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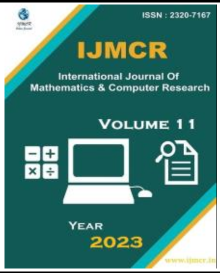


# Two Topological Indices of Two New Variants of Graph Products

**M. Durga[1], Dr.S.Nagarajan[2]**

[1,2] Department of mathematics, Kongu Arts and Science college, Erode, Tamil nadu.

| ARTICLE INFO | ABSTRACT |
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| Published Online: **29 August 2023** Corresponding Author: **M. Durga** | Graph operations are essential to developing advanced network structures from simple graphs. In [12] they defined two new variants of corona product and discovered their topological indices. In this Study, we extended the work and obtain the formulas of Y- Index and Redefined third Zagreb index for corona join product and Sub-division vertex join product of graphs |

**KEYWORDS:** Topological indices, join, Corona product, Sub-division vertex product, Graph operations

## I. INTRODUCTION

Graph theory is a fascinating and inviting branch of mathematics. The application of Graph Theory in the various fields like Network, Webpage, Neural networks, Chemistry

All the graph considered in this paper are simple and connected. Let G=(V(G),E(G)) be a graph with vertex set V(G) and edge set E(G). The number of vertices and number of edges are called the order n and size m respectively. A graph of order n and size m will be denoted by G(n,m). For a vertex $v \in V$, we denote the degree of $v$ by $d_G(V)$ or briefly $d(v)$ which is defined as the number of edges of G incident at a vertex V. For a simple graph G, The Sub-division of the graph G is denoted by $S(G)$ and obtained by inserting a new vertex on every edge of G.

Topological indices have been found to be useful in establishing relation between the Structure and the properties of molecules. Topological indices mainly used in Quantitative Structure Property Relationship (QSPR) and Quantitative Structure Activity Relationship (QSAR). Some Topological indices are degree based and some are distance based.

The Zagreb indices were introduced more than thirty years ago by Gutman and Trinajstic[7]

The First and Second Zagreb indices are defined as

$$M_1(G) = \sum_{v \in V(G)} d_G(V)^2 = \sum_{uv \in E(G)} [d_G(u) + d_G(v)]$$

$$M_2(G) = \sum_{uv \in E(G)} d_G(u) d_G(v)$$

These indices were introduced to study the Structure – dependency of the total $\pi$-electron energy $(\varepsilon)$. It was found that the $\varepsilon$ depends on $M_1(G)$ and thus provides a measure of carbon skeleton of the underlying molecules.

The Y-index is defined as

$$Y(G) = \sum_{u \in V(G)} d_G{}^4(u) = \sum_{uv \in E(G)} [d_G{}^3(u) + d_G{}^3(v)]$$

The Redefined third Zagreb index is defined as

$$\operatorname{Re} ZG_3(G) = \sum_{e=uv \in E(G)} (d(u)d(v))(d(u)+d(v))$$

The Redefined third Zagreb index was also independently defined by Mansour and Song [11]. Moreover, the generalized version was presented in[11]

Let $G_1(n_1, m_1)$ and $G_2(n_2, m_2)$ be two connected simple graphs. Corona product of graphs $G_1$ and $G_2$ , denoted by $G_1 \circ G_2$ is obtained by taking one copy of $G_1$ and $n_1$ copies of $G_2$ and joining each vertex of i-th copy of $G_2$ to i-th vertex of $G_1$ [9]

The Sub-division vertex variant of corona of $G_1$ and $G_2$ is attained from $S(G_1)$ and $n_1$ copies of $G_2$ by joining the $i^{th}$ vertex of a $V(G_1)$ to every vertex in the $i^{th}$ copy of $G_2$

The Join graph of $G_1$ and $G_2$ is obtained by joining each vertex of $G_1$ to each vertex $G_2$ and it is denoted by $G_1 + G_2$ [11]

