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Availability Evaluation of Warm Standby System with Fault Detection Delay and General Repair Times

Kanta[1], Sanjay Chaudhary[2]

[1,2]Department of mathematics, Dr. Bhimrao Ambedkar University, Agra, U.P. 282002, India

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Corresponding Author:
Kanta

ABSTRACT

This study analyzed the availability of a warm standby system that works with fault detection delay and general repair times. The time-to-detection delay is also considered as exponentially distributed. The detection state is used to detect the faults in the failed unit. The steady state availability of the system is obtained by using supplementary variable technique. Three types of repair time distributions are compared to find the best one.

KEYWORDS: Availability; Repair time distributions; Warm standby; Detection delay; Supplementary variable technique

1. INTRODUCTION

Availability has increased tremendously in accordance with the present day applications. It is used extensively in various field of engineering, such as production system, manufacturing system, parallel redundant system, multiprogramming system, and industrial system. The motivation of the system used in this study is from power plants, solar energy systems, steam turbine power plants as used in-house in distillery industries and other industries. The standby units are used to enhance the availability of systems. There are three types of standby units: cold standby, warm standby and hot standby. One of the ways increasing system availability is to allowed repair of primary units as well as standby units.

A lot of work has been done in this context; Cox (1955) introduced supplementary variable technique. According to this technique, a non-Markovian process is made Markovian by the inclusion of supplementary variables. Gupta and Rao (1994) gave an explanation of steady state probabilities for different models. Trivedi (2002) explained detection delay for repairable systems. Wang and Pearn (2003) analyzed cost benefit of various systems with warm standbys. Wang and Chen (2009) compared steady state availability of three systems that worked with switching failures. El-Sherbeny (2012) discussed series systems with Erlang repair time distribution. Singh et al. (2013) obtained MTTF and availability of a system. They also analyzed cost benefits of that system. Levitin et al. (2014) introduced reliability of non-Coherent warm standby system. Yu et al. (2014) optimized availability by dependency modeling. Ke and Liu (2014) expressed a system with reboot. Wang et al. (2014) obtained availability of M/G/1 model with imperfect coverage. Adlakha et al. (2017) considered reliability of a system which worked with two cold standby systems. This system used for communication. Kim (2018) worked on optimization of reliability of a system with component sequencing.

Ke et al. (2018) discussed a model with standbys. In this model unreliable repairman facility was also available. Wang et al. (2018) expressed a cold standby system with maintenance. Patowary et al. (2019) discussed redundancy modeling of hot standby system. Yen et al. (2020) compared availability of different systems with general repair time and detection delay. Lv (2021) examined a system with unreliable server. Tenekedjiev et al. (2021) evaluated reliability of warm standby system with switching.

The purpose of this study is to achieve three objectives. The first one is to derive steady state availability of the system by exploiting the supplementary variable technique (remaining repair time treated as supplementary variable). Second one is to derive explicit expressions for steady state availability for three different repair time distributions: exponential (M), k-stage Erlang, (E_K) and deterministic (D), respectively. Third one is to perform numerical analysis in terms of availability (A_V) for three different repair time distributions.

2. 2. SYSTEM DESCRIPTION

The system studied here is useful for many industries where availability of system plays an important role. Electricity distribution is one of them, to make electricity available without break; we consider a warm standby system with detection delay and general repair times. The system worked under the statement that the times to failure and repair of units (primary and warm standby) are dispersed exponentially and generally, respectively. The primary and warm standby units are considered with failure rate λ and α $(0 < \alpha < \lambda < 1)$ respectively. A fault detecting method is used in the system to detect fault whenever warm standby or primary unit fails. If a unit fails, it is instantaneously inferred a detection delay. Detection delay is the time from detecting the fault to replace the failed unit to primary unit. Detection delay rate also has an exponential distribution with parameter δ. It is assumed that units can be repaired. We suppose that times to repair of failed units are random variables (i.i.d.) which having the distribution $B(u)$ $(u \geq 0)$ with a probability density function $b(u)$ $(u \geq 0)$ and mean repair time b_1. When a primary unit fails it is replaced immediately by a warm standby and failed unit will go to repair after fault detection as FCFS.

