International Journal of Mathematics and Computer Research

ISSN: 2320-7167

Volume 11 Issue 12 December 2023, Page no. – 3902-3905

Index Copernicus ICV: 57.55, Impact Factor: 7.362

DOI: 10.47191/ijmcr/v11i12.04

The Effect of Viscous Dissipation on the Onset of Instability of Magneto Hydrodynamic Flow in a Porous Medium

Orukari, Mercy A.

Niger Delta University, Wilberforce Island. Bayelsa State. Nigeria

ARTICLE INFO

Published Online:
19 December 2023

Corresponding Author:
Orukari, Mercy A.

ABSTRACT

The study of flow of fluid with its motion was study in the nineteenth century by Maurice Couette which was as a result of two parallel plates in which the movement are relative, such that the surfaces is moving laterally. The plates with varying radii, could be flat, parallel or two concentric which is generally referred to as plane Couette. Characteristics of fluid at rest or in motion may undergo changes and become unstable. The critical value or range of the parameters of flow which will give rise to instabilities is one of the problems of stability analysis. The stability of fluid motion may be tested by perturbing the fluid with a small sinusoidal disturbances on the parameter. In plane Couette where moving plates drive the flow and plane Poiseuille flow where a pressure gradient drives the flow, in the situation of weak flow and strong distortional elasticity, an asymptotic analysis will yields closed - form steady solutions with identical wall conditions which will focus on simulations will expose the effects due to wall anchoring conflicts and illustrate the induced morphology of the orientation distribution, stored viscoelastic stresses and non - Newtonian flow.

KEYWORDS: Viscous, Dissipation, Stability, MHD, Porous Medium

INTRODUCTION

The hydrodynamic stability of Poiseuille flow in a curved channel was an early study for a channel formed by two concentric cylinders, with spacing (d) small compared with the radius of the inner cylinder (R_1). The flow was found to be unstable when $Re\left(\frac{d}{R_1}\right)^{0.5}$ exceeds a value of about 36, where Re is the Reynolds number based on the mean velocity of the unperturbed flow. The linear stability of the inertialess, pressure-driven Poiseuille flow of an oldroyd-B fluid through a slightly curved channel, the flow is shown to be unstable in certain flow parameters regimes and the instability is a stationary mode in MHD plane Poiseuille flow. Stability of flow problems was very well accepted and it started with Rayleigh (1892) and there has been increase in different ways and methods in its configurations (Hassard et al 1999, Orzag and Kelys 1980, Waters and Keeley 1987, Gupta 1999).

The effects of permeability and radiation that was studied by (Ngiangia and Wonu, 2007) on the stability of Couette flow in a porous medium, on which the both parameters had an independently affect the stability of Couette flow but with a high Reynolds number the effect of radiation was prominent.

The study of scaling properties with steady structure of nematic polymers in Couette cels and plane Poiseuille flow was done by (Cui et al (2006) and Forest et al (2004)). It was assumed that the fluid properties are constants, but Boussinesq relation that approximated the body force buoyancy term in the Navier - Stokes equation. The viscous dissipation that was generated by heat is very small and as such negligible which then reduces the function to $\mu\left(\frac{\partial V}{\partial Z}\right)^2$, this implies that in terms of velocity the flow will be fully developed. This work is a continuation of an earlier work of (Orukari (2012) by looking at the effects of viscous dissipation and magnetic field to his problem of study.

Most of the studies of this nature concentrated mainly in pipes and concentric cylinders of varying radii. The attempt therefore, is to investigate the effect of the given parameters in two infinitely long parallel plates in a porous medium.

MATHEMATICAL FORMULATION

We consider plane Couette - Poiseuille flow in an infinite parallel plate, the motion of the fluid produces by pressure gradient of the Poiscuille flow has a relative movement in Couette flow of the plates. The basic equations governing the

the problem following the (Orukari et al (2011), and Mebine (2007)):

$$\nabla . V = 0 \quad (1)$$

$$\rho\left(\frac{\partial V}{\partial t} + (V.\nabla)V\right) = -\nabla P + \mu\nabla^2\nabla + \rho g - \frac{\upsilon}{k}V - \frac{\sigma\mu^2 H_0 V}{\rho_\infty} \quad (2)$$

$$\left(\frac{\partial T}{\partial t} + (V.\nabla)T\right) = \alpha^2\nabla^2 T - \frac{1}{\rho c_p}\nabla . q_2 + \frac{\mu}{p c_p}\left(\frac{\partial V}{\partial Z}\right)^2 \quad (3)$$

