kenmotsu manifold with semi-symmetic metric connection.We have proved that 3-dimensional α-para Kenmotsu manifold with semi -symmetric metric connection is an η-Einstein manifold and the Ricci soliton defined on this manifold is named expanding and steady with respect to the value of λ constant.It is proved that Conharmonically flat α-para Kenmotsu manifold with semi-symmetric metric connection is η-Einstein manifold.

KEYWORDS: Curvature tensor, Ricci solitons, semi-symmetric metric connection.
Mathematics Subject Classification:.53C15,53C25,53C40,53C50

1 INTRODUCTION

In 1972 Kemmotsu[18] studied a class of contact Riemannian manifolds satisfying some special conditions and this manifolds is known as Kenmotsu manifold.Sharma and Sinha[15] started to study of the Ricci solitons in contact geometry in 1983.Later Mukut Mani Tripathi,Cornelia Livia Bejan and Mircea Crasmareanu[3],[17] and others extensively studied Ricci solitons in contact metric manifolds.In 1985, almost paracontact geometry was introduced by kaneyuki and williams[7] and then it was continued by many authors. Nagaraja ve premalatha[11] studied exclusively about Ricci solitons on Kenmotsu manifold in 2012 .Agashe and Chafle ,Liang,pravonovic and Sengupta,Yildiz and Cetinkaya [1],[9],[12],[14] and [19] studied semi-symmetric non-metric connection in different ways

A systematic study of almost paracontact metric manifolds was carried out by Zamkovoy[21].However such structures were also studied by Buchner and Rosca. Rossca and Venhecke[13]. Further almost Para-Hermitian Structure on the tangent of an almost Para-Co hermitian manifolds was studied by Bejan[3].A class of α-para kenmotsu manifolds was studied by srivastva and srivastva[16]. We can observe that the concircular curvature tensor on Pseudo-Riemannian manifold to be of constant curvature. Hayden[9] introduced Semi-symmetric linear connection on a Riemannian manifold.Let M be an n-dimensional Riemannian manifold of class C-endowed with the Riemannian metric g and ∇ be the Levi-Civita Connection on M^n. A linear connection $\bar{\nabla}$ defined on M^n is said to be semi symmetric[8] if its torsion tensor T is of the form

$$T(X,Y) = \eta(Y)X - \eta(X)Y$$

where ξ is a vector field and η is a 1-form defined by

$$g(X,\xi) = \eta(X)$$

for all vector field X $\in \chi(M^n)$ where, $\chi(M^n)$ is the set of all differentiable vector fields on M^n. A relation between the semi-symmetric metric connection $\bar{\nabla}$ and the Levi-Civita connection ∇ on M^n has been obtained by Yano[20] which is given as

$$\bar{\nabla}_X Y = \nabla_X Y + \eta(Y)X - g(X,Y)\xi \tag{1.1}$$

2 Preliminaries

A differentiable manifold M^n of dimension n is said to have an almost paracontact (ϕ,ξ,η)-structure if it admits an (1,1) tensor field ϕ,a unique vector field ξ,1-form η such that :

$$\phi^2 = I - \eta \otimes \xi,$$

$$\phi\xi = 0,$$

$$\eta \circ \phi = 0 \tag{2.1}$$

$$\eta(\xi) = 1 \tag{2.2}$$

for any vector field X,Y on M^n .The manifold M^n equipped with an almost paracontact structure (ϕ,ξ,η) is called almost paracontact manifold . In addition,if an almost paracontact manifold admits a pseudo-Riemannian metric satisfying

$$g(X,\xi) = \eta(X) \quad (2.3)$$

$$g(\phi X,\phi Y) = -g(X,Y) + \eta(X)\eta(Y) \quad (2.4)$$

$$g(\phi X,Y) = -g(X,\phi Y) \quad (2.5)$$

for any vector field X,Y on M^n,where ϕ is a (1,1) tensor field, ξ is a vector field, η is a 1-form and g is the Riemannian metric.Then M is called an almost contact manifold.For an almost contact manifold M,it follows that [9]

$$(\nabla_X \phi)Y = \nabla_X \phi Y - \phi(\nabla_X Y) \quad (2.6)$$

$$(\nabla_X \eta)Y = \nabla_X \eta(Y) - \eta(\nabla_X Y) \quad (2.7)$$

Let R be Riemann curvature tensor, S Ricci curvature tensor,Q Ricci operator we have

