

## The Comparison Some Extended Weibull Distribution (Weibull (W), Exponentiated Weibull (EW), Exponentiated Exponential Weibull (EEW) and Additive Weibull (AW)) for the wind Speed Probability Modelling

Elpa Sugian<sup>1</sup>, Rado Yendra<sup>2</sup>, Muhammad Marizal<sup>3</sup>, Rahmadeni<sup>4</sup>

<sup>1,2,3,4</sup>Department of Mathematics, Faculty of Science and Technology, Universitas Islam Negeri Sultan Syarif Kasim Riau

ARTICLE INFO	ABSTRACT
Published Online: 06 December 2023	Accurate wind speed modeling is critical in estimating wind energy potential for harnessing wind power effectively. One of the bases for assessment of wind energy potential for a specified region is the probability distribution of wind speed, therefore wind speed data is needed to produce statistical modeling, especially in determining the best probability distribution. For this purpose, several modified weibull distributions will be used and tested to determine the best model to describe wind speed in Pekanbaru. The main goal of this study is to find the best fitting distribution to the daily wind speed measured over Pekanbaru region for the years 1999-2020 by using the four modified weibull distributions, namely Weibull (W), Exponentiated Weibull (EW), Additive Weibull (AW) and Exponentiated Exponential Weibull (EEW). The maximum likelihood method will be used to get the estimated parameter value from the distribution used in this study. Furthermore, the graphical inspection (density-density plot and cumulative plot) and numerical criteria (Akaike's information criterion (AIC), Bayesian Information Criteria (BIC), - log likelihood (- l)) were used to determine the best fit model. In most cases, the results produced by the graphical inspection were similar, and differed from the numerical criteria. The best fit result was chosen as the distribution with the lowest values of AIC, BIC and - l. In general, the Exponentiated Exponential Weibull (EEW) distribution has been selected as the best model.
Corresponding Author: <b>Elpa Sugian</b>	
<b>KEYWORDS:</b> Weibull, Exponentiated Weibull, Exponentiated Exponential Weibull, Additive Weibull, Wind Speed, Wind Energy.	

### I. INTRODUCTION

In this decade, Indonesia produces energy up to 96% of energy usage is based on fossil fuel, with the remaining 4% is renewable. Energy requirement is an important criteria in almost every aspect of daily life, fossil energy fuels are used. However, the increase in the use of fossil fuels poses a threat for people by way of air pollution, climate change and carbon emission [1]. This, therefore, led to the creation of a policy by the government that aims to increase the usage of renewable energy resources to 17% by 2025 [2]. Indonesia, as an archipelago country, has great potential for wind power generation, due to its high intensity rate in most regions. Wind energy as a new source, is rapidly developing in the world due to its natural abundance, renewability, low cost, and inability to pollute the environment. There are several important factors

in obtaining wind speed power. One of which is determine the characteristics of wind speed statistically [3]. It is well-known that wind energy potential can be estimated by using the distribution of the wind speed. Over the past years, research activity in the area of wind-speed distribution modelling has increased considerably. For the prediction of wind-speed distributions statistical models, several statistical distributions have been used for the description of the wind speed distribution. Determining the distribution of wind speed is quite effective in calculating the wind force [4]. In the literature, Weibull distribution has been used commonly in modelling the wind speed data [5-9]. There are also studies in which different distributions are preferred in modelling these data [10-13]. Morgan et al. [14] have used Generalized Rayleigh, Bimodal Weibull Mixture, Kappa and Wakaby

distributions for modelling the wind speed data. In this study, preliminary study on wind energy potential was evaluated statistically using the daily wind speed data between 1999 and 2020 in Pekanbaru. The objective of this study is to propose four extended Weibull distributions namely of The Weibull (W), Exponentiated Weibull (EW), Exponentiated Exponential Weibull (EEW), and Additive Weibull (AW) distributions were used for modelling the wind speed data of Pekanbaru. Comparison of the proposed distributions with existing distribution functions is done to demonstrate their suitability in describing wind speed characteristics. Unknown parameter estimations were obtained with the Maximum Likelihood Method. Graphical methods and Numerical criteria such as AIC, BIC, and  $-l$  were used to obtain the distribution which provides the best fit the wind speed data. In the following section, the distributions to be used in modelling the wind speed data are included

II. DATA SET AND STUDY AREA

Data used in this work were downloaded from the website of the Federal Office of Meteorology and Climatology of Indonesia (BMKG). The original data consisted of daily wind speed records from 1999 to 2020, which were provided by the BMKG of Pekanbaru city, Indonesia. The data and the histogram or characteristic wind speed are presented in Figure 1. Pekanbaru City located in Riau Province, with a tropical climate that has wind speeds that vary from 0.2 m/s to 15.8 m/s. Initial information on wind speed in the city of Pekanbaru can be seen in the descriptive statistics for daily wind speed are presented in Table 1. The variations of wind speed data that are not so large (1,514 m/s) indicate that this wind speed is quite stable and very good as a source of electrical energy for household purposes. The average wind speed which is not too high (2.088) has added to the belief that the wind speed in the Pekanbaru area cannot be used as a source of electrical energy for industrial purposes