Where T is the temperature, V is fluid velocity, ρ is fluid density, P is Pressure, μ is absolute viscosity g is acceleration due to gravity, υ *is* kinematic viscosity, k is permeability of the medium, H_0 is magnetic field, ρ_∞ is porous medium density ,α , is thermal diffusivity and q_2 is radiative term

$$\frac{\partial^2 q_2}{\partial z^2} - 3\alpha^2 q_2 - 16\alpha T_\infty^3 \frac{\partial T}{\partial z} = 0 \quad (4)$$

A formulation in terms intego-differential equations is a treatment of radiative transfer. The approximation theories that were developed permit a formulation involving differential equation. The formulation of the problem is a modification by radiative term which will reduce equation (4) to

$$\frac{\partial q_2}{\partial Z} = 4\delta^2(T - T_\infty) \quad (5)$$

Where $\delta^2 = \int_0^\infty \left(\alpha_{k^*}\frac{\partial \Lambda}{\partial T}\right) dk^*$ (6)

Where Λ is the Planck's function, α_{k^*} is the absorption coefficient, k^* is the frequency of radiation and T is the temperature. Using Boussinesq approximation and substituting equation (5) in equation (3) and this will course the effect of variation of density with temperature exclusively to the body force term. The assumptions, gives the flow equation that describe the physical situation below:

$$\frac{\partial V}{\partial Z} = 0 \quad (7)$$

$$\rho\frac{\partial V}{\partial t} = -\frac{\partial P}{\partial Z} + \frac{\mu\partial^2 V}{\partial Z^2} + g\rho_0\xi(T - T_0) - \frac{\upsilon}{k}V - \frac{\sigma_0\mu^2 H_0^2 V}{\rho_\infty} \quad (8)$$

$$\frac{\partial T}{\partial t} = \frac{\alpha^2\partial^2 T}{\partial Z^2} - \frac{4\delta^2(T - T_0)}{\rho c_\rho} + \frac{\mu}{\rho c_\rho}\left(\frac{\partial V}{\partial Z}\right)^2 \quad (9)$$

Where ξ is coefficient of volume expansion.

PERTURBATION

The disturbance in the velocity field, temperature field, and pressure field are denoted by

$$V^1 = V - V_e, T^1 = T - T_e, P^1 = P - P_e \quad (10)$$

Where, e denotes equilibrium values.

If we put (10) in (7), (8) and (9) and keeping unity terms only, the following linearized equations are obtained

$$\frac{\partial V^1}{\partial Z} = 0 \quad (11)$$

$$\rho\frac{\partial V^1}{\partial t} = -\frac{\partial P^1}{\partial Z} + \frac{\mu\partial^2 V^1}{\partial Z^2} - g\rho_0\xi(T^1 - T_0^1) - \frac{\upsilon}{k}V^1 - \frac{\sigma_0\mu^2 H_0^1 V^1}{\rho_\infty} \quad (12)$$

$$\frac{\partial T^1}{\partial t} = \frac{\alpha^2\partial^2 T^1}{\partial Z^2} - \frac{4\delta^2(T^1 - T_0^1)}{\rho c_\rho} + \frac{\mu}{\rho c_\rho}\left(\frac{\partial V^1}{\partial Z}\right)^2 \quad (13)$$

Non - dimensional analysis

For dimensional homogeneity, we substitute the following expressions

$$Z = \frac{V^1 t}{d}, P = \frac{P^1}{\rho V^2}, \alpha^2 = \frac{4\delta^2\rho_\infty c_\infty d^2}{\rho c_\rho \upsilon}, K^\alpha = \frac{\upsilon\mu d^2}{k\rho}$$

$$V = \frac{V^1}{U}, \beta^2 = \frac{\alpha^2\rho t}{T_\infty}, g = \frac{gd}{V^2}, \theta = \frac{T^1 - T_\infty^1}{T - T_\infty}$$

$$Re^{-1} = \frac{\mu}{Vd\rho}, t = \frac{Vd}{t}, M^2 = \frac{\sigma_0\mu^2 H_0^1\upsilon}{\rho_\infty U^2}, Gr = \left[g\zeta\frac{(T - T_0)d^3}{V^2}\right]$$