$$S(X,Y) = \sum_{i=1}^{n} g(R(e_i,X)Y,e_i) \quad (2.8)$$

$$QX = -\sum_{i=1}^{n} R(e_i,X)e_i \quad (2.9)$$

and

$$S(X,Y) = g(QX,Y) \quad (2.10)$$

for any vector field X,Y on M^n,then (ϕ,ξ,η,g),is called an almost paracontact metric structure and the manifold M equipped with an almost paracontact metric structure is called an almost paracontact metric manifold.Further in addition,if the structure (ϕ,ξ,η,g) satisfies

$$d\eta(X,Y) = g(X,\phi Y) \quad (2.11)$$

for any vector fields X,Y on M^n .Then the manifold is called paracontact metric manifold and the corresponding structure (ϕ,ξ,η,g) , is called a paracontact structure with the associated metric g [10]. On an almost paracontact metric manifold,the (1,2) tensor field N_ϕ defined as

$$N_\phi = [\phi,\phi] - 2d\eta \otimes \xi \quad (2.12)$$

Where $[\phi,\phi]$ is the nijenhuis tensor of ϕ.If N vanishes identically,then we say that the manifold M^n is a normal almost parametric metric manifold. The normality condition implies that the almost paracomplex structure J defined on M^n ×R

$$J(X,\lambda\frac{d}{dt}) = (\phi X + \lambda(\xi),\eta(X)\frac{d}{dt}),$$

is integrable . Here X is tangent to M^n , t is the coordinate of R and λ is a differentiable function on M^n ×R.

For an almost paracontact metric 3-dimensional manifold M^3 , the following three conditions are mutually equivalent :

(i) there exist smooth functions α,β on M^3 such that

$$(\nabla_X \phi)Y = \alpha(g(\phi X,Y)\xi - \eta(Y)\phi X) + \beta(g(X,Y)\xi) - \eta(Y)X) \quad (2.13)$$

(ii) M^3 is normal,

(iii) there exist smooth functions α,β on M^3 such that

$$\nabla_X \xi = \alpha(X - \eta(X)\xi) + \beta\phi X \quad (2.14)$$

where ∇ is Levi-Civita connection of pseudo-Riemannian metric g.

A normal almost paracontact metric 3-dimensional manifold is called

(a) Para-Cosymplectic manifold if $\alpha = \beta = 0$,

(b) quasi-para Sasakian manifold if and only if α =0 and $\beta \neq 0$,

(c) β-para Sasakian manifold if and only if α = 0 and β is a non- zero constant,in particular para-Sasakian manifold if β = -1

(d) α-para Kenmotsu manifold if α is a non-zero constant and β =0 in particular para-Kenmotsu manifold if α=1.

3 On 3-dimensional α-para Kenmotsu manifold with semi-symmetric metric connection

In 3-dimensional α-para Kenmotsu manifold, the Ricci tensor S of Levi-Civita connection ∇ is given by

$$S(X,Y) = g(R(e_1,X)Y,e_1) - g(R(\phi e_1,X)Y,\phi e_1) + g(R(\xi,X)Y,\xi).$$

Let M^3 (ϕ,ξ,η,g) be an α-para Kenmotsu manifold [13],then we have

$$R(X,Y)Z = (\frac{r}{2} + 2\alpha^2)[g(Y,Z)X - g(X,Z)Y] - (\frac{r}{2} + 3\alpha^2)[\eta(X)g(Y,Z) - \eta(Y)g(X,Z)]\xi + (\frac{r}{2} + 3\alpha^2)[\eta(X)Y - \eta(Y)X]\eta(Z) \quad (3.1)$$

Replace Z = ξ in equation (3.1), we get

$$R(X,Y)\xi = \alpha^2\eta(X)Y - \eta(Y)X, \quad (3.2)$$

$$S(X,Y) = (\frac{r}{2} + 2\alpha^2)g(X,Y) - (\frac{r}{2} + 3\alpha^2)\eta(X)\eta(Y) \quad (3.3)$$

$$S(X,\xi) = -2\alpha^2\eta(X) \quad (3.4)$$

$$(\nabla_X \phi)Y = \alpha(g(\phi X,Y)\xi - \eta(Y)\phi X) \quad (3.5)$$

$$\nabla_X \xi = \alpha(X - \eta(X)\xi) \quad (3.6)$$

Let $\bar{\nabla}$ be a linear connection and ∇ be a Riemann connection of an α-para Kenmotsu manifold M. This $\bar{\nabla}$