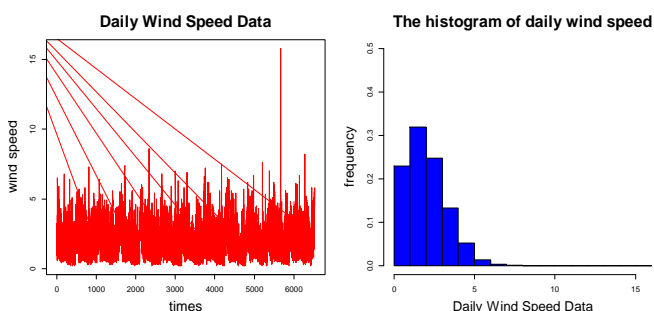


Fig. 1. Plot and histogram daily wind speed data on Pekanbaru respectively

Table 1: The descriptive statistics for daily wind speed.

Statistics	Value
Mean	2.088
Varians	1.51451
Minimum	0.200
Maximum	15.800

III. METHODOLOGY

The primary tools to describe wind speed characteristics are probability density functions. Weibull (W), Exponentiated Weibull (EW), Exponentiated Exponential Weibull (EEW), and Additive Weibull (AW) are used to describe wind speed characteristics. Parameters defining each distribution function are calculated using maximum likelihood method

A. Weibull Distribution (W).

In wind energy applications, two parameters Weibull distribution (W) is the widely used and accepted distribution for estimating wind energy potential thanks to W’s computable and flexible mathematical form [15, 16]. The probability density function (pdf) and cumulative distribution function (cdf) of the W respectively is given as follows:

$$f(x) = \frac{\alpha}{\beta} \left(\frac{x}{\beta}\right)^{\alpha-1} \exp\left(-\left(\frac{x}{\beta}\right)^\alpha\right), \quad \alpha, \beta > 0, x > 0$$

$$F(x) = 1 - \exp\left(-\left(\frac{x}{\beta}\right)^\alpha\right), \quad \alpha, \beta > 0, x > 0$$

where  $\alpha$  and  $\beta$  are shape and scale parameters, respectively

B. Exponentiated Weibull (EW)

The EW distribution is proposed by Mudholkar and Srivastava [17] and studied first by Mudholkar et al. [18] and further by Mudholkar and Hutson [19]. The density function (pdf) and The cumulative distribution function (CDF) of the EW distribution denoted by EW are respectively

$$f(x) = \lambda \alpha \theta x^{\theta-1} \exp(-\alpha x^\theta) \left(1 - \exp(-\alpha x^\theta)\right)^{\lambda-1}, \quad \lambda, \alpha, \theta > 0, x > 0$$

$$F(x) = \left(1 - \exp(-\alpha x^\theta)\right)^\lambda, \quad \lambda, \alpha, \theta > 0, x > 0$$

C. Additive Weibull (AW)

The additive Weibull (AddW) distribution has four parameters  $\alpha, \beta, \theta$  and  $\gamma$ . This distribution is first introduced by Xie and Lai [20] and is denoted by AW. The density function and cumulative distribution function of the AW, respectively is defined as follows:

$$f(x) = (\alpha x^{\theta-1} + \beta x^{\gamma-1}) \exp(-\alpha x^\theta - \beta x^\gamma), \quad \alpha, \theta, \beta > 0, \gamma < 1, x > 0$$

$$F(x) = 1 - \exp(-\alpha x^\theta - \beta x^\gamma), \quad \alpha, \theta, \beta > 0, \gamma < 1, x > 0$$

D. Exponentiated Exponential Distribution (EEW)

The Exponentiated Exponential Distribution (EEW) distribution has four parameters  $a, b, \alpha$  and  $\lambda$ . The density and cumulative distribution function of the EEW are respectively defined as follows:

$$f(x) = a x^b \exp\left(-a x^b (1 - \exp(-a x))^\lambda\right) \left(\frac{a \lambda}{\exp(a x) - 1} + \frac{b}{x}\right), \quad a, b, \alpha, \lambda > 0, x > 0$$

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$$F(x) = 1 - \exp\left(-ax^b(1 - \exp(-\alpha x))^\lambda\right),$$

$$a, b, \alpha, \lambda > 0, x > 0$$

The procedure of goodness of fit tests for model selection, both graphically and numerically. The formulations of Numerical criteria such as AIC and BIC for model evaluation are given in Table 2.