$$Pr = \frac{\mu c_\rho}{\alpha^2\rho c_v}, Ec = \frac{U^2}{c_\rho(T - T_0)}$$

into (11) to (13), which results in

$$\frac{\partial V}{\partial Z} = 0 \quad (14)$$

$$\rho\frac{\partial V}{\partial t} = -\frac{\partial P}{\partial Z} + Re^{-1}\frac{\partial^2 T}{\partial Z^2} + Gr\theta - K''V - M^2 V \quad (15)$$

$$\frac{\partial\theta}{\partial t} = \beta^2\frac{\partial^2\theta}{\partial Z^2} - \alpha^2\theta + Pr\,Ec\left(\frac{\partial V}{\partial Z}\right)^2 \quad (16)$$

Where the parameter M is dimensional magnetic field, Pr is Prandtl number, Ec is Eckert number and Gr is Grashof number. Equations (15) and (16) are subject to the boundary conditions

$\theta(0) = 1, \theta(\infty) = 0$

$V(0) = 0, V(d) = U$ for Couette flow

$v(0) = 0, V(d) = 0$ for Poiseuille flow

When we assume, that the fluid velocity at the wall of the plates is equal to the wall velocity and that the condition is no- slip one.

METHOD OF SOLUTION

The problem of in equations (15) and (16) are non-linear equations and a step by step numerical integration will be involved. However, analytical solution is possible, if we assume small Re as that of (Gbadeyan & Idowu 2006) and by adopting regular perturbation by (Israel- Cookey et al 2003)

$$V(Z,t) = V_0(Z) + ReV_1(Z)e^{t\omega t} \quad (17a)$$

$$\theta(Z,t) = \theta_0(Z) + Re\theta_1(Z)e^{t\omega t} \quad (17b)$$

Substituting equation (17) into equations (15) and (16), neglecting (θRe^2) and simplifying, we will get sequence of approximations below after collecting terms of the same order:

$$Re^{-1}V_0^{11}(Z) + Gr\theta_0(Z) - M^2 V_0(Z) - k_\rho = 0 \quad (18)$$

$$\beta^2\theta_0^{11}(Z) - \alpha^2\theta_0(Z) + Pr\,EcV_0^1(Z) = 0 \quad (19)$$

Subject to

$\theta(0) = 1, \theta_0(\infty) = 0$

$V_0(0) = 0, V_0(d) = U$ (20)

Therefore 0(1) equations, and

$$i\omega V_1(Z) = Re^{-1}V_1^{11}(Z) + Gr\theta_1(Z) - K^{\alpha}V_1(Z) - M^2V_1(Z) \quad (21)$$

$$i\omega\theta_1(Z) = \beta^2\theta_1^{11}(Z) - \alpha^2\theta_1(Z) + 2Pr\, EcV_0^1(Z)V_1^1(Z) \quad (22)$$

Subject to

$V_1(0) = 0, V_1(d) = U$

$\theta_1(0) = 1, \theta_1(\infty) = 0$ (23)

For 0(Re) equations.

Where $k_\rho = \frac{1}{\rho}\frac{\partial P}{\partial Z}$ is a constant pressure gradient

To solve the nonlinear- coupled equations of (18) - (23) we will assume the Eckert number (Ec) is small, and so, an asymptotic expansion for the flow with temperature and velocity will be as follows

$$V_0(Z) = V_{01}(Z) + EcV_{02}(Z) \quad (24a)$$

$$\theta_0(Z) = \theta_{01}(Z) + Ec\theta_{02}(Z) \quad (24b$$

$$V_1(Z) = V_{11}(Z) + EcV_{12}(Z) \quad (24c)$$

$$\theta_1(Z) = \theta_{11}(Z) + Ec\theta_{12}(Z) \quad (24d)$$

Substituting equation (24) into equations (16) - (23), we will have the sequence of approximations below;

$$Re^{-1}V_{01}^{11}(Z) + Gr\theta_{01}(Z) - (K^{\alpha} + M^2)V_{01}(Z) - k_\rho = 0 \quad (25)$$

