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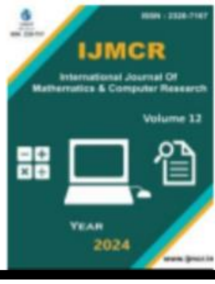

An Algebraic Solution of Dual Fuzzy Complex Linear Systems

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ABSTRACT

The main aims of this research is to solve the dual fuzzy complex linear system $A\tilde{X} + \tilde{C} = B\tilde{X} + \tilde{D}$, where A, B are crisp coefficient matrices and $\tilde{C}, \tilde{D}$ are fuzzy number matrices. The research findings indicate that the system of two fuzzy complex linear equations can be solved under the conditional that $(A - B)^{-1}$ and $(|B| - |A|)^{-1}$ exists.

KEYWORDS: Algebraic solutions; Dual fuzzy linear systems; Fuzzy complex numbers; Matrix

I. INTRODUCTION

In mathematics, one of the common problems encountered is the system of linear equations. Linear equation systems are found in almost all branches of science [1]. The application of linear equation systems in scientific fields can be seen in transportation planning [2], optimization finance, [3], business, physics, management [4], current flow and control theory [5], economics, sociology, and electronics. In its application, the system of linear equations involves not only real coefficients and variables but also can take the form of fuzzy numbers or complex numbers. Therefore, linear systems of equations involving fuzzy coefficients or variables can be solved in the form of fuzzy linear equation systems. Thus, understanding and developing methods for solving fuzzy linear equations are highly important [6].

Zadeh was the first to introduce and explore the concept of fuzzy numbers, along with the arithmetic operations associated with them [7]. The general form of a fuzzy linear equation system can be written as $A\tilde{X} = \tilde{Y}$ where $\tilde{X}$ and $\tilde{Y}$ are parameters within a certain interval [8]. Various solution method of fuzzy linear systems can be observed in [9],[10],[11],[12] [13],[14],[15],[16],[17],[18].

Many previous studies have discussed the resolution of complex fuzzy linear equation systems and dual fuzzy linear equation systems, one of which is [19] investigates the solution of linear equation systems in fuzzy complex numbers. The method used is the Gauss-Jordan elimination method. The research results indicate that the fuzzy complex linear equation system $C\tilde{Z} = \widetilde{W}$ is transformed into matrix form, which is then solved using the Gauss-Jordan elimination method. The research on dual fuzzy linear equation systems was first discussed in [20]. In [21] the dual fuzzy linear equation system is algebraically solved using dual fuzzy matrices, which can be written in the form $A\tilde{X} + \tilde{Y} = B\tilde{X} + \tilde{Z}$ where A, B are crisp coefficient matrices and $\tilde{Y}, \tilde{Z}$ are fuzzy number matrices. In this method, the dual fuzzy linear system does not need to be transformed into a dual crisp linear system. The research results indicate that the proposed method appears to be more efficient. For in [6] with the same linear equation system as [21] is solved algebraically involving triangular fuzzy number matrices.

From several previous studies, there has been no discussion regarding the solution of dual fuzzy complex linear equation systems. The dual fuzzy complex linear equation system to be solved can be written as $A\tilde{X} + \tilde{C} = B\tilde{X} + \tilde{D}$ where A, B are crisp coefficient matrices and $\tilde{C}, \tilde{D}$ are fuzzy complex number matrices. The objective of this research is to solve the dual fuzzy complex linear equation system.

