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Delta Banhatti-Sombor Indices of Certain Networks

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ABSTRACT

Recently, a novel degree concept has been defined in Graph Theory: δ vertex degree of a vertex in a graph. In this paper, the first, second, third, fourth, fifth and sixth delta Banhatti-Sombor indices of a graph are defined by using δ vertex degree concept. Furthermore, we compute these newly defined delta Banhatti-Sombor indices for four families of networks.

KEYWORDS : δ vertex degree, delta Banhatti-Sombor indices, network.

1. INTRODUCTION

In this paper, G denotes a finite, simple, connected graph, *V(G) and E(G)* denote the vertex set and edge set of *G*. The degree *d(u)* of a vertex *u* in *G* is the number of vertices adjacent to *u*. For undefined notations and terminologies, we refer the book [1].

Chemical Graph Theory is a branch of Mathematical Chemistry which has an important effect on the development of Chemical Sciences. A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. Topological indices are useful for establishing correlation between the structure of a molecular compound and its physicochemical properties. Numerous topological indices have been considered in Chemistry and have found some applications, especially in QSPR/QSAR research [2, 3].

The δ vertex degree was introduced in [4] and it is defined as

$$\delta_u = d(u) - \delta(G) + 1.$$

The delta Sombor index [4] of a graph *G* is defined as

$$\delta SO(G) = \sum_{uv\in E(G)} \sqrt{\delta_u^2 + \delta_v^2}$$

$$= \sqrt{\left(d(u) - \delta(G) + 1\right)^2 + \left(d(v) - \delta(G) + 1\right)^2}.$$

Recently, some delta indices were studied, for example, in [5, 6, 7, 8, 9, 10, 11].

We introduce the following delta Banhatti-Sombor indices and they are defined as follow:

The first delta Banhatti-Sombor index of a graph *G* is defined as

$$\delta BSO_1(G) = \sum_{uv\in E(G)} \frac{\left|\delta_u^2 - \delta_v^2\right|}{2}.$$

The second Banhatti-Sombor index of a graph *G* is defined as

$$\delta BSO_2(G) = \sum_{uv\in E(G)} \frac{\left|\delta_u^2 - \delta_v^2\right|}{\delta_u^2 + \delta_v^2}.$$

The third Banhatti-Sombor index of a graph *G* is defined as

$$\delta BSO_3(G) = \sum_{uv\in E(G)} \sqrt{2}\pi \frac{\delta_u^2 + \delta_v^2}{\delta_u + \delta_v}.$$

The fourth Banhatti-Sombor index of a graph *G* is defined as

$$\delta BSO_4(G) = \sum_{uv\in E(G)} \frac{\pi}{2}\left(\frac{\delta_u^2 + \delta_v^2}{\delta_u + \delta_v}\right)^2.$$

The fifth Banhatti-Sombor index of a graph *G* is defined as

$$\delta BSO_5(G) = \sum_{uv\in E(G)} 2\pi \frac{\left|\delta_u^2 - \delta_v^2\right|}{\sqrt{2} + 2\sqrt{\delta_u^2 + \delta_v^2}}.$$

The sixth Banhatti-Sombor index of a graph *G* is defined as

$$\delta BSO_6(G) = \sum_{uv\in E(G)} \pi \left(\frac{\left|\delta_u^2 - \delta_v^2\right|}{\sqrt{2} + 2\sqrt{\delta_u^2 + \delta_v^2}} \right)^2$$

Recently, some Sombor indices were studied, for example, in [12-31].

In this paper, we determine the delta Banhatti-Sombor indices of some networks such as oxide networks, hexagonal networks, honeycomb networks and silicate networks.

II. RESULTS FOR OXIDE NETWORKS

The oxide networks are of vital importance in the study of silicate networks. An oxide network of dimension n is denoted by of OX_n. An oxide network of dimension five is shown in Figure 1.