As per state transition diagram, the initial state of the system is (3, 0) with one primary and two warm standby units. On the failing of primary unit, the system goes to state (2, 1) which is known as detection state. A fault will be detected in this state. After fault detection system goes from state (2, 1) to state (2, 0) as working state or to state (1, 2). In state (2, 0) the failed unit is under repair, one unit is primary and another as warm standby. State (1, 2) is a detection state where one primary and one warm standby unit are failed and one unit as warm standby. Now from state (2, 0) on failure of a unit system goes to state (1, 1) known as detection state one failed unit and one as standby. From state (1, 2) system goes to state (1, 1) or system failure state (SF). After this from state (1, 1) system may go to state (1, 0) where two failed units are in repair and one as primary unit, no unit remains as warm standby or to system failure state (SF). At last from state (1, 0) system goes to system failure state (SF) after failing of one remaining primary unit. In the system, state (3, 0), (2, 0) and (1, 0) are working states and failure unit is being repaired. The state (2, 1), (1, 2) and (1, 1) are known as detection states for fault detection and (SF) is a system failure state.

3. NOTATIONS

Following notations are used in the derivation.

M (t): Number of working units,

N (t): Number of detecting units,

U (t): Lasting repair time for the units being repaired,

δ: Detection delay rate,

λ: Failure rate of primary units in the system,

α: Failure rate of a warm standby units in the system,

B (u): Repair time distribution function,

b (u): Probability density function of repair time.

$P_{m,n}(t)$: The probability at time t, where m units are operating and n units are under detection respectively at time $t \geq 0$,

$P^{*}_{m,n}(s)$: Laplace-Stieltjes transformation of $P_{m,n}(t)$,

$P^{*(1)}_{m,n}(s)$: First order derivative of $P_{m,n}(t)$ with respect to s

4. STATE TRANSITION RATE DIAGRAM

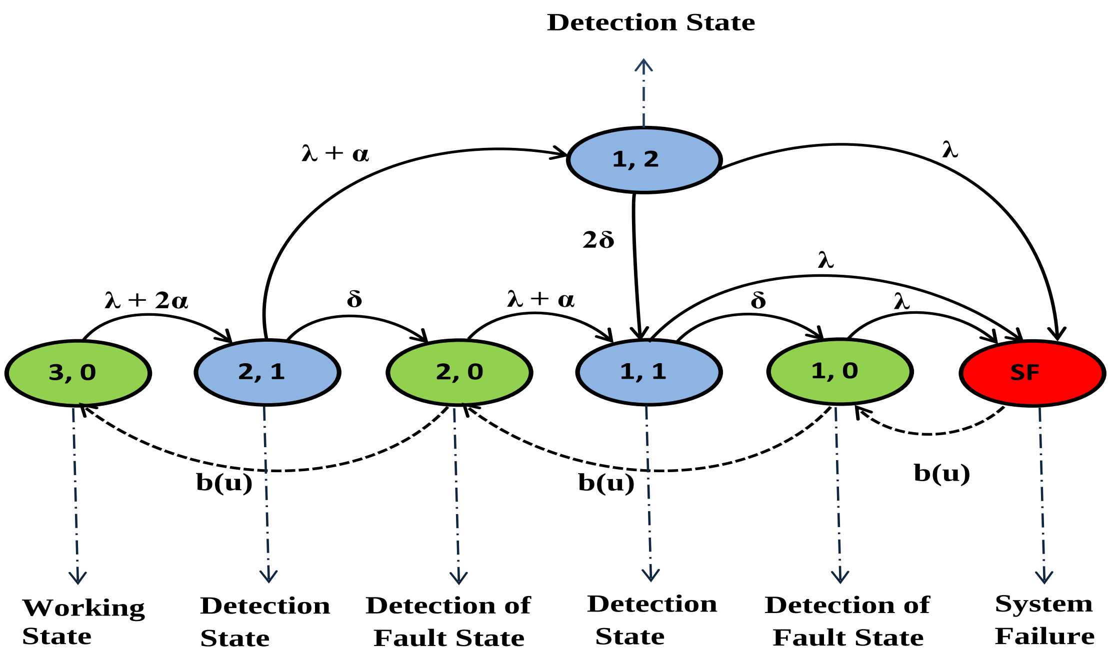

FIG 3.1 STATE TRANSITION DIAGRAM

5. System Equations:

The differential equations of each state are given as following.