Abdu Alameri [8] computed Y-index for some special graphs that have been applied to compute the Y-index for Nano-tube and Nano-torus. Wei Gao [] investigated the Redefined First, Second and Third Zagreb indices of Titania Nanotubes Tio2 [m, n] some graph operations and their topological indices are presented in [12]-[15]

## 1. CORONA JOIN PRODUCT

Let $G_1(n_1,m_1)$ and $G_2(n_2,m_2)$ be simple connected graphs and the corona join graph of $G_1$ and $G_2$ is obtained by taking one copy of $G_1$, $n_1$ copies of $G_2$, and joining each vertex of the $i^{th}$ copy of $G_2$ with all vertices of $G_1$. The Corona join product of $G_1$ and $G_2$ is denoted by $G_1 \oplus G_2$ and shown in fig 1

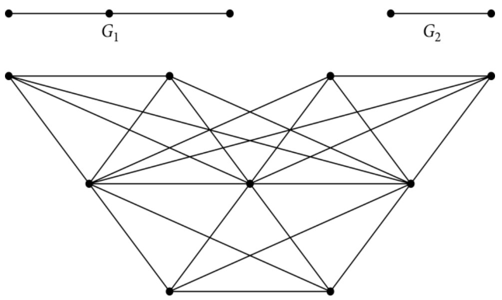
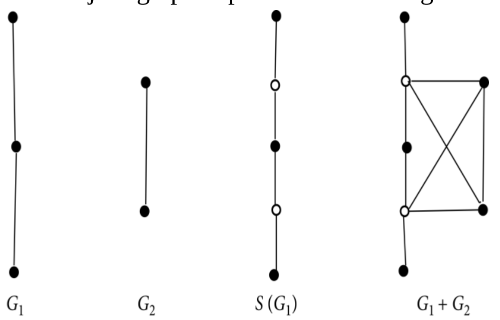



**Figure 1: Corona join product** $G_1 \oplus G_2$

## 2, SUBDIVISION VERTEX JOIN PRODUCT

Let $G_1=(n_1,m_1)$, $G_2=(n_2,m_2)$ and $S(G_1)=(n_1^{'},m_1^{'})$ be three simple connected graphs. The Sub-division vertex join graph is obtained by joining the each new vertex of $S(G_1)$ to all vertices of $G_2$ and it is denoted by $G_1+G_2$. The Sub-division vertex join graph is presented in the fig 2




## II. MAIN RESULTS

Throughout this part, we present the main results. The following lemma's can be used to obtain exact expressions of topological indices of two new graph variants of products. The proofs of the following two lemmas are directly from the Corona join product $G_1 \oplus G_2$ and Sub-division vertex join $G_1+G_2$

**Lemma.1**

$G_1=(n_1,m_1)$ and $G_2=(n_2,m_2)$ be two graphs ; then the degree behavior of vertices in the graph $G_1 \oplus G_2$ is

$$d_{G_1\oplus G_2}(v)=\begin{cases} d_{G_1}(v)+n_1n_2, & if\ v\in V(G_1) \\ d_{G_2}(v)+n_1, & if\ v\in (G_2) \end{cases}$$

**Lemma.2**

Let we have three simple connected graphs $G_1=(n_1,m_1)$, $G_2=(n_2,m_2)$ and $S(G_1)=(n_1',m_2')$ then the degree behavior of vertices in the graph $G_1+G_2$ is

$$d_{G_1+G_2}(v)=\begin{cases} d_{G_1}(v) & if\ v\in V(G_1) \\ 2+n_2 & if\ v\in V_s(G_1) \\ d_{G_1}(v)+m_1 & if\ v\in V(G_2) \end{cases}$$

**THEOREM.3**

Let we have two simple connected graphs $G_1=(n_1,m_1)$, and $G_2=(n_2,m_2)$ then the Y-index of corona join product $G_1 \oplus G_2$ is given as

$$Y(G_1\oplus G_2)=Y(G_1)+4F(G_1)n_1n_2+6M_1(G_1)n_1^2n_2^2+8m_1n_1^3n_2^3+n_1Y(G_1)$$
$$+4F(G_1)n_1^2+6M_1(G_1)n_1^3+8m_2n_1^4+n_1^5n_2(n_2^3+1)$$