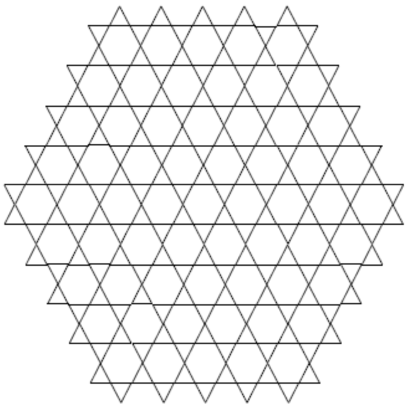

Figure 1. Oxide network of dimension 5

Let G be the graph of an oxide network OX_n. We obtain that G has $9n^2 + 3n$ vertices and $18n^2$edges. In G, there are two types of vertices as follows:

$V_1 = \{u \in V(G) \mid d_G(u) = 2\}$, $|V_1| = 6n$.
$V_2 = \{u \in V(G) \mid d_G(u) = 4\}$, $|V_2| = 9n^2 - 3n$.

Therefore, we have $\delta(G)=2$ and hence $\delta_u = d_G(u) - \delta(G) + 1 = d_G(G) - 1$.

In G, there are two types of edges based on degrees of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) =2, d_G(v) = 4\}$, $|E_1| = 12n$.
$E_2 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 4\}$, $|E_2| = 18n^2 - 12n$.

Hence there are 2 types of δ-edges as given in Table 1.

Table 1: δ-edge partition of OX_n

$\delta_u, \delta_v \backslash uv \in E(G)$	Number of edges
(1, 3)	$6n$
(3, 3)	$9n^2 - 3n$

Theorem 1. Let G be the graph of a nanotube OX_n. Then

(i) $\delta BSO_1(G) = 24n.$

(ii) $\delta BSO_2(G) = \frac{24n}{5}.$

(iii) $\delta BSO_3(G) = 27\sqrt{2}\pi n^2 + 6\sqrt{2}\pi n.$

(iv) $\delta BSO_4(G) = \frac{81}{2}\pi n^2 + \frac{21}{4}\pi n.$

(v) $\delta BSO_5(G) = \frac{96}{\sqrt{2} + 2\sqrt{10}}\pi n.$

(vi) $\delta BSO_6(G) = 6n\pi \left(\frac{8}{\sqrt{2} + 2\sqrt{1^2 + 3^2}} \right)^2.$

Proof: From definitions and by using Table 1, we deduce

(i) $$\delta BSO_1(G) = \sum_{uv\in E(G)} \frac{\left|\delta_u^2 - \delta_v^2\right|}{2}$$
$$= 6n\frac{|1^2 - 3^2|}{2} + (9n^2 - 3n)\frac{|3^2 - 3^2|}{2}$$
$$= 24n.$$

(ii) $$\delta BSO_2(G) = \sum_{uv\in E(G)} \frac{\left|\delta_u^2 - \delta_v^2\right|}{\delta_u^2 + \delta_v^2}$$
$$= 6n\frac{|1^2 - 3^2|}{1^2 + 3^2} + (9n^2 - 3n)\frac{|3^2 - 3^2|}{3^2 + 3^2}$$
$$= \frac{24n}{5}.$$

(iii) $$\delta BSO_3(G) = \sum_{uv\in E(G)} \sqrt{2}\pi \frac{\delta_u^2 + \delta_v^2}{\delta_u + \delta_v}$$
$$= \sqrt{2}\pi 6n\frac{1^2 + 3^2}{1+3} + \sqrt{2}\pi(9n^2 - 3n)\frac{3^2 + 3^2}{3+3}$$
$$= 27\sqrt{2}\pi n^2 + 6\sqrt{2}\pi n.$$

(iv) $$\delta BSO_4(G) = \sum_{uv\in E(G)} \frac{\pi}{2}\left(\frac{\delta_u^2 + \delta_v^2}{\delta_u + \delta_v} \right)^2$$
$$= 6n\frac{\pi}{2}\left(\frac{1^2 + 3^2}{1+3} \right)^2 + (9n^2 - 3n)\frac{\pi}{2}\left(\frac{3^2 + 3^2}{3+3} \right)^2$$
$$= \frac{81}{2}\pi n^2 + \frac{21}{4}\pi n.$$