II. PRELIMINARIES

Definition 2.1 [22] A fuzzy number is the fuzzy set $\tilde{x}$ with membership function $\mu_{\tilde{x}}: \mathbb{R} \to [0,1]$, if

i. There exists $t_0 \in \mathbb{R}$ such that $\mu_{\tilde{x}}(t_0) = 1$, i.e. $\tilde{x}$ is normal.

ii. For any $\lambda \in [0,1]$ and $s, t \in \mathbb{R}$, we have $\mu_{\tilde{x}}(\lambda s + (1 - \lambda)t) \geq \min\{\mu_{\tilde{x}}(s), \mu_{\tilde{x}}(t)\}$, i.e. $\tilde{x}$ is a convex fuzzy set.

iii. For any $s \in \mathbb{R}$, the set $\{t \in \mathbb{R} : \mu_{\tilde{x}}(t) > s\}$ is an open set in $\mathbb{R}$, i.e. $\mu_{\tilde{x}}$ is upper semi-continuous on $\mathbb{R}$.

iv. The closure of the set $\overline{\{t \in \mathbb{R} : \mu_{\tilde{x}}(t) > 0\}}$ is a compact set in $\mathbb{R}$, where $\bar{A}$ denotes the closure of A.

The set of all fuzzy numbers is represented by the letter $\mathbb{F}$ in this paper. If the real line $\mathbb{R}$ as $\mathbb{R}\ \{X_{\{t\}} : t$ is a real number$\}$, we can clearly derive $\mathbb{R} \subset \mathbb{F}$ [21]. Moreover, for $0 < \alpha \leq 1$, α-level of the fuzzy number $\tilde{x}$ is defined as $[\tilde{x}]_\alpha = \{t \in R : \mu_{\tilde{x}}(t) \geq \alpha\}$ and for $\alpha = 0$ it is defined as $[\tilde{x}]_0 =$

$\overline{\{t \in R : \mu_{\tilde{x}}(t) > 0\}}$. Usually, the support of the fuzzy set is defined as

$$supp(\tilde{x}) = [\tilde{x}]_0 = \overline{\{t \in R : \mu_{\tilde{x}}(t) > 0\}}.$$

Lemma 2.1. [22] Let $\{[\underline{x}(\alpha), \overline{x}(\alpha)] : 0 \le \alpha \le 1\}$ be a certain non-empty set in $\mathbb{R}$. If

i. $[\underline{x}(\alpha), \overline{x}(\alpha)]$ is a bounded closed interval, for each $\alpha \in [0,1]$,

ii. $[\underline{x}(\alpha_1), \overline{x}(\alpha_1)] \supseteq [\underline{x}(\alpha_2), \overline{x}(\alpha_2)]$ fot all $0 \le \alpha_1 \le \alpha_2 \le 1$,

iii. $\left[\lim_{k\to\infty} \underline{x}(\alpha_k), \lim_{k\to\infty} \overline{x}(\alpha_k)\right] = [\underline{x}(\alpha), \overline{x}(\alpha)]$ whenever $\{\alpha_k\}$ is a non-decreasing sequence in [0,1] converging to α

Definition 2.2 [19] For any fuzzy complex number $\tilde{z}$ expressed as $\tilde{z} = \tilde{x} + i\tilde{y}$, where $\tilde{x} = [\underline{x}(\alpha), \overline{x}(\alpha)]$ and $\tilde{y} = [\underline{y}(\alpha), \overline{y}(\alpha)]$, $0 \le \alpha \le 1$, it can be written as

$$\tilde{z} = [\underline{x}(\alpha), \overline{x}(\alpha)] + i[\underline{y}(\alpha), \overline{y}(\alpha)]$$
$$= \left[\left(\underline{x}(\alpha) + i\underline{y}(\alpha)\right), \left(\overline{x}(\alpha) + i\overline{y}(\alpha)\right)\right].$$

Furthermore, the arithmetic of fuzzy complex numbers, as described in [23],[24] and [25] is discussed as in Definition 2.3.

