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On Alpha and Gamma Gourava Indices

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ARTICLE INFO	ABSTRACT
Published Online: **11 April 2024** Corresponding Author: **V. R. Kulli**	In this paper, we introduce the alpha Gourava and gamma Gourava indices of a graph. Also we compute the alpha and gamma Gourava indices of some standard graphs, armchair polyhex nanotubes and zigzag polyhex nanotubes.

KEYWORDS: alpha Gourava index, gamma Gourava index, nanotube.

I. INTRODUCTION

The graph *G= (V(G), E(G)),* where *V(G)* be the vertex set and *E(G)* be the edge set. Let d_u be the degree of a vertex *u*. For undefined term and notation, we refer the book [1].

Graph indices [2] have their applications in various disciplines of Science and Engineering. A molecular structure is a graph whose vertices correspond to the atoms and edges to the bonds. Studying molecular structures is a constant focus in Chemical Graph Theory: an effort to better understand molecular structure of a molecule.

Recently some graph indices were studied in [3, 4, 5, 6].

The first and second Zagreb indices [7] of a graph *G* are defined as

$$M_1(G)=\sum_{uv\in E(G)}(d_u+d_v)$$

$$M_2(G)=\sum_{uv\in E(G)}d_u d_v.$$

The F index [8] of a graph *G* is defined as

$$F(G)=\sum_{uv\in E(G)}\left(d_u^2+d_v^2\right).$$

The Y index [9] of a graph *G* is defined as

$$Y(G)=\sum_{uv\in E(G)}\left(d_u^3+d_v^3\right).$$

In [10], the generalization of first Zagreb index was proposed, defined as

$$M_1^{a+1}(G)=\sum_{u\in V(G)}d_G(u)^{a+1}=\sum_{uv\in E(G)}\left[d_u^a+d_v^a\right].$$

The first and second Gourava indices [11] of a graph *G* are defined as

$$GO_1(G)=\sum_{uv\in E(G)}\left[d_u+d_v+d_u d_v\right].$$

$$GO_2(G)=\sum_{uv\in E(G)}(d_u+d_v)(d_u d_v)$$

$$=\sum_{uv\in E(G)}\left(d_u^2 d_v+d_u d_v^2\right).$$

Recently some Gourava indices were studied, for example, in [12-25].

Motivated by the definitions of the first and second Gourava indices, we define the first and second alpha Gourava indices of a graph *G* as

$$AGO_1(G)=\sum_{uv\in E(G)}\left[d_u^2+d_v^2+d_u d_v\right],$$

$$AGO_2(G)=\sum_{uv\in E(G)}\left(d_u^2+d_v^2\right)(d_u d_v)$$

$$=\sum_{uv\in E(G)}\left(d_u^3 d_v+d_u d_v^3\right).$$

Also we define the first and second gamma Gourava indices of a graph *G* as

$$GGO_1(G)=\sum_{uv\in E(G)}\left[d_u^3+d_v^3+d_u d_v\right],$$

$$GGO_2(G)=\sum_{uv\in E(G)}\left(d_u^3+d_v^3\right)(d_u d_v)$$

$$=\sum_{uv\in E(G)}\left(d_u^4 d_v+d_u d_v^4\right).$$

May be the following generalizations of Gourava indices would make sense:

$$GO_1^a(G) = \sum_{uv \in E(G)} \left[d_u^a + d_v^a + d_u d_v\right].$$

$$GO_2^a(G) = \sum_{uv \in E(G)} \left(d_u^a + d_v^a\right)\left(d_u d_v\right)$$

$$= \sum_{uv \in E(G)} \left(d_u^{a+1} d_v + d_u d_v^{a+1}\right)$$

where a is a real number.

In this paper, we compute the alpha and gamma Gourava indices of some standard graphs, armchair polyhex nanotubes and zigzag polyhex nanotubes.

II. RESULTS FOR SOME STANDARD GRPHS

Proposition 1. If G is an r-regular graph with n vertices, then

$$AGO_1(G) = \frac{3}{2} nr^5.$$

Proof: If G is an r-regular graph with n vertices, then G has $\frac{nr}{2}$ edges. The degree of each vertex of G is r.

$$AGO_1(G) = \sum_{uv \in E(G)} \left[d_u^2 + d_v^2 + d_u d_v\right]$$

$$= \frac{1}{2} nr\left[\left(r^2 + r^2\right) + r^2\right] = \frac{3}{2} nr^5.$$

Corollary 1.1. Let C_n be a cycle with $n \geq 3$ vertices. Then $AGO_1(C_n) = 48n.$

Corollary 1.2. Let K_n be a complete graph with $n \geq 2$ vertices. Then $AGO_1(K_n) = \frac{3}{2} n(n-1)^5$.