(v) $$\delta BSO_5(G) = \sum_{uv\in E(G)} 2\pi \frac{\left|\delta_u^2 - \delta_v^2\right|}{\sqrt{2} + 2\sqrt{\delta_u^2 + \delta_v^2}}$$
$$= 6n2\pi\frac{|1^2 - 3^2|}{\sqrt{2} + 2\sqrt{1^2 + 3^2}} + (9n^2 - 3n)2\pi\frac{|3^2 - 3^2|}{\sqrt{2} + 2\sqrt{3^2 + 3^2}}$$
$$= \frac{96}{\sqrt{2} + 2\sqrt{10}}\pi n.$$

(vi) $\delta BSO_6(G) = \sum_{uv \in E(G)} \pi \left(\frac{|\delta_u^2 - \delta_v^2|}{\sqrt{2} + 2\sqrt{\delta_u^2 + \delta_v^2}} \right)^2$

$= 6n\pi \left(\frac{|1^2 - 3^2|}{\sqrt{2} + 2\sqrt{1^2 + 3^2}} \right)^2 + (9n^2 - 3n)\pi \left(\frac{|3^2 - 3^2|}{\sqrt{2} + 2\sqrt{3^2 + 3^2}} \right)^2$

$= 6n\pi \left(\frac{8}{\sqrt{2} + 2\sqrt{1^2 + 3^2}} \right)^2.$

III. RESULTS FOR HEXAGONAL NETWORKS

It is known that there exist three regular plane tilings with composition of some kind of regular polygons such as triangular, hexagonal and square. Triangular tiling is used in the construction of hexagonal networks. This network is denoted by HX_n. A hexagonal network of dimension six is shown in Figure 2.

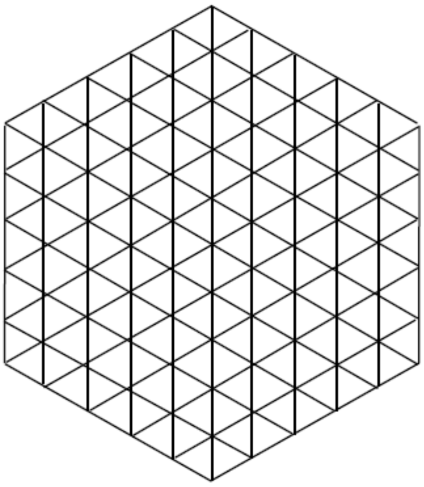

Figure 2. Hexagonal network of dimension six

Let G be the graph of a hexagonal network HX_n. By calculation, we obtain that G has $3n^2 - 3n+1$ vertices and $9n^2 - 15n+6$ edges. In G, there are three types of vertices as follows:

$V_1 = \{u \in V(G) \mid d_G(u) = 3\}$, $|V_1| = 6$.
$V_2 = \{u \in V(G) \mid d_G(u) = 4\}$, $|V_2| = 6n - 12$.
$V_3 = \{u \in V(G) \mid d_G(u) = 6\}$, $|V_3| = 3n^2 - 9n + 7$.

Thus $\delta(G)=3$ and hence $\delta_u = d_G(u) - \delta(G) + 1 = d_G(u) - 2$.

In G, there are five types of edges based on degrees of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 4\}$,
$|E_1| = 12$.
$E_2 = \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 6\}$,
$|E_2| = 6$.
$E_3 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 4\}$,
$|E_3| = 6n - 18$.
$E_4 = \{uv \in E(G) \mid d_G(u) = 4, d_G(v) = 6\}$,
$|E_4| = 12n - 24$.
$E_5 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 6\}$,
$|E_5| = 9n^2 - 33n + 30$.

Hence there are 5 types of δ-edges as given in Table 2.