Definition 2.3 [25] For any two arbitrary fuzzy complex numbers $\tilde{z}_1 = \tilde{x}_1 + i\tilde{y}_1$ and $\tilde{z}_2 = \tilde{x}_2 + i\tilde{y}_2$ with a complex number $c = a + ib$, $\alpha - cut$ of the sum $\tilde{z}_1 + \tilde{z}_2$ and the product $c \cdot \tilde{z}_1$ Are determined based on interval arithmetic as follows:

$$[\tilde{z}_1 + \tilde{z}_2]_\alpha = ([\tilde{x}_1]_\alpha + [\tilde{x}_2]_\alpha) + i([\tilde{y}_1]_\alpha + [\tilde{y}_2]_\alpha)$$
$$= [\underline{x_1}(\alpha) + \underline{x_2}(\alpha), \overline{x_1}(\alpha) + \overline{x_2}(\alpha)]$$
$$+ i[\underline{y_1}(\alpha) + \underline{y_2}(\alpha), \overline{y_1}(\alpha) + \overline{y_2}(\alpha)],$$

and

$$[c \cdot \tilde{z}_1]_\alpha = [(a + ib) \cdot \tilde{z}_1]_\alpha = (a + ib) \cdot ([\tilde{x}_1]_\alpha + i[\tilde{y}_1]_\alpha)$$
$$= (a[\tilde{x}_1]_\alpha - b[\tilde{y}_1]_\alpha) + i([\tilde{x}_1]_\alpha + b[\tilde{y}_1]_\alpha).$$

Definition 2.4 [21] We say that two fuzzy complex numbers $\tilde{x} = \tilde{a}_1 + i\tilde{b}_1$ and $\tilde{y} = \tilde{a}_2 + i\tilde{b}_2$ are equal if and only if for any $t \in \mathbb{R}, \mu\tilde{x}(t) = \mu\tilde{y}(t), [\tilde{x}]_\alpha = [\tilde{y}]_\alpha$ for any $\alpha \in [0,1]$. Also $\tilde{x} \subseteq \tilde{y} \Leftrightarrow [\tilde{x}]_\alpha \subseteq [\tilde{y}]_\alpha, \quad \forall \alpha \in [0,1]$.

The two concepts below will be used in this paper.

Definition 2.5 [16] The α-center of the fuzzy complex number center $\tilde{x}$ denoted by $\tilde{z} = \tilde{x} + i\tilde{y}$ is defined as

$$[\tilde{z}]^C_\alpha = \left(\frac{\overline{x}(\alpha)+\underline{x}(\alpha)}{2}\right) + i\left(\frac{\overline{y}(\alpha)+\underline{y}(\alpha)}{2}\right) \qquad \alpha \in [0,1],$$

where $[\tilde{x}]_\alpha = [\overline{x}(\alpha) + \underline{x}(\alpha)]$ and $[\tilde{y}]_\alpha = [\overline{y}(\alpha) + \underline{y}(\alpha)]$.

Definition 2.6 [16] The α-radius of the fuzzy complex number $\tilde{z} = \tilde{x} + i\tilde{y}$ is defined as

$$[\tilde{z}]^R_\alpha = \left(\frac{\overline{x}(\alpha)+\underline{x}(\alpha)}{2}\right) + i\left(\frac{\overline{y}(\alpha)+\underline{y}(\alpha)}{2}\right) \qquad \alpha \in [0,1],$$

where $[\tilde{x}]_\alpha = [\overline{x}(\alpha) + \underline{x}(\alpha)]$ and $[\tilde{y}]_\alpha = [\overline{y}(\alpha) + \underline{y}(\alpha)]$.

Remark 2.6.1 [21] Clearly, α-center and α-radius of any arbitrary fuzzy complex number are continuous real functions of α.

Remark 2.6.2 [21] For a fuzzy complex number $\tilde{x}$, if for any $\alpha \in [0,1], x^R(\alpha) = 0$ then it can be easily concluded that $\tilde{x}$ is a crisp real number.

Remark 2.6.3 [21] For a fuzzy complex number $\tilde{x}$, if for any $\alpha \in [0,1], x^C(\alpha) = \underline{x}(\alpha)$ or $x^C(\alpha) = \overline{x}(\alpha)$, by Remark 2.6.2, it can be shown that $\tilde{x}$ is a crisp real number.