$$P'_{3,0}(t) = -(\lambda+2\alpha)P_{3,0}(t) + P_{2,0}(0,t) \quad \dots (1)$$

$$P'_{2,1}(t) = -(\lambda+\alpha+\delta)P_{2,1}(t) + (\lambda+2\alpha)P_{3,0}(t) \quad \dots (2)$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial u}\right)P_{2,0}(u,t) = -(\lambda+\alpha)P_{2,0}(u,t) + \delta P_{2,1}(u,t) + b(u)P_{1,0}(u,t) \quad \dots (3)$$

$$P'_{1,1}(t) = -(\lambda+\delta)P_{1,1}(t) + (\lambda+\alpha)P_{2,0}(t) + 2\delta P_{1,2}(t) \quad \dots (4)$$

$$P'_{1,2}(t) = -(\lambda+2\delta)P_{1,2}(t) + (\lambda+\alpha)P_{2,1}(t) \quad \dots (5)$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial u}\right)P_{1,0}(u,t) = -\lambda P_{1,0}(u,t) + \delta P_{1,1}(u,t) + b(u)P_{sf}(u,t) \quad \dots (6)$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial u}\right)P_{sf}(u,t) = \lambda P_{1,1}(u,t) + \lambda P_{1,2}(u,t) + \lambda P_{1,0}(u,t) \quad \dots (7)$$

We get the following steady state equations from differential difference equations.

$$0 = -(\lambda+2\alpha)P_{3,0} + P_{2,0}(0) \quad \dots (8)$$

$$0 = -(\lambda+\alpha+\delta)P_{2,1} + (\lambda+2\alpha)P_{3,0} \quad \dots (9)$$

$$-\frac{\partial}{\partial u}P_{2,0}(u) = -(\lambda+\alpha)P_{2,0}(u) + \delta P_{2,1}(u) + b(u)P_{1,0}(0) \quad \dots (10)$$

$$0 = -(\lambda+\delta)P_{1,1} + (\lambda+\alpha)P_{2,0} + 2\delta P_{1,2} \quad \dots (11)$$

$$0 = -(\lambda+2\delta)P_{1,2} + (\lambda+\alpha)P_{2,1} \quad \dots (12)$$

$$-\frac{\partial}{\partial u}P_{1,0}(u) = -\lambda P_{1,0}(u) + \delta P_{1,1}(u) + b(u)P_{sf}(0) \quad \dots (13)$$

$$-\frac{\partial}{\partial u}P_{sf}(u) = \lambda P_{1,1}(u) + \lambda P_{1,2}(u) + \lambda P_{1,0}(u) \quad \dots (14)$$

In steady state, we define further more

$$P_{2,1}(u) = b(u)P_{2,1}$$

$$P_{1,1}(u) = b(u)P_{1,1}$$

$$\& \quad P_{1,2}(u) = b(u)P_{1,2}$$

Further, we describe the following LST expressions.

$$B^*(S) = \int_0^\infty e^{-su}\,dB(u) = \int_0^\infty e^{-su}b(u)\,du$$

$$P^*_{n,m}(S) = \int_0^\infty e^{-su}P_{n,m}(u)\,du$$

$$P_{n,m} = P^*_{n,m}(0) = \int_0^\infty P_{n,m}\,du$$

$$\int_0^\infty e^{-su}\frac{d}{dt}P_{n,m}(u)\,du = SP^*_{n,m}(S) - P_{n,m}(0)$$

It follows from Equations (8), (9) & (12).

$$P_{2,0}(0) = (\lambda+2\alpha)P_{3,0} \quad \dots (15)$$

$$P_{2,1} = \frac{(\lambda+2\alpha)}{(\lambda+\alpha+\delta)}P_{3,0} \quad \dots (16)$$

$$P_{1,2} = \frac{(\lambda+\alpha)}{(\lambda+2\delta)}P_{2,1} \quad \dots (17)$$

We obtain the following value after putting Equation (16) into Equation (17).

$$P_{1,2} = \frac{(\lambda+\alpha)(\lambda+2\alpha)}{(\lambda+2\delta)(\lambda+\alpha+\delta)}P_{3,0} \quad \dots (18)$$

On taking the LST on both sides of Equation (10) and using $P_{2,1}(u) = b(u)P_{2,1}$ we obtain,

$$(\lambda+\alpha-S)P^*_{2,0}(s) = \delta B^*(s)P_{2,1} + B^*(s)P_{1,0}(0) - P_{2,0}(0) \quad \dots (19)$$