$\delta_u, \delta_v \backslash uv \in E(G)$	Number of edges
(1, 2)	12
(1, 4)	6
(2, 2)	$6n - 18$
(2, 4)	$12n - 24$
(4, 4)	$9n^2 - 33n+30$

Table 2: δ-edge partition of HX_n

Theorem 2. Let G be the graph of a nanotube HX_n. Then

(i) $\delta BSO_1(G) = 72n - 81.$

(ii) $\delta BSO_2(G) = \frac{36n}{5} - \frac{162}{85}.$

(iii) $\delta BSO_3(G) = \sqrt{2}\pi \left(36n^2 - 80n + \frac{222}{5} \right).$

(iv) $\delta BSO_4(G) = \frac{\pi}{2} \left(144n^2 - \frac{1112}{3}n + \frac{23702}{75} \right).$

(v) $\delta BSO_5(G) = 2\pi \left(\frac{36}{\sqrt{2} + 2\sqrt{5}} + \frac{90}{\sqrt{2} + 2\sqrt{17}} \right)$
$-2\pi \left(\frac{288}{\sqrt{2} + 2\sqrt{20}} - \frac{144n}{\sqrt{2} + 2\sqrt{20}} \right).$

(vi) $\delta BSO_6(G) = \pi \left(\frac{108}{22 + 4\sqrt{10}} + \frac{1350}{70 + 4\sqrt{34}} \right)$
$-\pi \left(\frac{3456}{82 + 8\sqrt{10}} - \frac{1728n}{82 + 8\sqrt{10}} \right).$

Proof: From definitions and by using Table 2, we deduce

(i) $\delta BSO_1(G) = \sum_{uv \in E(G)} \frac{|\delta_u^2 - \delta_v^2|}{2}$

$= 12\frac{|1^2 - 2^2|}{2} + 6\frac{|1^2 - 4^2|}{2} + (6n - 18)\frac{|2^2 - 2^2|}{2}$

$+(12n - 24)\frac{|2^2 - 4^2|}{2} + (9n^2 - 33n + 39)\frac{|4^2 - 4^2|}{2}$

$= 72n - 81.$

(ii) $\delta BSO_2(G) = \sum_{uv \in E(G)} \frac{|\delta_u^2 - \delta_v^2|}{\delta_u^2 + \delta_v^2}$

$$=12\frac{|1^2-2^2|}{1^2+2^2}+6\frac{|1^2-4^2|}{1^2+4^2}+(6n-18)\frac{|2^2-2^2|}{2^2+2^2}$$
$$+(12n-24)\frac{|2^2-4^2|}{2^2+4^2}+(9n^2-33n+30)\frac{|4^2-4^2|}{4^2+4^2}$$
$$=\frac{36n}{5}-\frac{162}{85}.$$

(iii) $$\delta BSO_3(G)=\sum_{uv\in E(G)}\sqrt{2}\pi\frac{\delta_u^2+\delta_v^2}{\delta_u+\delta_v}$$
$$=\sqrt{2}\pi 12\frac{1^2+2^2}{1+2}+\sqrt{2}\pi 6\frac{1^2+4^2}{1+4}+\sqrt{2}\pi(6n-18)\frac{2^2+2^2}{2+2}$$
$$+\sqrt{2}\pi(12n-24)\frac{2^2+4^2}{2+4}+\sqrt{2}\pi(9n^2-33n+30)\frac{4^2+4^2}{4+4}$$
$$=\sqrt{2}\pi\left(36n^2-80n+\frac{222}{5}\right).$$

(iv) $$\delta BSO_4(G)=\sum_{uv\in E(G)}\frac{\pi}{2}\left(\frac{\delta_u^2+\delta_v^2}{\delta_u+\delta_v}\right)^2$$
$$=\frac{\pi}{2}12\left(\frac{1^2+2^2}{1+2}\right)^2+\frac{\pi}{2}6\left(\frac{1^2+4^2}{1+4}\right)^2+\frac{\pi}{2}(6n-18)\left(\frac{2^2+2^2}{2+2}\right)^2$$
$$+\frac{\pi}{2}(12n-24)\left(\frac{2^2+4^2}{2+4}\right)^2+\frac{\pi}{2}(9n^2-33n+30)\left(\frac{4^2+4^2}{4+4}\right)^2$$
$$=\frac{\pi}{2}\left(144n^2-\frac{1112}{3}n+\frac{23702}{75}\right).$$