linear connection defined by

$$\bar{\nabla}_X Y = \nabla_X Y + \eta(Y)X - g(X,Y)\xi. \quad (3.7)$$

For α-para Kenmotsu manifold with semi-symmetric metric connection ,using (2.6),(3.5) and (3.7) we have

$$(\bar{\nabla}_X \phi)Y = \alpha[g(\phi X,Y)\xi - \eta(Y)\phi X] + \eta(Y)\phi X \quad (3.8)$$

from equation (3.7),we have

$$\bar{\nabla}_X \xi = (1+\alpha)(X - \eta(X)\xi) \quad (3.9)$$

Let M^3 be a 3-dimensional α-para Kenmotsu manifold .The curvature tensor $\bar{R}$ of M^3 with respect to the semi-symmetric metric connection $\bar{\nabla}$ is defined by

$$\bar{R}(X,Y)Z = \bar{\nabla}_X\bar{\nabla}_Y Z - \bar{\nabla}_Y\bar{\nabla}_X Z - \bar{\nabla}_{[X,Y]} Z, \quad (3.10)$$

with the help of (3.7) and (3.9), we get

$$\begin{aligned}\bar{\nabla}_X\bar{\nabla}_Y Z &= \nabla_X\nabla_Y Z + X\eta(Z)Y + \eta(Z)\nabla_X Y - Xg(Y,Z)\xi \\ &-\alpha g(Y,Z)X + \alpha g(Y,Z)\eta(X)\xi \\ &+\eta(\nabla_Y Z)\eta(Z)\eta(Y)X - g(Y,Z)X \\ &-g(X,\nabla_Y Z)\xi - \eta(Z)g(X,Y)\xi + g(Y,Z)\eta(X)\xi\end{aligned} \quad (3.11)$$

$$\begin{aligned}\bar{\nabla}_Y\bar{\nabla}_X Z &= \nabla_Y\nabla_X Z + Y\eta(Z)X + \eta(Z)\nabla_Y X - Yg(X,Z)\xi \\ &-\alpha g(X,Z)Y + \alpha g(X,Z)\eta(Y)\xi \\ &+\eta(\nabla_X Z)\eta(Z)\eta(X)Y - g(X,Z)Y \\ &-g(Y,\nabla_X Z)\xi - \eta(Z)g(Y,X)\xi + g(X,Z)\eta(Y)\xi\end{aligned} \quad (3.12)$$

and

$$\begin{aligned}-\bar{\nabla}_{[X,Y]} Z &= -\nabla_{[X,Y]} - \eta(Z)\nabla_X Y + \eta(Z)\nabla_Y X \\ &-g(\nabla_X Y,Z)\xi + g(\nabla_Y X,Z)\xi.\end{aligned} \quad (3.13)$$

By using equations (3.7),(2.2),(2.3),(3.6),(3.9)(3.10),(3.11),(3.12) and(3.13) ,we get

$$\bar{R}(X,Y)Z = R(X,Y)Z - (1+2\alpha)[g(Y,Z)X - g(X,Z)Y] + (1+\alpha)[\eta(Y)X - \eta(X)Y]\eta(Z) + (1+\alpha)[\eta(X)g(Y,Z) - \eta(Y)g(X,Z)]\xi \quad (3.14)$$

Replace Z = ξ in equation (3.14),using (2.3) and (3.2),we have

$$\bar{R}(X,Y)\xi = \alpha(1+\alpha)(\eta(X)Y - \eta(Y)X). \quad (3.15)$$

Replace Y = ξ in equation (3.15) and using equation (2.3),we get

$$\bar{R}(X,\xi)\xi = \alpha(1+\alpha)(\eta(X)\xi - X). \quad (3.16)$$

In (3.15) taking the inner product with Z,we have

$$g(\bar{R}(X,Y)\xi,Z) = \alpha(1+\alpha)(\eta(X)g(Y,Z) - \eta(Y)g(X,Z)). \quad (3.17)$$

Thus we have

Lemma 3.1 Let M be a 3-dimensional α-para Kenmotsu manifold with the semi-symmetric metric connection,$\bar{S}$ Ricci curvature tensor and $\bar{Q}$ Ricci operator .Then

$$\bar{S}(X,\xi) = -2\alpha(1+\alpha)\eta(X) \quad (3.18)$$

and

$$\bar{Q}\xi = -2\alpha(1+\alpha)\xi \quad (3.19)$$

Proof. Contracting with Y and Z in (3.17) and summing over i=1,2,...,n,from (2.8) expression

$$\sum g(\bar{R}(e_i,Y)\xi,e_i) = \alpha(1+\alpha)[\sum \eta(e_i)g(Y,e_i) - \eta(Y)\sum g(e_i,e_i)]$$

the proof of (3.18) is completed.Then also usnig (2.10) and (2.1),(2.2),(2.3) ,the proof of (3.19) is completed.