Proposition 2. Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$. Then

$$AGO_1(K_{m,n}) = \left[\left(m^2 + n^2\right) + (m \times n)\right] mn.$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$. Then $K_{m,n}$ has $m+n$ vertices and mn edges such that $|V_1| = m$, $|V_2| = n$, $V(K_{m,n}) = V_1 \cup V_2$. Clearly every vertex of V_1 is adjacent with n vertices and every vertex of V_2 is adjacent with m vertices.

We have

$$AGO_1(K_{m,n}) = \sum_{uv \in E(K_{m,n})} \left[d_u^2 + d_v^2 + d_u d_v\right]$$

$$= \left[\left(m^2 + n^2\right) + (m \times n)\right] mn.$$

Corollary 2.1. Let $K_{n,n}$ be a complete bipartite graph with $n \geq 1$ vertices. Then

$$AGO_1(K_{n,n}) = 3n^4.$$

Corollary 2.2. Let $K_{1,n}$ be a complete bipartite graph with $n \geq 1$ vertices. Then

$$AGO_1(K_{1,n}) = n\left(1 + n + n^2\right).$$

Proposition 3. Let P_n be a path with $n \geq 3$ vertices. Then $AGO_1(P_n) = 12n - 8.$

Proof: Let $G = P_n$ be a path with $n \geq 3$ vertices. We obtain two partitions of edge set of P_n as follows:

$E_3 = \{uv \in E(G) \mid d_G(u) = 1, d_G(v) = 2\}$, $|E_3| = 2$.

$E_4 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_4| = n - 3$.

We have

$$AGO_1(G) = \sum_{uv \in E(G)} \left[d_u^2 + d_v^2 + d_u d_v\right]$$

$$= \left[\left(1^2 + 2^2\right) + (1 \times 2)\right] 2 + \left[\left(2^2 + 2^2\right) + (2 \times 2)\right](n-3)$$

$$= 12n - 8.$$

Similarly the second alpha Gourava index of some standard classes of graphs are computed

Proposition 4.

(1) Let G is an r-regular graph with n vertices. Then
$AGO_2(G) = nr^5$.

(2) Let C_n be a cycle with $n \geq 3$ vertices, then
$AGO_2(C_n) = 32n$.

(3) Let K_n be a complete graph with $n \geq 2$ vertices. Then
$AGO_2(K_n) = n(n-1)^5$.

(4) Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$. Then
$AGO_2(K_{m,n}) = (m^2 + n^2)\, m^2 n^2$.

(5) Let P_n be a path with $n \geq 3$ vertices. Then
$AGO_2(P_n) = 32n - 76$.

Proposition 5. If G is an r-regular graph with n vertices, then

$$GGO_1(G) = \frac{1}{2} nr^3 (2r + 1).$$

Proof: If G is an r-regular graph with n vertices, then G has $\frac{nr}{2}$ edges. The degree of each vertex of G is r.

$$GGO_1(G) = \sum_{uv \in E(G)} \left[d_u^3 + d_v^3 + d_u d_v\right]$$

$$= \frac{1}{2} nr\left[\left(r^3 + r^3\right) + r^2\right] = \frac{1}{2} nr^3 (2r + 1).$$

Corollary 5.1. Let C_n be a cycle with $n \geq 3$ vertices. Then $GGO_1(C_n) = 20n.$

Corollary 5.2. Let K_n be a complete graph with $n \geq 2$ vertices.

Then $GGO_1(K_n) = \frac{1}{2} n(n-1)^3 (2n-1)$.

Proposition 6. Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$. Then

$$GGO_1(K_{m,n}) = \left[\left(m^3 + n^3\right) + (m \times n)\right] mn.$$

Proof: Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$. Then $K_{m,n}$ has $m+n$ vertices and mn edges such that $|V_1| = m$, $|V_2| = n$, $V(K_{m,n}) = V_1 \cup V_2$. Clearly every vertex of V_1 is adjacent with n vertices and every vertex of V_2 is adjacent with m vertices.