(v) $$\delta BSO_5(G)=\sum_{uv\in E(G)}2\pi\frac{|\delta_u^2-\delta_v^2|}{\sqrt{2}+2\sqrt{\delta_u^2+\delta_v^2}}$$
$$=2\pi 12\frac{|1^2-2^2|}{\sqrt{2}+2\sqrt{1^2+2^2}}+2\pi 6\frac{|1^2-4^2|}{\sqrt{2}+2\sqrt{1^2+4^2}}$$
$$+2\pi(6n-18)\frac{|2^2-2^2|}{\sqrt{2}+2\sqrt{2^2+2^2}}+2\pi(12n-24)\frac{|2^2-4^2|}{\sqrt{2}+2\sqrt{2^2+4^2}}$$
$$+2\pi(9n^2-33n+30)\frac{|4^2-4^2|}{\sqrt{2}+2\sqrt{4^2+4^2}}$$
$$=2\pi\left(\frac{36}{\sqrt{2}+2\sqrt{5}}+\frac{90}{\sqrt{2}+2\sqrt{17}}-\frac{288}{\sqrt{2}+2\sqrt{20}}+\frac{144n}{\sqrt{2}+2\sqrt{20}}\right).$$

(v) $$\delta BSO_6(G)=\sum_{uv\in E(G)}\pi\left(\frac{|\delta_u^2-\delta_v^2|}{\sqrt{2}+2\sqrt{\delta_u^2+\delta_v^2}}\right)^2$$
$$=\pi 12\left(\frac{|1^2-2^2|}{\sqrt{2}+2\sqrt{1^2+2^2}}\right)^2+\pi 6\left(\frac{|1^2-4^2|}{\sqrt{2}+2\sqrt{1^2+4^2}}\right)^2$$
$$+\pi(6n-18)\left(\frac{|2^2-2^2|}{\sqrt{2}+2\sqrt{2^2+2^2}}\right)^2$$
$$+\pi(12n-24)\left(\frac{|2^2-4^2|}{\sqrt{2}+2\sqrt{2^2+4^2}}\right)^2$$
$$+\pi(9n^2-33n+30)\left(\frac{|4^2-4^2|}{\sqrt{2}+2\sqrt{4^2+4^2}}\right)^2$$
$$=\pi 12\left(\frac{|1^2-2^2|}{\sqrt{2}+2\sqrt{1^2+2^2}}\right)^2+\pi 6\left(\frac{|1^2-4^2|}{\sqrt{2}+2\sqrt{1^2+4^2}}\right)^2$$
$$+\pi(12n-24)\left(\frac{|2^2-4^2|}{\sqrt{2}+2\sqrt{2^2+4^2}}\right)^2$$
$$=\pi\left(\frac{108}{22+4\sqrt{10}}+\frac{1350}{70+4\sqrt{34}}-\frac{3456}{82+8\sqrt{10}}+\frac{1728n}{82+8\sqrt{10}}\right).$$

IV. RESULTS FOR HONEYCOMB NETWORKS

If we recursively use hexagonal tiling in particular pattern, honeycomb networks are formed. These networks are very useful in Chemistry and also in Computer Graphics. A honeycomb network of dimension n is denoted by HC_n. A honeycomb network of dimension four is shown in Figure 3.

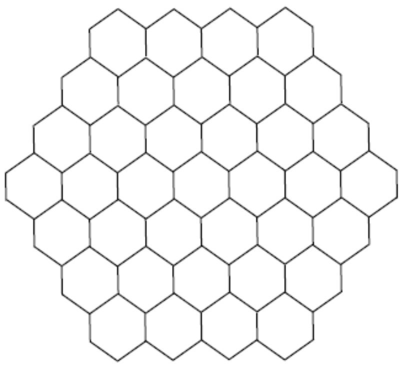

Figure 3. Honeycomb network of dimension four

Let G be the graph of a honeycomb network HC_n. By calculation, we obtain that G has $6n^2$ vertices and $9n^2 - 3n$ edges. In G, there are two types of vertices as follows:

$V_1 = \{u \in V(G) \mid d_G(u) = 2\}$, $|V_1| = 6n$.
$V_2 = \{u \in V(G) \mid d_G(u) = 3\}$, $|V_2| = 6n^2 - 6n$.