Lemma 3.2 Let M be a 3-dimensional α-para Kenmotsu manifold with the semi-symmetric metric connection, r scalar curvature tensor,$\bar{S}$(X,Y) Ricci curvature tensor and $\bar{Q}$X Ricci operator.Then it follows that

$$\bar{S}(X,Y) = (-1+\tfrac{r}{2} - 3\alpha + \alpha^2)g(X,Y) + (1 - \tfrac{r}{2} + \alpha - 3\alpha^2)\eta(X)\eta(Y) \quad (3.20)$$

And

$$\bar{Q}X = (-1+\tfrac{r}{2} - 3\alpha + \alpha^2)X + (1 - \tfrac{r}{2} + \alpha - 3\alpha^2)\eta(X)\xi \quad (3.21)$$

Proof. Taking inner product of equation (3.14) with U and using equation (2.3) we have

$$\begin{aligned}g(\bar{R}(X,Y)Z,U) &= g(R(X,Y)Z,U) - (1+2\alpha)[g(Y,Z)g(X,U) - g(X,Z)g(Y,U)] \\ &+(1+\alpha)[\eta(Y)g(X,U) - \eta(X)g(Y,U)]\eta(Z) \\ &+(1+\alpha)[\eta(X)g(Y,Z) - \eta(Y)g(X,Z)]\eta(U)\end{aligned} \quad (3.22)$$

Let $\{e_1, \phi e_1, \xi\}$ be a local orthonormal ϕ-basis of vector fields on α-para Kenmotsu manifold M^3.Then, we get

$$\bar{S}(X,Y) = (-1+\tfrac{r}{2} - 3\alpha + \alpha^2)g(X,Y) + (1 - \tfrac{r}{2} + \alpha - 3\alpha^2)\eta(X)\eta(Y) \quad (3.23)$$

from equation (3.23) ,we have

$$\bar{r} = -2 + r - 8\alpha \quad (3.24)$$

where $\bar{r}$ is the scalar curvature with semi-symmetric metric connection.

using (3.23) and (2.10),it's verified that

$$g(\bar{Q}X,Y) = g((-1+\tfrac{r}{2} - 3\alpha + \alpha^2)X + (1 - \tfrac{r}{2} + \alpha - 3\alpha^2)\eta(X)\xi),Y) \quad (3.25)$$

from equation (3.25),we get

$$\bar{Q}X = (-1+\tfrac{r}{2} - 3\alpha + \alpha^2)X + (1 - \tfrac{r}{2} + \alpha - 3\alpha^2)\eta(X)\xi \quad (3.26)$$

the proof of (3.21) is completed.

4 Ricci solitons in α-para kenmotsu Manifold with semi-symmetric metric connection

Let M be a 3-dimensional α-para Kenmotsu manifold with the semi-symmetric metric connection and V be pointwise collinear with ξ (i.e.V =bξ , where b is a function).Then

$$(L_V g + 2S + 2\lambda g)(X,Y) = 0$$

implies

$$0 = bg(\overline{\nabla}_X \xi, Y) + (Xb)\eta(Y) + bg(X, \overline{\nabla}_Y \xi) + (Yb)\eta(X) + 2\bar{S}(X,Y) + 2\lambda g(X,Y) \quad (4.1)$$

using (3.9) in (4.1) , we get

$$0 = 2b(1+\alpha)g(X,Y) - 2b(1+\alpha)\eta(X)\eta(Y) + (Xb)\eta(Y) + (Yb)\eta(X) + 2\bar{S}(X,Y) + 2\lambda g(X,Y) \quad (4.2)$$

With the substitution of Y with ξ in (4.2) , it follows that

$$(Xb) + (\xi b)\eta(X) + 2\lambda\eta(X) - 4\alpha(1+\alpha)\eta(X) = 0 \quad (4.3)$$