Setting S=λ+α And S=0 in Equation (19). We obtain,

$$P_{1,0}(0) = \frac{P_{2,0}(0) - \delta B^*(\lambda+\alpha)P_{2,1}}{B^*(\lambda+\alpha)} \quad \dots (20)$$

$$P_{2,0} = P^*_{2,0}(0) = \frac{\delta P_{2,1} + P_{1,0}(0) - P_{2,0}(0)}{(\lambda+\alpha)} \quad \dots (21)$$

After substituting Equations (15) & (16) into Equation (20), we get the following result.

$$P_{1,0}(0) = \frac{(\lambda+2\alpha)\{(\lambda+\alpha+\delta) - \delta B^*(\lambda+\alpha)\}}{(\lambda+\alpha+\delta)B^*(\lambda+\alpha)}P_{3,0} \quad \dots (22)$$

$$P_{1,0}(0) = \phi P_{3,0} \quad \dots (23)$$

Where

$$\phi = \frac{(\lambda+2\alpha)\{(\lambda+\alpha+\delta) - \delta B^*(\lambda+\alpha)\}}{(\lambda+\alpha+\delta)B^*(\lambda+\alpha)}$$

This implies from after putting Equations (15), (16) and (22) into Equation (21).

$$P_{2,0} = \frac{(\lambda+2\alpha)\{1 - B^*(\lambda+\alpha)\}}{(\lambda+\alpha)B^*(\lambda+\alpha)}P_{3,0} \quad \dots (24)$$

Substituting Equations (18) and (24) into Equation (11). We obtain,

$$P_{1,1} = \psi P_{3,0} \quad \dots (25)$$

Where

$$\psi=\frac{(\lambda+2\alpha)\left[(\lambda+2\delta)(\lambda+\alpha+\delta)-\left\{\lambda^2+\alpha\lambda+\lambda\delta+2\delta^2\right\}B^*(\lambda+\alpha)\right]}{(\lambda+\delta)(\lambda+2\delta)(\lambda+\alpha+\delta)B^*(\lambda+\alpha)}$$

Again taking LST of Equation (13) on both sides and using $P_{1,1}(u)=b(u)P_{1,1}$

$$(\lambda-s)P^*_{1,0}(s)=\delta B^*(s)P_{1,1}+B^*(s)P_{sf}(0)-P_{1,0}(0) \quad \dots (26)$$

Setting S=λ and S=0 in Equation (26). We obtain,

$$P_{sf}(0)=\frac{P_{1,0}(0)-\delta B^*(\lambda)P_{1,1}}{B^*(\lambda)} \quad \dots (27)$$

Substituting Equations (23) and (25) into Equation (27). We get,

$$P_{sf}(0)=\frac{\left\{\phi-\delta B^*(\lambda)\psi\right\}}{B^*(\lambda)}P_{3,0} \quad \dots (28)$$

And S=0 into Equation (26)

$$P^*_{1,0}(0)=\frac{\delta P_{1,1}+P_{sf}(0)-P_{1,0}(0)}{\lambda} \quad \dots (29)$$

We get after substituting Equations (23), (25) and (28) into Equation (29).

$$P_{1,0}=P^*_{1,0}(0)=\frac{\phi\left\{1-B^*(\lambda)\right\}}{\lambda B^*(\lambda)}P_{3,0} \quad \dots (30)$$

After taking LST on both sides of Equation (14) and using $P_{1,1}(u)=b(u)P_{1,1}$ & $P_{1,2}(u)=b(u)P_{1,2}$. We get,

$$sP^*_{sf}(s)=P_{sf}(0)-\lambda B^*(s)P_{1,1}-\lambda B^*(s)P_{1,2}-\lambda P^*_{1,0}(s) \quad \dots (31)$$

We get the result on differentiating Equation (31) with respect to S and putting S=0.

$$P^*_{sf}(0)=-\lambda P^{*(1)}_{1,0}(0)+\lambda b_1P_{1,1}+\lambda b_1P_{1,2} \quad \dots (32)$$

Where $b_1=-B^{*(1)}(0)$,

Now differentiating Equation (26) with respect to S and after this putting S=0 into obtained result

$$P^{*(1)}_{1,0}(0)=\frac{P^*_{1,0}(0)-\delta b_1P_{1,1}-b_1P_{sf}(0)}{\lambda} \quad \dots (33)$$