Thus $\delta(G)=2$ and hence $\delta_u = d_G(u) - \delta(G) + 1 = d_G(u) - 1$.

By calculation, in G, there are three types of edges based on degrees of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_1| = 6$.
$E_2 = \{uv \in E(G) \mid d_G(u) = 2, d_G(v) = 3\}$, $|E_2| = 12n - 12$.
$E_3 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_3| = 9n^2 - 15n+6$.

Hence there are 3 types of δ-edges as given in Table 3.

Table 3: δ**-edge partition of** HC_n

$\delta_u, \delta_v \backslash uv \in E(G)$	Number of edges
(1, 1)	6
(1, 2)	12n-12
(2, 2)	$9n^2 - 15n+6$

Theorem 3. Let G be the graph of a nanotube OX_n. Then

(i) $\delta BSO_1(G) = 18n - 18.$

(ii) $\delta BSO_2(G) = \frac{36n}{5} + \frac{36}{5}.$

(iii) $\delta BSO_3(G) = \sqrt{2}\pi\left(\frac{9}{2}n^2 + \frac{25}{2}n - 11\right).$

(iv) $\delta BSO_4(G) = \frac{\pi}{2}\left(\frac{9}{4}n^2 + \frac{355}{12}n - \frac{155}{6}\right).$

(v) $\delta BSO_5(G) = \frac{72\pi(n-1)}{\sqrt{2}+2\sqrt{5}}.$

(vi) $\delta BSO_6(G) = \frac{108\pi(n-1)}{22+4\sqrt{10}}.$

Proof: From definitions and by using Table 3, we deduce

(i) $$\delta BSO_1(G) = \sum_{uv\in E(G)} \frac{|\delta_u^2 - \delta_v^2|}{2}$$

$$= 6\frac{|1^2-1^2|}{2} + (12n-12)\frac{|1^2-2^2|}{2} + (9n^2-15n+6)\frac{|2^2-2^2|}{2}$$
$$= 18n - 18.$$

(ii) $$\delta BSO_2(G) = \sum_{uv\in E(G)} \frac{|\delta_u^2 - \delta_v^2|}{\delta_u^2+\delta_v^2}$$

$$= 6\frac{|1^2-1^2|}{1^2+1^2} + (12n-12)\frac{|1^2-2^2|}{1^2+2^2} + (9n^2-15n+6)\frac{|2^2-2^2|}{2^2+2^2}$$
$$= \frac{36n}{5} + \frac{36}{5}.$$

(iii) $$\delta BSO_3(G) = \sum_{uv\in E(G)} \sqrt{2}\pi\frac{\delta_u^2+\delta_v^2}{\delta_u+\delta_v}$$

$$= \sqrt{2}\pi 6\frac{1^2+1^2}{1+1} + \sqrt{2}\pi(12n-12)\frac{1^2+2^2}{1+2}$$
$$+\sqrt{2}\pi(9n^2-15n+6)\frac{2^2+2^2}{2+2}$$
$$= \sqrt{2}\pi\left(\frac{9}{2}n^2 + \frac{25}{2}n - 11\right).$$

(iv) $$\delta BSO_4(G) = \sum_{uv\in E(G)} \frac{\pi}{2}\left(\frac{\delta_u^2+\delta_v^2}{\delta_u+\delta_v}\right)^2$$

$$= \frac{\pi}{2}6\left(\frac{1^2+1^2}{1+1}\right)^2 + \frac{\pi}{2}(12n-12)\left(\frac{1^2+2^2}{1+2}\right)^2$$
$$+\frac{\pi}{2}(9n^2-15n+6)\left(\frac{2^2+2^2}{2+2}\right)^2$$
$$= \frac{\pi}{2}\left(\frac{9}{4}n^2 + \frac{355}{12}n - \frac{155}{6}\right).$$