**Proof**: From the definition Y-index, we have

$$Y(G_1\oplus G_2)=\sum_{v\in V(G_1\oplus G_2)} d_{G_1\oplus G_2}(v)^4$$

Now we apply the lemma 1

$$=\sum_{v\in V(G_1)}(d_{G_1}(v)+n_1n_2)^4+\sum_{v\in V(G_1)}\sum_{v\in V(G_2)}\left(d_{G_2}(v)+n_1\right)^4$$

$$=\sum_{v\in V(G_1)}\begin{bmatrix} d_{G_1}(v)^4+4d_{G_1}(v)^3(n_1n_2)+6d_{G_1}(v)^2(n_1n_2)^2 \\ +4d_{G_1}(v)(n_1n_2)^3+(n_1n_2)^4 \end{bmatrix}$$

$$+\sum_{v\in V(G_1)}\sum_{v\in V(G_2)}\begin{bmatrix} d_{G_2}(v)^4+n_1^4+4d_{G_2}(v)^3(n_1) \\ +6d_{G_2}(v)^2(n_1)^2+4d_{G_2}(v)(n_1)^3 \end{bmatrix}$$

$$=\sum_{v\in V(G_1)}d_{G_1}(v)^4+4\sum_{v\in V(G_1)}d_{G_1}(v)^3(n_1n_2)+6\sum_{v\in V(G_1)}d_{G_1}(v)^2(n_1n_2)^2$$
$$+4\sum_{v\in V(G_1)}d_{G_1}(v)\ (n_1n_2)^3+\sum_{v\in V(G_1)}(n_1n_2)^4+n_1$$
$$+n_1\left(\begin{matrix}\sum_{v\in V(G_2)}d_{G_2}(v)^4+\sum_{v\in V(G_2)}n_1^4+4\sum_{v\in V(G_2)}d_{G_2}(v)^3(n_1) \\ +6\sum_{v\in V(G_2)}d_{G_2}(v)^2(n_1)^2+4\sum_{v\in V(G_2)}d_{G_2}(v)\ (n_1)^3)\end{matrix}\right)=$$

$$=Y(G_1)+4F(G_1)\ n_1n_2+6M_1(G_1)n_1^2n_2^2+8m_1n_1^3n_2^3+n_1Y(G_1)$$
$$+4F(G_1)n_1^2+6M_1(G_1)n_1^3\ +8m_2n_1^4+n_1^5n_2(n_2^3+1)$$

Hence we get the result

**EXAMPLE: By** using the statement of theorem 3 , we get

$$Y(P_n\oplus C_m)=16n-30+4nm(8n-14)+6n^2m^2(4n-6)+8n^3m^3(n-1)$$
$$+n(16n-30)\ +4n^2(8n-14)+4n^3(4n-6)+8mn^4+n^5m(m^3+1)$$

**THEOREM. 4**

Let we have three simple connected graphs $G_1=(n_1,m_1)$, $G_2=(n_2,m_2)$ and $S(G_1)=(n_1',m_1')$, the Y-index of Subdivision-vertex join $G_1+G_2$ is given as

$$Y(G_1+G_2)=Y(G_1)+Y(G_2)+m_1(2+n_2)^4+4F(G_2)m_1$$
$$+6M_1(G_2)m_1^2+8m_2m_1^3+n_2m_1^4$$