Substituting the value of Equations (25), (28) and (30) into Equation (33). We obtain,

$$P^{*(1)}_{1,0}(0)=\frac{\phi\left\{1-\lambda b_1-B^*(\lambda)\right\}}{\lambda^2B^*(\lambda)}P_{3,0} \quad \dots (34)$$

Applying Equations (18), (25) and (34) to Equation (32) yielded

$$P_{sf}=P^*_{sf}(0)=\frac{\Omega}{\lambda(\lambda+2\delta)(\lambda+\alpha+\delta)B^*(\lambda)}P_{3,0} \quad \dots (35)$$

Where

$$\Omega=\phi(\lambda+2\delta)(\lambda+\alpha+\delta)\left\{\lambda b_1-1+B^*(\lambda)\right\}+\lambda^2b_1\psi(\lambda+2\delta)(\lambda+\alpha+\delta)B^*(\lambda)+\lambda^2b_1(\lambda+\alpha)(\lambda+2\alpha)B^*(\lambda)$$

The normalizing condition is given below. We obtain $P_{3,0}$ with the help of this condition.

$$P_{3,0}+P_{2,1}+P_{2,0}+P_{1,2}+P_{1,1}+P_{1,0}+P_{sf}=1 \quad \dots (36)$$

We can't show this expression here because it is too ample. There are three detection states which are (2, 1), (1, 2) and (1, 1). These states are considered as system down states. Therefore we get the following steady state availability.

$$A_V=P_{3,0}+P_{2,0}+P_{1,0} \quad \dots (37)$$

Now substituting Equations (24) and (30) into Equation (37). We obtain,

$$A_V=\left[1+\frac{(\lambda+2\alpha)\left\{1-B^*(\lambda+\alpha)\right\}}{(\lambda+\alpha)B^*(\lambda+\alpha)}+\frac{\phi\left\{1-B^*(\lambda)\right\}}{\lambda B^*(\lambda)}\right]P_{3,0} \quad \dots (38)$$

6. Special cases:

In this exploration, three different repair time distributions such as exponential (M), k-stage Erlang (E_k), and deterministic (D) are investigated. Following are the explicit expressions for the repair time distributions as mentioned above.

6.1 Exponential repair time distribution:

This distribution contains the value of $b_1=1/\mu$ which is mean repair time. Where μ is repair rate of primary and standby units. We obtain the following expressions after taking the LST.

$$B^*(\lambda)=\frac{\mu}{\mu+\lambda} \quad \dots (39)$$

$$B^*(\lambda+\alpha)=\frac{\mu}{\mu+\lambda+\alpha} \quad \dots (40)$$

From Equation (38),

$$A_{VM}=\left[\frac{\mu+\lambda+2\alpha+\phi_1}{\mu}\right]P_{3,0} \quad \dots (41)$$

Where ϕ_1 is obtained by substituting the value of $B^*(\lambda+\alpha)=\frac{\mu}{\mu+\lambda+\alpha}$

$$\phi_1=\frac{(\lambda+2\alpha)\left[(\lambda+\alpha+\delta)(\mu+\lambda+\alpha)-\delta\mu\right]}{\mu(\lambda+\alpha+\delta)} \quad \dots (42)$$

6.2 K-stage Erlang repair time distribution:

This distribution consists of k stages that are independent and identical respectively, each stage with same mean $b_1 = 1/k\mu$. After taking the LST, we get the following results and we set the mean repair time $b_1 = 1/\mu$.

$$B^*(\lambda)=\left(\frac{k\mu}{k\mu+\lambda}\right)^k \quad \ldots (43)$$

$$B^*(\lambda+\alpha)=\left(\frac{k\mu}{k\mu+\lambda+\alpha}\right)^k \quad \ldots (44)$$

From equation (38),

$$A_{VE}=\left[1+\frac{(\lambda+2\alpha)\left\{(k\mu+\lambda+\alpha)^k-(k\mu)^k\right\}}{(\lambda+\alpha)(k\mu)^k}+\frac{\phi_2\left\{(k\mu+\lambda)^k-(k\mu)^k\right\}}{\lambda(k\mu)^k}\right]P_{3,0} \quad \ldots (45)$$

Where ϕ_2 is obtained by substituting the value of $B^*(\lambda+\alpha)=\left(\frac{k\mu}{k\mu+\lambda+\alpha}\right)^k$.