Remark 2.6.4 [21] Let $\tilde{z}_1, \tilde{z}_2, \dots, \tilde{z}_n \in \mathbb{F}_c, c_1, c_2, \dots, c_n \in \mathbb{R}$ and also $\tilde{u} = \sum_{i=1}^n c_i \tilde{z}_i$ Then

$$u^C(\alpha) = \sum_{i=1}^n c_i {z_i}^C(\alpha), \qquad u^R(\alpha) = \sum_{i=1}^n |c_i| {z_i}^R(\alpha).$$

Definition 2.7 [21] We define a vector valued fuzzy complex number as $\tilde{X} = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n)^T$, where, $i = 1,2,\dots,n$, is a fuzzy number. Also, we denote $[\tilde{X}]_\alpha$ by $[\tilde{X}]_\alpha = ([\tilde{x}_1]_\alpha, [\tilde{x}_2]_\alpha, \dots, [\tilde{x}_n]_\alpha)^T$ and consequently $x^C(\alpha) = \left(x_1^C(\alpha), x_2^C(\alpha), \dots, x_n^C(\alpha)\right)^T$ and $x^R(\alpha) = \left(x_1^R(\alpha), x_2^R(\alpha), \dots, x_n^R(\alpha)\right)^T$

Moreover, we establish definitons for two for two fuzzy complex numbers valued vector denoted as $\tilde{X}$ and $\tilde{Y}$, we define:

$$\tilde{X} \subseteq \tilde{Y} \Leftrightarrow [\tilde{X}]_\alpha \subseteq [\tilde{Y}]_\alpha, \quad \forall \alpha \in [0,1]$$
$$\Leftrightarrow [\tilde{x}_i]_\alpha \subseteq [\tilde{y}_i]_\alpha, \quad i = 1,2,\dots,n, \quad \forall \alpha \in [0,1].$$

Theorem 2.1 [21] Let $A = (a_{ij})_{n\times n}$ is an arbitrary crisp-valued matrix and $\tilde{X} = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n)^T$ is an arbitrary vector valued fuzzy complex number. Then,

$$\left(A \cdot \tilde{X}\right)^C(\alpha) = A \cdot X^C(\alpha),$$

and

$$\left(A \cdot \tilde{X}\right)^R(\alpha) = |A| \cdot X^R(\alpha),$$

where $|A| = (|a_{ij}|)_{n\times n}$.

Proof. The implementation follows Remark 2.6.4. and Definition 2.7.■

Complex dual fuzzy linear systems are one of the intriguing topics in fuzzy mathematics that have numerous applications in various branches of science. Here is the definition of this system.

Definition 2.8 [21] The $n \times n$ linear systems

$$\begin{aligned} a_{11}\tilde{x}_1 + a_{12}\tilde{x}_2 + \dots + a_{1n}\tilde{x}_n + \tilde{c}_1 &= b_{11}\tilde{x}_1 + b_{12}\tilde{x}_2 + \dots + b_{1n}\tilde{x}_n + \tilde{d}_1, \\ a_{21}\tilde{x}_1 + a_{22}\tilde{x}_2 + \dots + a_{2n}\tilde{x}_n + \tilde{c}_1 &= b_{21}\tilde{x}_1 + b_{22}\tilde{x}_2 + \dots + b_{2n}\tilde{x}_n + \tilde{d}_1, \\ &\vdots \\ a_{n1}\tilde{x}_1 + a_{n2}\tilde{x}_2 + \dots + a_{nn}\tilde{x}_n + \tilde{c}_1 &= b_{n1}\tilde{x}_1 + b_{n2}\tilde{x}_2 + \dots + b_{nn}\tilde{x}_n + \tilde{d}_1, \end{aligned} \qquad (2.1)$$