$$\beta^2\theta_{01}^{11}(Z) - \alpha^2\theta_{01}(Z) = 0 \quad 26)$$

$$Re^{-1}V_{02}^{11}(Z) + Gr\theta_{02}(Z) - (K^{\alpha} + M^2)V_{02}(Z) = 0 \quad 27)$$

$$\beta^2\theta_{02}^{11}(Z) - \alpha^2\theta_{02}(Z) = 0 \quad (28)$$

Subject to

$$V_{01}(0) = 0, V_{01}(d) = U, V_{02}(0) = 0, V_{02}(d) = U \quad (29)$$

$\theta_{01}(0) = 1, \theta_{01}(\infty) = 0, \theta_{02}(0) = 1, \theta_{02}(\infty) = 0$

For 0(1) equations, and

$$i\omega V_{11}^1(Z) = Re^{-1}V_{11}^{11}(Z) + Gr\theta_{11}(Z) - (K^{\alpha} + M^2)V_{11}(Z) \quad (30)$$

$$\iota\omega\theta_{11}(Z) = \beta^2\theta_{11}^{11}(Z) - \alpha^2\theta_{11}(Z) \quad (31)$$

$$i\omega V_{11}^1(Z) = Re^{-1}V_{12}^{11}(Z) + Gr\theta_{12}(Z) - (K^{\alpha} + M^2)V_{12}(Z) \quad (32)$$

$$\iota\omega\theta_{12}(Z) = \beta^2\theta_{12}^{11}(Z) - \alpha^2\theta_{12}(Z) \quad (33)$$

Subject to

$V_{11}(0) = 0, V_{11}(d) = U, V_{12}(0) = 0, V_{12}(d) = U$

$$\theta_{11}(0) = 1, \theta_{11}(\infty) = 0, \theta_{12}(0) = 1, \theta_{12}(\infty) = 0 \quad (34)$$

For 0(Ec) equations.

Solving equation (28), we assume a solution of the form

$$\theta_{01}(Z) = e^{\lambda Z} \quad (35)$$

Substituting equation (35) into equation (28) with the B.C of (29), we will get

$$\theta_{02}(Z) = e^{m_1Z} \quad (36)$$

If we substitute equation (36) into equation (27) and simplify, we obtain

$$V_{02}^{11}(Z) - AV_{02}(Z) = -A_2e^{m_1Z} \quad (37)$$

Using the boundary conditions of equation (29) in equation (37) gives

$$V_{01}^{11}(Z) = A_1e^{m_6Z} + Ue^{-m_6Z} - A_2e^{m_1Z} \quad (38)$$

From equation (28), we can get the solution of equation (34) as

$$\theta_{12}(Z) = e^{m_1Z} \quad (39)$$

Substituting equation (39) into equation (32) and simplifying, results

$$V_{12}^{11}(Z) - A_3V_{12}^1(Z) - AV_{12}(Z) = -A_2e^{m_2Z} \quad (40)$$

From equation (28), the solution to equation (31) can be written as

$$\theta_{11}(Z) = e^{m_2Z} \quad (41)$$

Substituting equation (41) into equation (30) and rearrangement results in

$$V_{11}^{11}(Z) - A_3V_{11}^1(Z) - AV_{11}(Z) = -A_2e^{m_2Z} \quad (42)$$

The solution of equation (40) and equation (42) with the boundary conditions of (34) is

$$V_1(Z) = (C_1 + U)e^{m_3Z} + Ue^{m_4Z} + C_1e^{m_2Z} \quad (43)$$

To determine the complete solution of equation **(4.10)** using the same method) can be written as

$$\theta_{01}(Z) = e^{m_1Z} \quad (44)$$

We substitute equation (44) into equation (25) and after simplification, we get

$$V_{01}^{11}(Z) - AV_{01}(Z) = A_4 - A_2e^{m_1Z} \quad (45)$$

To determine the complimentary function of equation (37), and the solution of equation (29) for the particular integral of the same equation, is given by

$$V_{01}(Z) = (U - C_3)e^{m_5Z} + (C_2 + U)e^{-m_5Z} - C_2 - C_3e^{m_1Z} \quad (46)$$

Substituting equations (38) and (46) into equations (24a), gives

$$V_0(Z) = (U - C_3)e^{m_5Z} + (C_2 + U)e^{-m_5Z} - C_2 - C_3e^{m_1Z} + Ec(A_1e^{m_6Z} + Ue^{m_6Z} - A_2e^{m_1Z}) \quad (47)$$