**Proof**: From the definition of Y-index, we have

$$Y(G_1+G_2)=\sum_{v\in V(G_1+G_2)}d_{G_1+G_2}(v)^4$$

Now we apply the lemma 2

$$=\sum_{v\in V(G_1)}d_{G_1}(v)^4+\sum_{v\in V_s(G_1)}(2+n_2)^4+\sum_{v\in V(G_2)}(d_{G_1}(v)+m_1)^4$$

$$= Y(G_1) + (2+n_2)^4 \sum_{v\in V_s(G_1)} 1 + \sum_{v\in V(G_2)} d_{G_1}(v)^4 + 4\sum_{v\in V(G_2)} d_{G_2}(v)^3 m_1$$

$$+6\sum_{v\in V(G_2)} d_{G_2}(v)^2 m_1^2 + 4\sum_{v\in V(G_2)} d_{G_2}(v)\ m_1^3 + \sum_{v\in V(G_2)} m_1^4$$

$$= Y(G_1) + m_1(2+n_2)^4 + Y(G_2) + 4F(G_2)m_1 + 6M_1(G_2)m_1^2 + 8m_2m_1^3 + n_2m_1^4$$

Which is our required result

**EXAMPLE:** By using the statement of theorem 4, we get

$Y(P_n \oplus C_m)$

$$= 16n - 30 + 4nm(8n-14) + 6n^2m^2(4n-6) + 8n^3m^3(n-1)$$
$$+ n(16n-30) + 4n^2(8n-14) + 4n^3(4n-6) + 8mn^4 + n^5m(m^3+1)$$

**THEOREM.5**

Let we have the two simple connected graphs $G_1 = (n_1, m_1)$, and $G_2 = (n_2, m_2)$ then the Redefined third Zagreb index of corona join product $G_1 \oplus G_2$ is given as

**Proof:** By the definition of Redefined third Zagreb index, we have

$$ReZG_3(G_1 \oplus G_2) = \sum_{uv\in E(G_1\oplus G_2)} (d_{G_1\oplus G_2}(u)\ d_{G_1\oplus G_2}(v))\ (d_{G_1\oplus G_2}(u) + d_{G_1\oplus G_2}(v))$$

$$= \sum_{uv\in E(G_1)} \big((d_{G_1}(u)+n_1n_2)(d_{G_1}(v)+n_1n_2)\big)\big((d_{G_1}(u)+n_1n_2)+(d_{G_1}(v)+n_1n_2)\big)$$
$$+ n_1 \sum_{uv\in E(G_2)} \sum_{uv\in E(G_2)} \big(d_{G_2}(u)+n_1)(d_{G_2}(v)+n_1)\big)\big((d_{G_2}(u)+n_1)+(d_{G_2}(v)+n_1)\big)$$
$$+ n_1 \sum_{\substack{uv\in E(G_1\oplus G_2)\\ u\in V(G_1)\\ v\in V(G_2)}} \big((d_{G_1}(u)+n_1n_2)(d_{G_2}(v)+n_1)\big)\big((d_{G_1}(u)+n_1n_2)+(d_{G_2}(v)+n_1\big)$$

$$= \sum_{uv\in E(G_1)} \Big[(d_{G_1}(u)\cdot(d_{G_1}(v))(d_{G_1}(u)+d_{G_1}(v)) + 2n_1n_2((d_{G_1}(u)\cdot(d_{G_1}(v))$$
$$+ n_1n_2((d_{G_1}(u)+d_{G_1}(v)) + 2n_1^2n_2^2((d_{G_1}(u)+d_{G_1}(v))$$
$$+ n_1^2n_2^2((d_{G_1}(u)+d_{G_1}(v)) + 2n_1^3n_2^3\Big]$$

$$+ n_1 \sum_{uv\in E(G_2)} \Big[(d_{G_2}(u)\cdot(d_{G_2}(v))(d_{G_2}(u)+d_{G_2}(v)) + 2n_1(d_{G_2}(u)\cdot(d_{G_2}(v))$$
$$+ n_1(d_{G_2}(u)+d_{G_2}(v))^2\ + 2n_1^2(d_{G_2}(u)+d_{G_2}(v)) + n_1^2(d_{G_2}(u)+d_{G_2}(v)) + 2n_1^3\Big]$$