$$\phi_2=\frac{(\lambda+2\alpha)\left\{(\lambda+\alpha+\delta)(k\mu+\lambda+\alpha)^k-\delta(k\mu)^k\right\}}{(\lambda+\alpha+\delta)(k\mu)^k} \quad \ldots (46)$$

6.3 Deterministic repair time distribution:

The distribution function of the repair time has the following Laplace transformation. We set the mean repair time $b_1 = 1/\mu$.

$$B^*(\lambda)=e^{-\lambda/\mu} \quad \ldots (47)$$

$$B^*(\lambda+\alpha)=e^{-(\lambda+\alpha)/\mu} \quad \ldots (48)$$

On substituting these values into equation (38), we obtain

$$A_{VD}=\left[1+\frac{(\lambda+2\alpha)\left\{e^{(\lambda+\alpha)/\mu}-1\right\}}{(\lambda+\alpha)}+\frac{\phi_3\left\{e^{\lambda/\mu}-1\right\}}{\lambda}\right]P_{3,0} \quad \ldots (49)$$

Where ϕ_3 is obtained by substituting the value of $B^*(\lambda+\alpha)=e^{-(\lambda+\alpha)/\mu}$

$$\phi_3=\frac{(\lambda+2\alpha)\left[(\lambda+\alpha+\delta)e^{(\lambda+\alpha)/\mu}-\delta\right]e^{(\lambda+\alpha)/\mu}}{(\lambda+\alpha+\delta)e^{(\lambda+\alpha)/\mu}} \quad \ldots (50)$$

7. COMPARISON OF AVAILABILITY

Availability is compared on the basis of different repair time distributions. Three types of distribution are used for repair time as exponential, three-stage Erlang and deterministic. The values of different parameter are set as following.

$\frac{1}{\lambda}=1000$ days, 10000 days, 100000 days; $\frac{1}{\alpha}=2000$ days; $\frac{1}{\mu}=10$ days; $\frac{1}{\delta}=\frac{10}{24}$ days

i.e. λ = 0.001, 0.0001, 0.00001; $\alpha=0.0005$; $\mu=0.1$; $\delta=2.4$

In the following three cases α, μ and δ are keep fixed. We see variation in λ.

Case a: The value of λ vary from 0.001 to 0.01.

Case b: The value of λ vary from 0.0001 to 0.001.

Case c: The value of λ vary from 0.00001 to 0.0001.

Symbols used for different availability are as follows

A_{VM} – Availability for exponential repair time distribution,

A_{VE} – Availability for Erlang repair time distribution,

A_{VD} – Availability for deterministic repair time distribution.

The comparisons of availability A_{VM}, A_{VE} and A_{VD} using numerical results are given as following in Tables 7.1 – 7.3 and fig.2 - fig.10.

Table 7.1 Comparison of availability for case a

λ	α	δ	μ	A_{VM}	A_{VE}	A_{VD}
0.001	0.0005	2.4	0.1	0.997532034	0.955308434	0.997530812
0.002	0.0005	2.4	0.1	0.996316349	0.934160471	0.996310044
0.003	0.0005	2.4	0.1	0.995121027	0.913780407	0.995103534
0.004	0.0005	2.4	0.1	0.993951784	0.894144748	0.993915016
0.005	0.0005	2.4	0.1	0.992814022	0.875229828	0.992748172
0.006	0.0005	2.4	0.1	0.991712836	0.857011971	0.991606635
0.007	0.0005	2.4	0.1	0.990653025	0.839467628	0.990493977
0.008	0.0005	2.4	0.1	0.9896391	0.82257349	0.989413713
0.009	0.0005	2.4	0.1	0.9886753	0.806306584	0.98836929
0.01	0.0005	2.4	0.1	0.987765597	0.79064435	0.987364084