We have

$$GGO_1(K_{m,n}) = \sum_{uv \in E(K_{m,n})} \left[d_u^3 + d_v^3 + d_u d_v\right]$$

$$= \left[\left(m^3 + n^3\right) + (m \times n)\right] mn.$$

Corollary 6.1. Let $K_{n,n}$ be a complete bipartite graph with $n \geq 1$ vertices. Then

$$GGO_1(K_{n,n}) = (2n+1) n^4.$$

Corollary 6.2. Let $K_{1,n}$ be a complete bipartite graph with $n \geq 1$ vertices. Then

$$GGO_1(K_{1,n}) = n\left(1 + n + n^3\right).$$

Proposition 7. Let P_n be a path with $n \geq 3$ vertices. Then $GGO_1(P_n) = 20n - 38$.

Proof: Let $G = P_n$ be a path with $n \geq 3$ vertices. We obtain two partitions of edge set of P_n as follows:

$E_3 = \{uv \in E(G) \mid d_G(u) = 1, d_G(v) = 2\}$, $|E_3| = 2$.

$E_4 = \{uv \in E(G) \mid d_G(u) = d_G(v) = 2\}$, $|E_4| = n - 3$.

We have

$$GGO_1(G) = \sum_{uv \in E(G)} \left[d_u^3 + d_v^3 + d_u d_v\right]$$

$$= \left[\left(1^3 + 2^3\right) + (1 \times 2)\right] 2 + \left[\left(2^3 + 2^3\right) + (2 \times 2)\right](n-3)$$

$$= 20n - 38.$$

Similarly the second alpha Gourava index of some standard classes of graphs are computed

Proposition 8.

(1) Let G is an r-regular graph with n vertices. Then
$GGO_2(G) = nr^6$.

(2) Let C_n be a cycle with $n \geq 3$ vertices, then
$GGO_2(C_n) = 64n$.

(3) Let K_n be a complete graph with $n \geq 2$ vertices. Then
$GGO_2(K_n) = n(n-1)^6$.

(4) Let $K_{m,n}$ be a complete bipartite graph with $1 \leq m \leq n$. Then
$GGO_2(K_{m,n}) = (m^3 + n^3)\, m^2 n^2$.

(5) Let P_n be a path with $n \geq 3$ vertices. Then
$GGO_2(P_n) = 64n - 156$.

III. SOME PROPERTIES OF ALPHA AND GAMMA INDICES

Theorem 1. Let G be a connected graph on $n \geq 2$ vertices. Then

$$AGO_1(G) = F(G) + M_2(G).$$

Proof: We have

$$AGO_1(G) = \sum_{uv \in E(G)} \left[d_u^2 + d_v^2 + d_u d_v\right]$$

$$= \sum_{uv \in E(G)} \left(d_u^2 + d_v^2\right) + \sum_{uv \in E(G)} d_u d_v$$

$$= F(G) + M_2(G).$$

From Theorem 1, the mathematical properties of AGO_1 can be directly deduced from these of $F(G)$ and $M_2(G)$.

Theorem 2. Let G be a connected graph on $n \geq 2$ vertices. Then

$$GGO_1(G) = Y(G) + M_2(G).$$

Proof: We have

$$GGO_1(G) = \sum_{uv \in E(G)} \left[d_u^3 + d_v^3 + d_u d_v\right]$$

$$= \sum_{uv \in E(G)} \left(d_u^3 + d_v^3\right) + \sum_{uv \in E(G)} d_u d_v$$

$$= Y(G) + M_2(G).$$

From Theorem 2, the mathematical properties of GGO_1 can be directly deduced from these of $Y(G)$ and $M_2(G)$.

Theorem 3. Let G be a connected graph on $n \geq 2$ vertices. Then

$$GO_1^a(G) = M_1^{a+1}(G) + M_2(G).$$

Proof: We have

$$GO_1^a(G) = \sum_{uv \in E(G)} \left[d_u^a + d_v^a + d_u d_v\right]$$

$$= \sum_{uv \in E(G)} \left(d_u^a + d_v^a\right) + \sum_{uv \in E(G)} d_u d_v$$

$$= M_1^{a+1}(G) + M_2(G).$$

From Theorem 3, the mathematical properties of GO_1^a can be directly deduced from these of $M_1^{a+1}(G)$ and $M_2(G)$.