(v) $$\delta BSO_5(G) = \sum_{uv\in E(G)} 2\pi\frac{|\delta_u^2-\delta_v^2|}{\sqrt{2}+2\sqrt{\delta_u^2+\delta_v^2}}$$

$$= 2\pi 6\frac{|1^2-1^2|}{\sqrt{2}+2\sqrt{1^2+1^2}} + 2\pi(12n-12)\frac{|1^2-2^2|}{\sqrt{2}+2\sqrt{1^2+2^2}}$$
$$+2\pi(9n^2-15n+6)\frac{|2^2-2^2|}{\sqrt{2}+2\sqrt{2^2+2^2}}$$
$$= \frac{72\pi(n-1)}{\sqrt{2}+2\sqrt{5}}.$$

(vi) $$\delta BSO_6(G) = \sum_{uv\in E(G)} \pi\left(\frac{|\delta_u^2-\delta_v^2|}{\sqrt{2}+2\sqrt{\delta_u^2+\delta_v^2}}\right)^2$$

$$= \pi 6\left(\frac{|1^2-1^2|}{\sqrt{2}+2\sqrt{1^2+1^2}}\right)^2 + \pi(12n-12)\left(\frac{|1^2-2^2|}{\sqrt{2}+2\sqrt{1^2+2^2}}\right)^2$$
$$+\pi(9n^2-15n+6)\left(\frac{|2^2-2^2|}{\sqrt{2}+2\sqrt{2^2+2^2}}\right)^2$$
$$= \frac{108\pi(n-1)}{22+4\sqrt{10}}.$$

V. RESULTS FOR SILICATE NETWORKS

Silicate networks are obtained by fusing metal oxide or metal carbonates with sand. A silicate network is symbolized by SL_n, where n is the number of hexagons between the center and boundary of SL_n. A 2-D silicate network is presented in Figure 4.

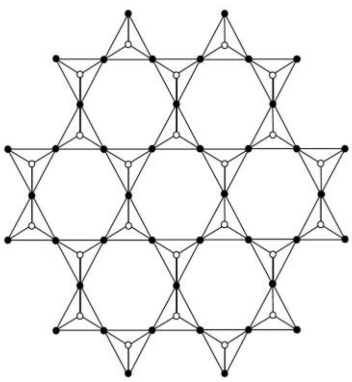

Figure 4. A 2-*D* silicate network

Let G be the graph of a silicate network SL_n. By calculation, we obtain that G has $15n^2+3n$ vertices and $36n^2$ edges. In G, there are two types of vertices as follows:

$V_1 = \{u \in V(G) \mid d_G(u) = 3\}$, $|V_1| = 6n^2 + 6n$.
$V_2 = \{u \in V(G) \mid d_G(u) = 6\}$, $|V_2| = 9n^2 - 3n$.

Therefore, we have $\delta(G)=3$ and hence $\delta_u = d_G(u) - \delta(G) + 1 = d_G(u) - 2$.

By calculation, in SL_n there are 3 types of edges based on degrees of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 3\}$, $|E_1| = 6n$.
$E_2 = \{uv \in E(G) \mid d_G(u) = 3, d_G(v) = 6\}$, $|E_2| = 18n^2 + 6n$.
$E_3 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 6\}$, $|E_3| = 18n^2 - 12n$.
Hence there are 3 types of δ-edges as given in Table 4.

Table 4: δ-edge partition of SL_n

$\delta_u, \delta_v \backslash uv \in E(G)$	Number of edges
(1, 1)	$6n$
(1, 4)	$18n^2 + 6n$
(4, 4)	$18n^2 - 12n$

Theorem 4. Let G be the graph of a nanotube SL_n. Then

(i) $\delta BSO_1(G) = 18n - 18.$

(ii) $\delta BSO_2(G) = \frac{36n}{5} + \frac{36}{5}.$

(iii) $\delta BSO_3(G) = \sqrt{2}\pi\left(\frac{9}{2}n^2 + \frac{25}{2}n - 11\right).$

(iv) $\delta BSO_4(G) = \frac{\pi}{2}\left(\frac{9}{4}n^2 + \frac{355}{12}n - \frac{155}{6}\right).$