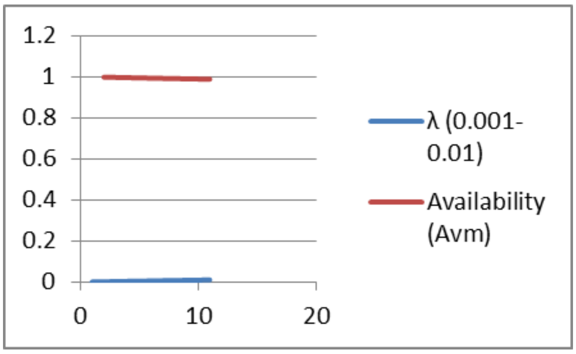

Fig. 2

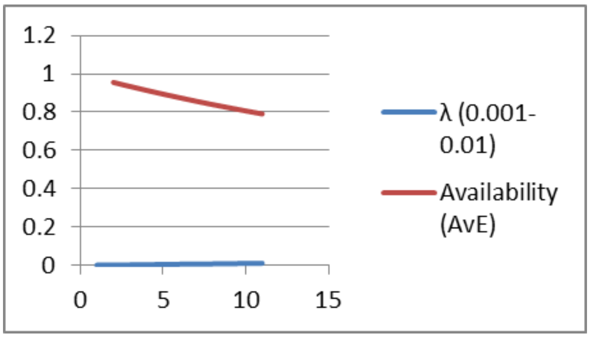

Fig. 3

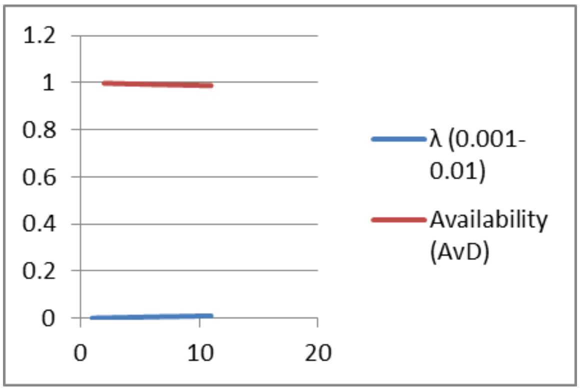

Fig. 4

Table 7.2 Comparison of availability for case b

λ	α	δ	μ	A_{VM}	A_{VE}	A_{VD}
0.0001	0.0005	2.4	0.1	0.998638574	0.975017283	0.998638577
0.0002	0.0005	2.4	0.1	0.998515194	0.972795253	0.999407183
0.0003	0.0005	2.4	0.1	0.998391907	0.970581316	0.998391815
0.0004	0.0005	2.4	0.1	0.998268719	0.968375447	0.998268549
0.0005	0.0005	2.4	0.1	0.998145637	0.996177626	0.998145366
0.0006	0.0005	2.4	0.1	0.998022667	0.963987829	0.998022268
0.0007	0.0005	2.4	0.1	0.997899815	0.961806034	0.99789926
0.0008	0.0005	2.4	0.1	0.997777089	0.959632218	0.997776345
0.0009	0.0005	2.4	0.1	0.997654492	0.957466359	0.997736764
0.001	0.0005	2.4	0.1	0.997532034	0.955308434	0.997530812