IV. RESULTS FOR ARMCHAIR POLYHEX NANOTUBES

Carbon polyhex nanotubes are the nanotubes whose cylindrical surface nanotubes are the nanotubes whose cylindrical surface is made up of entirely hexagons. These polyhex nanotubes exist in nature with remarkable stability and poses very interesting electrical, mechanical and thermal properties. Armchair polyhex nanotube is denoted by $TUAC_6[p, q]$, where p is the number of hexagons in a row and q is the number of hexagons in a column. A graph of $TUAC_6[p, q]$ is shown in Figure 1.

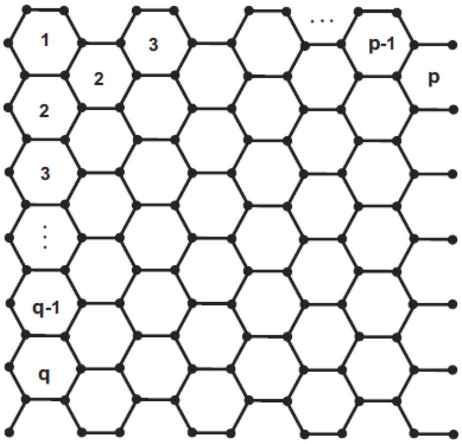

Figure 1. Graph of $TUAC_6[p, q]$

Let $G = TUAC_6[p, q]$ where $p, q \geq 1$. By calculation, G has $2p(q+1)$ vertices and $3pq + 2p$ edges. There are three types of edges based on degree of end vertices of each edge as given Table 1

Table 1. Edge partition of TUAC6[p, q]

$d_u, d_v \backslash uv \in E(G)$	Number of edges
(2, 2)	p
(2, 3)	$2p$
(3, 3)	$3pq - p$

Theorem 4. The general first Gourava index of $TUAC_6[p, q]$ is

$$GO_1^a(G) = p[2^{a+1} + 4] + 2p[2^a + 3^a + 6] + (3pq - p)[2 \times 3^a + 9] \quad (1)$$

Proof: Let $G = TUAC_6[p, q]$. By using equation (1) and Table 1, we deduce

$$GO_1^a(G) = \sum_{uv \in E(G)} [d_u^a + d_v^a + d_u d_v]$$

$$= p[2^a + 2^a + (2 \times 2)] + 2p[2^a + 3^a + (2 \times 3)]$$

$$+ (3pq - p)[3^a + 3^a + (3 \times 3)]$$

$$= p[2^{a+1} + 4] + 2p[2^a + 3^a + 6] + (3pq - p)[2 \times 3^a + 9].$$

We obtain the following results by using Theorem 4.

Corollary 4.1. The first Gourava index of $TUAC_6[p, q]$ is

$$GO_1(G) = 45pq + 15p.$$

Corollary 4.2. The first alpha Gourava index of $TUAC_6[p, q]$ is

$$AGO_1(G) = 81pq + 23p.$$

Corollary 4.3. The first gamma Gourava index of $TUAC_6[p, q]$ is

$$GGO_1(G) = 189pq + 39p.$$

Proof: Put a = 1, 2, 3 in equation (1), we get the desired results respectively.

Theorem 5. The general second Gourava index of $TUAC_6[p, q]$ is

$$GO_2^a(G) = 2^{a+3}p + 12(2^a + 3^a)p + 2 \times 3^{a+2}(3pq - p) \quad (2)$$

Proof: Let $G = TUAC_6[p, q]$. We have

$$GO_2^a(G) = \sum_{uv \in E(G)} (d_u^a + d_v^a)(d_u d_v)$$

$$= p[(2^a + 2^a)(2 \times 2)] + 2p[(2^a + 3^a)(2 \times 3)]$$

$$+ (3pq - p)[(3^a + 3^a)(3 \times 3)]$$

$$= 2^{a+3}p + 12(2^a + 3^a)p + 2 \times 3^{a+2}(3pq - p).$$

We obtain the following results by using Theorem 5.

Corollary 5.1. The second Gourava index of $TUAC_6[p, q]$ is

$$GO_2(G) = 162pq + 22p.$$

Corollary 5.2. The second alpha Gourava index of $TUAC_6[p, q]$ is

$$AGO_2(G) = 486pq + 26p.$$

Corollary 5.3. The second gamma Gourava index of $TUAC_6[p, q]$ is

$$GGO_2(G) = 1458pq - 2p.$$

Proof: Put a = 1, 2, 3 in equation (2), we get the desired results respectively.