(v) $\delta BSO_5(G) = \frac{72\pi(n-1)}{\sqrt{2}+2\sqrt{5}}.$

(vi) $\delta BSO_6(G) = \frac{108\pi(n-1)}{22+4\sqrt{19}}.$

Proof: From definitions and by using Table 4, we deduce

(i) $$\delta BSO_1(G) = \sum_{uv\in E(G)} \frac{\left|\delta_u^2 - \delta_v^2\right|}{2}$$
$$= 6n\frac{|1^2-1^2|}{2} + (18n^2+6n)\frac{|1^2-4^2|}{2} + (18n^2-12n)\frac{|4^2-4^2|}{2}$$
$$= 135n^2 + 45n.$$

(ii) $$\delta BSO_2(G) = \sum_{uv\in E(G)} \frac{\left|\delta_u^2 - \delta_v^2\right|}{\delta_u^2 + \delta_v^2}$$
$$= 6n\frac{|1^2-1^2|}{1^2+1^2} + (18n^2+6n)\frac{|1^2-4^2|}{1^2+4^2} + (18n^2-12n)\frac{|4^2-4^2|}{4^2+4^2}$$
$$= \frac{270}{17}n^2 + \frac{90}{17}n.$$

(iii) $$\delta BSO_3(G) = \sum_{uv\in E(G)} \sqrt{2}\pi\frac{\delta_u^2+\delta_v^2}{\delta_u+\delta_v}$$
$$= \sqrt{2}\pi 6n\frac{1^2+1^2}{1+1} + \sqrt{2}\pi(18n^2+6n)\frac{1^2+4^2}{1+4}$$
$$+\sqrt{2}\pi(18n^2-12n)\frac{4^2+4^2}{4+4}$$
$$= \sqrt{2}\pi\left(\frac{666}{5}n^2 - \frac{108}{5}n\right).$$

(iv) $$\delta BSO_4(G) = \sum_{uv\in E(G)} \frac{\pi}{2}\left(\frac{\delta_u^2+\delta_v^2}{\delta_u+\delta_v}\right)^2$$
$$= \frac{\pi}{2}6n\left(\frac{1^2+1^2}{1+1}\right)^2 + \frac{\pi}{2}(18n^2+6)\left(\frac{1^2+4^2}{1+4}\right)^2$$
$$+\frac{\pi}{2}(18n^2-12n)\left(\frac{4^2+4^2}{4+4}\right)^2$$
$$= \frac{\pi}{2}\left(\frac{12402}{25}n^2 - \frac{2916}{25}n\right).$$

(v) $$\delta BSO_5(G) = \sum_{uv\in E(G)} 2\pi\frac{\left|\delta_u^2-\delta_v^2\right|}{\sqrt{2}+2\sqrt{\delta_u^2+\delta_v^2}}$$
$$= 2\pi 6n\frac{|1^2-1^2|}{\sqrt{2}+2\sqrt{1^2+1^2}} + 2\pi(18n^2+6n)\frac{|1^2-4^2|}{\sqrt{2}+2\sqrt{1^2+6^2}}$$
$$+2\pi(18n^2-12n)\frac{|4^2-4^2|}{\sqrt{2}+2\sqrt{4^2+4^2}}$$
$$= \frac{180\pi(3n^2+n)}{\sqrt{2}+2\sqrt{5}}.$$

(vi) $$\delta BSO_6(G) = \sum_{uv\in E(G)} \pi\left(\frac{\left|\delta_u^2-\delta_v^2\right|}{\sqrt{2}+2\sqrt{\delta_u^2+\delta_v^2}}\right)^2$$
$$= \pi 6n\left(\frac{|1^2-1^2|}{\sqrt{2}+2\sqrt{1^2+1^2}}\right)^2 + \pi(18n^2+6n)\left(\frac{|1^2-4^2|}{\sqrt{2}+2\sqrt{1^2+4^2}}\right)^2$$
$$+\pi(18n^2-12n)\left(\frac{|4^2-4^2|}{\sqrt{2}+2\sqrt{4^2+4^2}}\right)^2$$
$$= \frac{1350\pi(3n^2+n)}{70+4\sqrt{34}}.$$

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