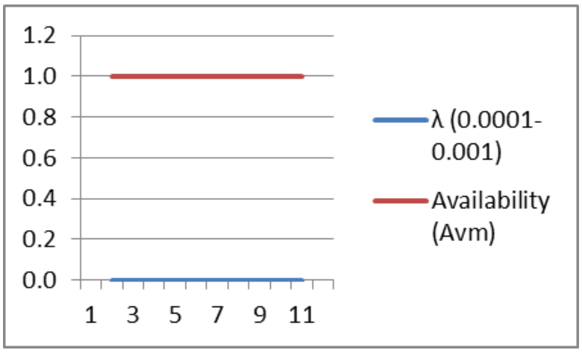

Fig. 5

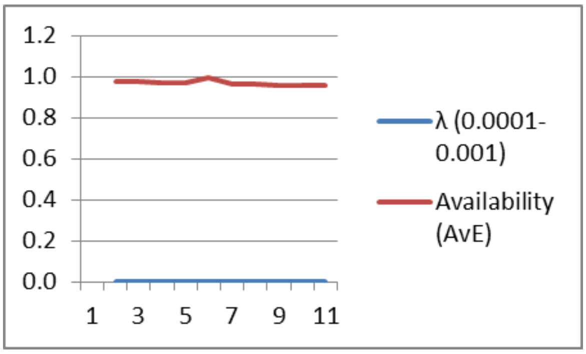

Fig. 6

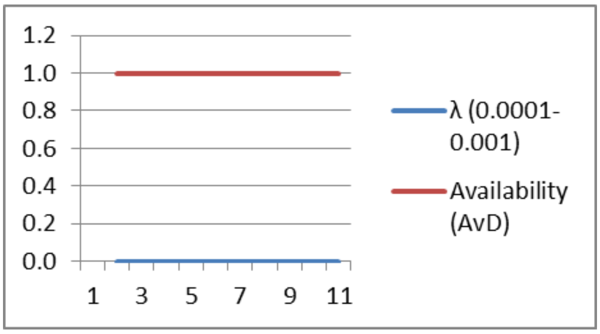

Fig. 7

Table 7.3 Comparison of availability for case c

λ	α	δ	μ	A_{VM}	A_{VE}	A_{VD}
0.00001	0.0005	2.4	0.1	0.998749691	0.977024046	0.998749714
0.00002	0.0005	2.4	0.1	0.998737341	0.976800748	0.998737363
0.00003	0.0005	2.4	0.1	0.998724992	0.976577531	0.998725013
0.00004	0.0005	2.4	0.1	0.998712645	0.976354394	0.998712662
0.00005	0.0005	2.4	0.1	0.998700297	0.976131339	0.998700314
0.00006	0.0005	2.4	0.1	0.998679664	0.975908365	0.998687965
0.00007	0.0005	2.4	0.1	0.998683893	0.975685473	0.998675617
0.00008	0.0005	2.4	0.1	0.998663261	0.975462662	0.998663269
0.00009	0.0005	2.4	0.1	0.998650917	0.975239932	0.998650923
0.0001	0.0005	2.4	0.1	0.998638574	0.975017283	0.998638577

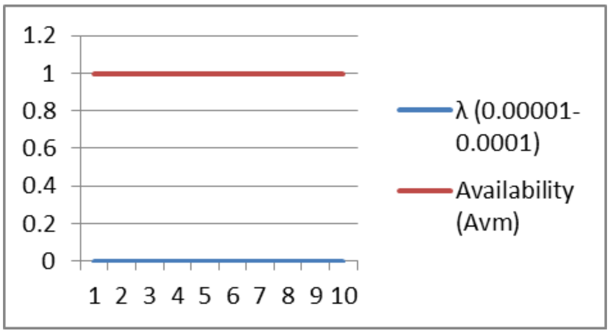

Fig. 8

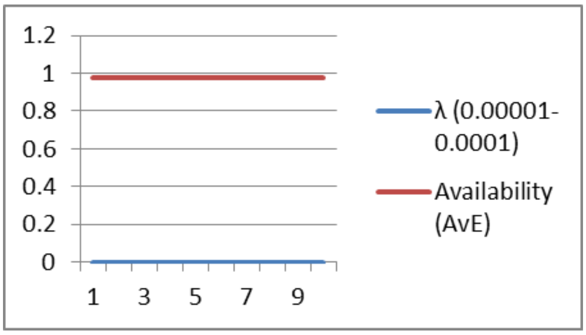

Fig. 9

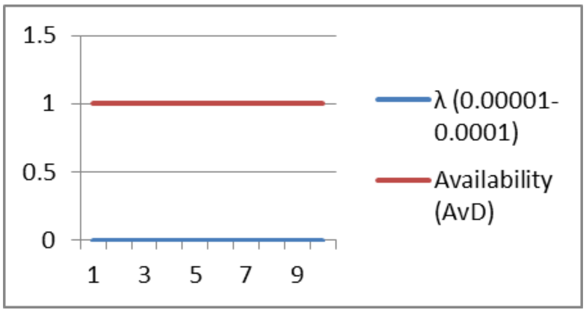

Fig. 10

8. CONCLUSION

In this study firstly, we obtain steady state availability of a warm standby system with fault detection delay and general repair times. The supplementary variable technique helps us very much to form a recursive method by which we able to get system availability. After this we derive expressions of availability for three types distribution of repair time as exponential, three-stage Erlang, and deterministic, respectively. We compare all three repair time distributions numerically, for this we consider three cases. In all cases three parameters α, μ and δ are keep fixed. We vary the value of λ from 0.001- 0.01in first case (table 7.1), from

0.0001- 0.001in second case (table 7.2) and from 0.00001-0.0001in third case (table 7.2). We also make comparison of availabilities of three cases with the help of graphs as shown in fig.2 - fig.10. On the basis of these cases, we find exponential repair time distribution is the best one in all three cases.

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