Again replacing X by ξ in (4.3) shows that

$$\xi b = -\lambda + 2\alpha(\alpha+1) \quad (4.4)$$

Putting (4.4) in (4.3), we obtain

$$b = (2\alpha(1+\alpha) - \lambda)\eta \quad (4.5)$$

By applying d in (4.5), we get

$$0 = (2\alpha(1+\alpha) - \lambda)d\eta \quad (4.6)$$

Since d$\eta \neq 0$ from , we have

$$2\alpha(1+\alpha) - \lambda = 0 \quad (4.7)$$

By using (4.5) and (4.7), we obtain that b is constant. Hence from (4.2) it is verified

$$\bar{S}(X,Y) = -b((1+\alpha)+\lambda)g(X,Y) + b(1+\alpha)\eta(X)\eta(Y) \quad (4.8)$$

which implies that M is an η-Einstein manifold. This leads to the following

Theorem 4.1 If in a 3-dimensional α-para Kenmotsu manifold with the semi symmetric metric connection , the metric g is a Ricci soliton and V is a pointwise collinear with ξ ,then V is a constant multiple of ξ and g is an η-Einstein manifold of the form (4.8) and Ricci soliton is steady and expanding according as λ = 2α(1+α) is zero and positive , respectively.

5 Conharmonically flat α-para Kenmotsu manifolds with the semi-symmetric metric connection

We have studied conharmonically flat α-para Kenmotsu manifolds with respect to the semi-symmetric metric connection. In a α-para Kenmotsu manifold the conharmonic curvature tensor with respect to the semi-symmetric metric connection is given by

$$\bar{K}(X,Y)Z = \bar{R}(X,Y)Z - [\bar{S}(Y,Z)X - \bar{S}(X,Z)Y + g(Y,Z)\bar{Q}X - g(X,Z)\bar{Q}Y]. \quad (5.1)$$

If $\bar{K}$=0 ,then the manifold M is called conharmonically flat manifold with respect to the semi- symmetric metric connection. Let M be a conharmonically flat manifold with respect to the semi-symmetric metric connection. from (5.1),we have

$$\bar{R}(X,Y)Z = \bar{S}(Y,Z)X - \bar{S}(X,Z)Y + g(Y,Z)\bar{Q}X - g(X,Z)\bar{Q}Y \quad (5.2)$$

using (3,14),(3.20)and (3.21) in (5.1) ,we get

$$R(X,Y)Z - (1+2\alpha)[g(Y,Z)X - g(X,Z)Y] + (1+\alpha)[\eta(Y)X - \eta(X)Y]\eta(Z) + (1+\alpha)[\eta(X)g(Y,Z) - \eta(Y)g(X,Z)]\xi = S(Y,Z)X - S(X,Z)Y + (\frac{r}{2} - 6\alpha - 2)[g(Y,Z)X - g(X,Z)Y] + (1+\alpha)[\eta(Y)X - \eta(X)Y]\eta(Z) + (1 - \frac{r}{2} + \alpha - 3\alpha^2)[g(Y,Z)\eta(X) - g(X,Z)\eta(Y)]\xi \quad (5.3)$$

Now putting X=ξ in (5.3), we obtain

$$R(\xi,Y)Z = S(Y,Z)\xi - S(\xi,Z)Y + (\frac{r}{2} - 1 - 4\alpha)[g(Y,Z)\xi - \eta(Z)Y] - (\frac{r}{2} + 3\alpha^2)[g(Y,Z) - \eta(Z)\eta(Y)]\xi, \quad (5.4)$$

using (3.1) and (3.4) in (5.4), we get

$$S(Y,Z)\xi - S(\xi,Z)Y + (-1 - 4\alpha - 2\alpha^2)g(Y,Z)\xi + (1 + 4\alpha + 2\alpha^2)\eta(Z)Y = 0 \quad (5.5)$$

Taking inner product with ξ in (5.5) , we get

$$S(Y,Z) = (1 + 4\alpha + 2\alpha^2)g(Y,Z) - (1 + 4\alpha + 4\alpha^2)\eta(Y)\eta(Z)) \quad (5.6)$$

Thus M is an η-Einstein manifold with respect to the Levi-Civita connection. This leads to the following

Theorem 5.1 If M is a conharmonically flat α-para Kenmotsu manifolds with respect to the semi-symmetric metric connection.Then the manifold M is an η-Einstein.