$$+ n_1\Bigg[\Bigg(\sum_{u\in V(G_1)} d_{G_1}(u) \sum_{u\in V(G_2)} d_{G_2}(v) + n_1n_2 \sum_{u\in V(G_1)} 1 \sum_{u\in V(G_2)} d_{G_2}(v) + n_1 \sum_{u\in V(G_1)} d_{G_1}(u) \sum_{u\in V(G_2)} 1$$
$$+ n_1^2n_2 \sum_{u\in V(G_1)} 1 \sum_{v\in V(G_1)} 1\Bigg)\Bigg(\sum_{u\in V(G_1)} d_{G_1}(u) \sum_{v\in V(G_2)} 1 + \sum_{u\in V(G_1)} 1 \sum_{u\in V(G_1)} d_{G_1}(v) + n_1(n_2+1) \sum_{u\in V(G_1)} 1 \sum_{v\in V(G_2)} 1\Bigg)\Bigg]$$

$$= ReZG_3(G_1) + 2n_1n_2M_2(G_1) + 2n_1^3n_2^3m_1 + n_1n_2HM_1(G_1) + 3n_1^2n_2^2M_1(G_1)$$
$$+ n_1\, ReZG_3(G_2) + 2n_1^2M_2(G_2) + n_1^2HM_1(G_2) + 3n_1^3M_1(G_2) + 2n_1^4m_2$$
$$+ (4m_1m_2n_1 + 2n_1^3n_2m_2 + 2m_1n_1^2n_2 + n_1^4n_2^2)(2m_1n_1\ n_2 + 2m_2n_1^2$$
$$+ n_1^3n_2(n_2+1))$$

**EXAMPLE:** By using the statement of theorem 5, we get

$$Re\,ZG_3(P_n \oplus C_m) = 2n^4m + 2n^4m^3 - 2n^3n^3 + 12n^3m(m+1)$$
$$-18n^2m^2 + 32n^2m - 30nm + 16n - 24 + n^4m^2$$
$$+ (2n^2m + 2n^3m(m+1) - 4nm)(4n^2m + n^3m(m+1) - 2nm)$$

**THEOREM.6**

Let we have three simple connected graphs $G_1 = (n_1, m_1), G_2 = (n_2, m_2)$ and $S(G_1) = (n', m_1')$ then the Redefined third Zagreb index of Sub-division vertex join $G_1 + G_2$ is given as

**Proof**: From the definition of Redefined Zagreb index, we get

$$Re\,ZG_3(G_1+G_2) = \sum_{uv\in E(G_1+G_2)} (d_{G_1+G_2}(u)\cdot d_{G_1+G_2}(v))\ (d_{G_1+G_2}(u) + d_{G_1+G_2}(v))$$

$$= \sum_{\substack{uv\in E(S(G_1))\\ u\in V(G_1)\\ v\in V_s(G_1)}} (d_{G_1}(u))(2+n_2))((d_{G_1}(u)) + (2+n_2))$$
$$+ \sum_{uv\in E(G_2)} \Big[(d_{G_2}(u)+m_1)(d_{G_2}(v)+m_1)(d_{G_2}(u)+d_{G_2}(v)+2m_1)\Big]$$
$$+ \sum_{\substack{uv\in E(G_1+G_2)\\ u\in V(G_1)\\ v\in V_s(G_1)}} \Big[(2+n_2)(d_{G_2}(v)+m_1)((d_{G_2}(v)+(m_1+n_2+2)\Big]$$