Also, further substituting equations (36) into equation (24) results,

$$\theta_0(Z) = e^{m_1Z} + Ece^{m_1Z} \quad (48)$$

Again, substituting equations (43) and (41) into equation (24c) results,

$$V_1(Z) = (C_1 + U)e^{m_3Z} + Ue^{m_4Z} + C_1e^{m_2Z} + Ec(C_1 + U)e^{m_3Z} + Ue^{m_4Z} + C_1e^{m_2Z} \quad (49)$$

Finally, putting equations (39) and (41) into equation (24d), we get

$$\theta_1(Z) = e^{m_1Z} + Ece^{m_1Z} \quad (50)$$

Similarly, if we put equations (47) and (49) into (17a) and equations (48) and (50) into (17b), the temperature and velocity profiles are obtain respectively as:

$$V(Z,t) = (U - C_3)e^{m_5Z} + (C_2 + U)e^{-m_5Z} - C_2 - C_3e^{m_1Z} + Ec(A_1e^{m_6Z} + Ue^{m_6Z} - A_2e^{m_1Z}) + Re(e^{m_1Z} + Ece^{m_1Z})e^{i\omega t} \quad (51)$$

$$\theta(Z,t) = (1 + Ec)e^{m_1 Z} + (Ree^{m_1 Z} + ReEce^{m_1 Z})e^{i\omega t} \quad (52)$$

CONCLUSION

Since the fluid properties are the heat generated by viscous shear viscous dissipation is not negligible and the function reduces to $\mu\left(\frac{\partial v}{\partial z}\right)^2$. The flow was fully developed in terms of velocity and the difference in temperature between the plates and that of the fluid is large enough for free convection to flow. This condition may prevail in practice and therefore is physically important.

REFERENCES

1. Cogley, A, C; Vincenti, W, G; Giles, E, S (1968). Differential approximation of a radiative heat transfer. ATAA Journal 6551-6554.
2. Cui, Z; Porest, M, G; Wang, Q; Zhou, H2006). On weak Plane Couette and Poiseuille flows of rigid rod and platelet ensembles. SIAM J. Appl. Math. 418 - 429.
3. Gbadeyan, J, A; Idowu, A, S (2006). Radiation effect of magnetohydrodynamic flow of gas between concentric spheres. J *of NAMP. Vol. 10, 305-314.*
4. Mebine, P (2007). Radiation effects on MHD Couette flow with heat transfer between two parallel plates. *Global Journal of Pure and Applied Mathematics. Vol. 3, 2, 191-202.*
5. Muhammad Ashraf, Asifa Ilyas, Zia Ullah and Aamir Ail (2022). *Proceeding of the Institution of Mechanical Engineer. Journal of Process Mechanical Engineering 236(6)*
6. Ngiangia, A; Ikata, E; Wonu, N (2008). Influence of radiation on the onset of instability of magneto hydrodynamic plane Poiseuille flow in a porous medium. J*ournal of the Mathematical Association of Nigeria.*
7. Orukari, M.A. (2012) The effects of Radiation Absorption and vertical magnetic field on Thermosolutal convection in a porous medium. *International Journal Of Applied Mathematical Sciences Vol. 5, (1), Pp 35-49.*
8. Orukari, M.A., Ngiangia, A.T. and Frimabo, L. J. G (2011)Influence of Viscous Dissipation and Radiation on MHD Coutte Flow in a Porous Medium. *Journal of Nigerian Mathematical Physics vol. 18, 2011, pp201-208*
9. Oladele, R, O; Gbadeyan, J, A; Taiwo, O, A (2006). Viscous dissipation effects on the flow of a radiating gas between concentric elliptic cylinders. *J of NAMP. Vol. 10, 299-304.*
10. Orzag, S, A; Kells, L, C (1980). Transition to turbulence in plane Poiseuille and plane Couette flow. *J. Fluid Mech. 96, 159 - 168.*
11. Spurk, J (1997). Fluid Mechanics. Springer - Verlag. Berlin Heidelberg. New York.
12. Zikanov, O, Y (1996). On the instability of pipe Poiseuille flow. *Phys. Fluids, Vol. 8, 1, 2923 - 2932.*

.