6 Example

(A 3-dimensional α-para Kenmotsu manifold with the semi-symmetric metric connection.) We consider the 3-dimensional manifold M = (x,y,z) $\in R^3, z \neq 0$, where (x,y,z) are the standard coordinates in R^3. The vector fields

$$e_1 = z^2\frac{\partial}{\partial x}, e_2 = z^2\frac{\partial}{\partial y}, e_3 = \frac{\partial}{\partial z}$$

are linearly independent at each point of M. Let g be the Riemannian metric defined by

$$g(e_1,e_3) = g(e_2,e_3) = g(e_1,e_2) = 0,$$

$$g(e_1,e_1) = g(e_2,e_2) = g(e_3,e_3) = 1.$$

Let η be the 1-form defined by η(Z) = g(Z, e_3) for any Z $\in$ χ(M) . Let ϕ be the (1,1) tensor field defined by $\phi(e_1)$ = -e_2 , $\phi(e_2)$ = e_1 , $\phi(e_3)$ = 0 .

Then using linearity of ϕ and g we have

$$\eta(e_3) = 1, \phi^2(Z) = -Z + \eta(Z)e_3$$

$$g(\phi Z, \phi W) = g(Z,W) - \eta(Z)\eta(W)$$

for any Z , W $\in$ χ(M) .Now , by direct computations we obtain

$$[e_1, e_2] = 0, \qquad [e_2, e_3] = -\frac{2}{z}e_2, \quad [e_1, e_3] = -\frac{2}{z}e_1$$

by using these above equations we get[18]

$$\nabla_{e_i} e_i = \frac{2}{z}e_3 \ and \ \nabla_{e_i} e_3 = -\frac{2}{z}e_1 \qquad (6.1)$$

$$\nabla_{e_2} e_1 = \nabla_{e_1} e_2 = \nabla_{e_3} e_1 = \nabla_{e_3} e_2 = \nabla_{e_3} e_3 = 0 \qquad (6.2)$$

Now we consider at this example for semi-symmetric metric connection . from (3.8) , (6.1) and (6.2)

$$\overline{\nabla}_{e_i} e_i = (\frac{2}{z} - 1)e_3 \quad and \quad \overline{\nabla}_{e_i} e_3 = (-\frac{2}{z} + 1)e_1 \qquad (6.3)$$

$$\overline{\nabla}_{e_i}\nabla_{e_j} = \overline{\nabla}_{e_3} e_j = 0 \ and \ \overline{\nabla}_{e_3} = 0 \qquad (6.4)$$

where i $\neq$ j =1,2 . it's known that

$$\bar{R}(X,Y)Z = \overline{\nabla}_X\overline{\nabla}_Y Z - \overline{\nabla}_Y\overline{\nabla}_X Z - \overline{\nabla}_{[X,Y]} Z. \qquad (6.5)$$

By using (6.3),(6.4) and (6.5) we obtain

$$\bar{R}(e_i, e_3)e_3 = (\frac{6}{z^2} + \frac{2}{z})e_i, \quad \bar{R}(e_i, e_j)e_3 = 0$$

$$\bar{R}(e_i, e_j)e_j = (\frac{4}{z} - \frac{4}{z^2} - 1)e_i, \ \bar{R}(e_i, e_3)e_j = 0 \quad (6.6)$$

$$\bar{R}(e_3, e_i)e_i = (\frac{2}{z} - \frac{6}{z^2})e_3$$

where i $\neq$ j = 1,2 . From (2.8) and (6.6)
it's verified that

$$S(e_1, e_1) = (\frac{-2}{z^2} + \frac{2}{z} - 1)$$

$$S(e_2, e_2) = (\frac{-10}{z^2} + \frac{6}{z} - 1) \qquad (6.7)$$

$$S(e_3, e_3) = (\frac{-12}{z^2} + \frac{4}{z})$$

7 CONCLUSION

If in a 3-dimensional α-para Kenmotsu manifold with the semi-symmetric metric connection , the metric g is a Ricci soliton and In this study , we gave some curvature conditions for 3-dimensional α-para Kenmotsu manifolds with semi-symmetric metric connection.In 3-dimensional α -para Kenmotsu manifolds with semi-symmetric metric connection is also an η-Einstein manifold and Ricci soliton defined steady or expanding on this manifold is named with respect to values of α and λ constant.We also proved that conharmonically flat α-para Kenmotsu manifolds with semi-symmetric metric connection is an η-Einstein manifold.

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