**Table 2. The formulas of numerical criteria for model evaluation**

Numerical Criteria	Formula
AIC	$-2l + 2q$
BIC	$-2l + q \log(n)$

$l = \log$  likelihood,  $q =$  Number of parameters,  $n =$  number of data

**IV. RESULT AND DISCUSSION**

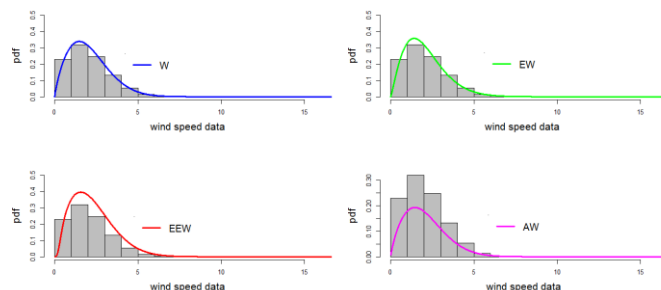
In this section, we analyze a daily wind speed data set to demonstrate the performance of the E, EW, EEW and AW distributions in practice. The fitting of that distributions was considered using data from the period between 1999 and 2020. Computed parameter values of different probability density function used for the BMKG Pekanbaru stations are presented in Table 3.

**Table 3. Computed parameter values of different probability density functions**

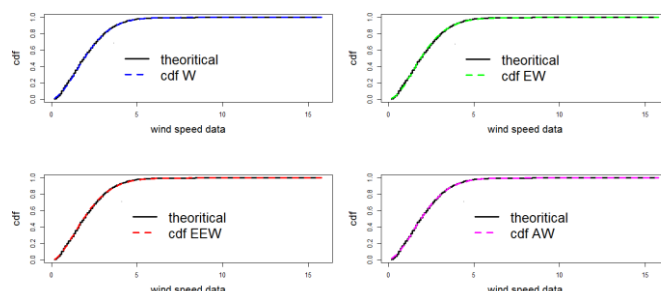
	W	EW	EEW	AW
$\alpha$	1.777	-	-	-
$\beta$	2.352	-	-	-
$\alpha$	-	0.377	-	-
$\theta$	-	1.468	-	-
$\lambda$	-	1.422	-	-
$a$	-	-	0.236	-
$b$	-	-	1.708	-
$\alpha$	-	-	5.422	-
$\lambda$	-	-	3.183	-
$\alpha$	-	-	-	0.224
$\theta$	-	-	-	1.756
$\beta$	-	-	-	$7.68 \times 10^{-8}$
$\gamma$	-	-	-	0.999

On the graphical presentation of modelling of the wind speed data, in other words on daily wind speed histogram, the density function curve for W, EW, EEW, and AW models are seen in Figure 2. When the density functions (pdf) are examined, it was determined that all four distributions have yielded similar results. Figure 3 shows the fitted four extended weibull distribution, based on cumulative distribution function (cdf) . From this figure W, EW, EEW and AW distribution model is able to provide a good result for wind speed data. However, instead of graphical

evaluation, Table 4 provides a more meaningful comparison using AIC, BIC, and  $-l$  values.



**Fig. 2 Fitted pdf plots of some extended Weibull distributions (W, EW, EEW and AW) for the given dataset.**



**Fig. 3 Fitted cdf plots of some extended Weibull distributions (W, EW, EEW and AW) for the given dataset**

Table 4 includes AIC, BIC and  $-l$  values test statistics values of the goodness of fit test for the fitness of daily wind speed data based on Maximum Likelihood Estimators for W, EW, EEW, and AW distributions. According to these results, although similar results are obtained for all four distributions, lowest AIC, BIC and  $-l$  value are obtained for EEW distribution. As a conclusion, it is seen that EEW distribution provides better modelling in terms of Numerical criteria

**Table 4. AIC, BIC, and – Log Likelihood (l) function values**

	W	EW	EEW	AW
AIC	19929.09	19898.09	<b>19800.56</b>	19934.58
BIC	19942.66	19918.44	<b>19827.69</b>	19961.72
$-l$	9962.545	9946.043	<b>9896.279</b>	9963.292

**V. CONCLUSION**

In the present article, a comparison of distribution models has been undertaken for describing wind regimes of Pekanbaru wind stations. Common conventional PDFs W and extended PDFs along with three proposed PDFs, viz., EW, EEW, and AW are used for wind speed modeling. It is shown that conventional PDFs, such as Weibull, is inadequate; hence, extended functions are used to model the observed wind speed distributions better. Results show clearly that proposed

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extended PDFs, EEW provide viable alternative to other PDFs in describing wind regimes

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