V. RESULTS FOR ZIGZAG POLYHEX NANOTUBES

Zigzag polyhex nanotube is denoted by $TUZC_6[p, q]$, where p is the number of hexagons in a row and q is the number of hexagons in a column. A graph of $TUZC_6\,[p, q]$ is presented in Figure 2.

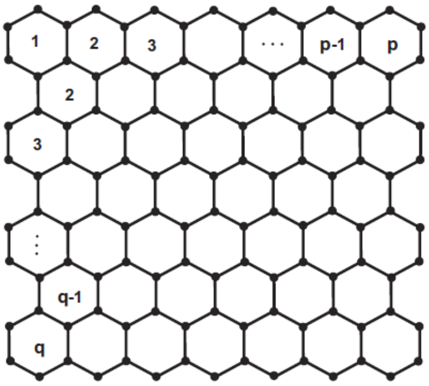

Figure 2. Graph of $TUZC_6$ [p, q]

Let $G = TUZC_6\,[p, q]$, where $p, q \geq 1$. By calculation, G has $2p(q+1)$ vertices and $3pq + 2p$ edges. There are two types of edges based on degree of end vertices of each edge as given in Table 2.

Table 2. Edge partition of $TUZC_6[p, q]$

$d_u, d_v \setminus uv \in E(G)$	(2, 3)	(3, 3)
Number of edges	$4p$	$3pq - 2p$

Theorem 6. The general first Gourava index of $TUZC_6\,[p, q]$ is

$$GO_1^a(G) = 4p\left[2^a + 3^a + 6\right] + (3pq - 2p)\left[2\times 3^a + 9\right]$$

(3)

Proof: Let $G = TUZC_6[p, q]$. We have

$$GO_1^a(G) = \sum_{uv\in E(G)}\left[d_u^a + d_v^a + d_u d_v\right]$$
$$= 4p\left[2^a + 3^a + (2\times 3)\right]$$
$$+(3pq - 2p)\left[3^a + 3^a + (3\times 3)\right]$$
$$= 4p\left[2^a + 3^a + 6\right] + (3pq - 2p)\left[2\times 3^a + 9\right].$$

We obtain the following results by using Theorem 6.

Corollary 6.1. The first Gourava index of $TUZC_6\,[p, q]$ is

$$GO_1(G) = 45pq + 14p.$$

Corollary 6.2. The first alpha Gourava index of $TUZC_6\,[p, q]$ is

$$AGO_1(G) = 81pq + 22p.$$

Corollary 6.3. The first gamma Gourava index of $TUZC_6\,[p, q]$ is

$$GGO_1(G) = 189pq + 38p.$$

Proof: Put a = 1, 2, 3 in equation (3), we get the desired results respectively.

Theorem 7. The general second Gourava index of $TUZC_6\,[p, q]$

$$GO_2^a(G) = 24\left(2^a + 3^a\right)p + 2\times 3^{a+2}(3pq - 2p) \qquad (4)$$

Proof: Let $G = TUZC_6\,[p, q]$. We have

$$GO_2^a(G) = \sum_{uv\in E(G)}\left(d_u^a + d_v^a\right)\left(d_u d_v\right)$$
$$= 4p\left[\left(2^a + 3^a\right)(2\times 3)\right] + (3pq - 2p)\left[\left(3^a + 3^a\right)(3\times 3)\right]$$
$$= 24\left(2^a + 3^a\right)p + 2\times 3^{a+2}(3pq - 2p).$$

We obtain the following results by using Theorem 7.

Corollary 7.1. The second Gourava index of $TUAC_6\,[p, q]$ is

$$GO_2(G) = 162pq + 12p.$$

Corollary 7.2. The second alpha Gourava index of $TUAC_6\,[p, q]$ is

$$AGO_2(G) = 486pq - 12p.$$

Corollary 7.3. The second gamma Gourava index of $TUAC_6\,[p, q]$ is

$$GGO_2(G) = 729pq + 354p.$$

Proof: Put a = 1, 2, 3 in equation (4), we get the desired results respectively.

VI. CONCLUSION

In this paper, two novel invariants are considered which are the alpha Gourava index and gamma Gourava index. The alpha Gourava and gamma Gourava indices of some standard graphs, armchair polyhex nanotubes and zigzag polyhex nanotubes are determined.

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