$$= \sum_{\substack{uv\in E(S(G_1))\\ u\in V(G_1)\\ v\in V_s(G_1)}} \Big[(2+n_2)(d_{G_1}(u))^2 + 4d_{G_1}(u)(n_2+1) + n_2^2(d_{G_1}(u))\Big]$$
$$+ \sum_{uv\in E(G_2)} \begin{bmatrix} (d_{G_2}(u))(d_{G_2}(v))(d_{G_2}(u)+(d_{G_2}(v)) \\ +2m_1(d_{G_2}(u))(d_{G_2}(v)+m_1(d_{G_2}(u)+(d_{G_2}(v))^2 \\ +3m_1^2(d_{G_2}(u)+(d_{G_2}(v))+2m_1^3 \end{bmatrix}$$
$$+ \sum_{\substack{uv\in E(G_1+G_2)\\ u\in V_s(G_1)\\ v\in V(G_2)}} \begin{bmatrix} (2+n_2)(d_{G_2}(v))^2 + 8m_1(d_{G_2}(v)) + 4n_2(d_{G_2}(v)+2m_1n_2(d_{G_2}(v)) \\ +n_2^2(d_{G_2}(v)+m_1) + m_1^2(n_2+2) + 4m_1(n_2+1) \end{bmatrix}$$

$$= \sum_{uv\in E(S(G_1))} \Big[(2+n_2)(d_{G_1}(u))^2 + 4d_{G_1}(u)(n_2+1) + n_2^2(d_{G_1}(u))\Big]$$
$$+ \sum_{uv\in E(G_2)} (d_{G_2}(u))(d_{G_2}(v))(d_{G_2}(u)) + (d_{G_2}(v))$$
$$+2m_1 \sum_{uv\in E(G_2)} (d_{G_2}(u))(d_{G_2}(v)) + m_1 \sum_{uv\in E(G_2)} (d_{G_2}(u)) + (d_{G_2}(v))^2$$
$$+3m_1^2 \sum_{uv\in E(G_2)} (d_{G_2}(u)) + (d_{G_2}(v)) + 2m_1^3 \sum_{uv\in E(G_2)} 1 + (2+n_2) \sum_{u\in V_s(G_1)} 1 \sum_{v\in V(G_2)} (d_{G_2}(v))^2$$

$$+8m_1 \sum_{u\in V_s(G_1)} 1 \sum_{v\in V(G_2)} d_{G_2}(v) + 4n_2 \sum_{u\in V_s(G_1)} 1 \sum_{v\in V(G_2)} d_{G_2}(v) + 2m_1n_2 \sum_{u\in V_s(G_1)} 1 \sum_{v\in V(G_2)} d_{G_2}(v)$$
$$+n_2^2 \sum_{u\in V_s(G_1)} 1 \sum_{v\in V(G_2)} (d_{G_2}(v)+m_1) + m_1^2(n_2+2) \sum_{u\in V_s(G_1)} 1 \sum_{v\in V(G_2)} 1$$
$$+4m_1((n_2+1) \sum_{u\in V_s(G_1)} 1 \sum_{v\in V(G_2)} 1$$

$$= \sum_{uv\in E(S(G_1))} \Big[(2+n_2)(d_{G_1}(u))^2 + 4d_{G_1}(u)(n_2+1) + n_2^2(d_{G_1}(u))\Big]$$
$$+ Re\,ZG_3(G_2) + 2m_1M_2(G_2) + m_1HM_1(G_2) + 3m_1^2M_1(G_2)$$
$$+ m_1^2m_2(2m_1+16) + M_1(G_2)m_1(n_2+2) + m_1n_2(8m_2 + 4m_1m_2$$
$$+ 2n_2m_2 + m_1n_2^2 + m_1^2(n_2+2) + 4m_1(n_2+1)$$

**EXAMPLE**

Using the statement of theorem 6, we obtain

$$Re\,ZG_3(P_n + C_m) = \sum_{uv\in E(S(P_n))} \Big[(2+m)(d_{p_n}(u))^2 + 4d_{p_n}(u)(m+1) + m^2 d_{p_n}(u)\Big]$$
$$+ 4n^3m - m^3 + n^2m^3 + 5n^2m^2 - 6n^2m$$
$$+ 2nm^2 + 36nm - 5m^2 + n^3m^2$$

## III. CONCLUSION

We proposed two variants of special graph generate and their exact formulations for Y-index and Redefined third Zagreb index .The results we obtained in this paper may help to build and investigate the Topological indices of complex network structures

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