where there are two $n\ x\ n$ crisp real matrices of coefficiets two real matrices $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ and two vectors $\tilde{C} = (\tilde{c}_1, \tilde{c}_2, \dots, \tilde{c}_n)^T$ and $\tilde{D} = (\tilde{d}_1, \tilde{d}_2, \dots, \tilde{d}_n)^T$ being fuzzy complex vectors in the form $\tilde{c} = \tilde{g} + i\tilde{h}$ and $\tilde{d} = \tilde{m} + i\tilde{n}$ respectively, where $\tilde{X} = (\tilde{x}_1, \tilde{x}_2, \dots, \tilde{x}_n)^T$ represents a fuzzy complex number vector with the form $\tilde{x} = \tilde{p} + i\tilde{q}$. Furthermore, if described as a dual fuzzy complex linear system. Definition 2.7 states that the dual fuzzy complex linear system (2.1) has the following matrix form:

$$A\tilde{X} + \tilde{C} = B\tilde{X} + \tilde{D}. \qquad (2.2)$$

In Defintion 2.7, matrices A, B and vectors $\tilde{X}, \tilde{C}$ and $\tilde{D}$ are defined. It is important to note that there is no element $\tilde{y} \in \mathbb{F}_{\mathbb{C}}$ such that $\tilde{x} + \tilde{y} = 0$ for any fuzzy complex number $\tilde{x} \in \mathbb{F}_{\mathbb{C}}$. This means that for every $\tilde{x} \in \mathbb{F}_{\mathbb{C}} - \mathbb{R}$, we have $\tilde{x} + (-\tilde{x}) \neq 0$, hence it is not possible to replace the dual fuzzy complex linear system (2,2) with the fuzzy complex linear system $(A - B)\tilde{X} = \tilde{Z} - \tilde{Y}$ equivalently. Therefore, the develop of mathematical methods is necessary to solve the dual fuzzy complex linear system (2,2).

Definition 2.9 [21] If for the system (2.2), $det\ (A - B) \neq 0$, then we define the extended solution of the system (2.2) as follows:

$$\tilde{X}_E = (A - B)^{-1}(\tilde{D} - \tilde{C}). \qquad (2.3)$$

Also, if $det(A - B) = 0$, then we say that there is no solution.

Definition 2.10 [21] A vector-valued fuzzy complex number $\tilde{X}_A = \left(\tilde{x}_{1_A}, \tilde{x}_{2_A}, \dots, \tilde{x}_{n_A}\right)^T$ is called an algebraic solution of the dual fuzzy complex linear system (2.1) or (2.2) if

$$A\tilde{X}_A + \tilde{C} = B\tilde{X}_A + \tilde{D},$$

or, in other words,

$$\sum_{j=1}^{n} a_{ij}([\tilde{p}_{jA}] + i[\tilde{q}_{jA}]) + ([\tilde{g}_i] + i[\tilde{h}_i]) = \sum_{j=1}^{n} B_{ij}([\tilde{p}_{jA}] + i[\tilde{q}_{jA}]) + ([\tilde{m}_i] + i[\tilde{n}_i]), \qquad \forall i = 1,2,\dots,n.$$

Renmark 2.10.1 [21] Based on Definitions 2.5 and 2.11, it is clear that $\tilde{X}_A = \left(\tilde{x}_{1_A}, \tilde{x}_{2_A}, \dots, \tilde{x}_{n_A}\right)^T$ is an algebraic solution of the system (2.1) or (2.2) if

$$\sum_{j=1}^{n} a_{ij}\left([\tilde{p}_{jA}]_\alpha + i[\tilde{q}_{jA}]_\alpha\right) + \left([\tilde{g}_i]_\alpha + i[\tilde{h}_i]_\alpha\right) = \sum_{j=1}^{n} B_{ij}\left([\tilde{p}_{jA}]_\alpha + i[\tilde{q}_{jA}]_\alpha\right) + ([\tilde{m}_i]_\alpha + i[n_i]_\alpha), \quad \forall i = 1,2,\dots,n.$$

For each $\alpha \in [0,1]$ and $i = 1,2,\dots,n$. Moreover, if $det\ (A - B) \neq 0$, based on Definitions 2.4 and 2.9, we have

$$[\tilde{X}_E]_\alpha = (A - B)^{-1}\left([\tilde{D}]_\alpha - [\tilde{C}]_\alpha\right),$$

and also, with Definition 2.9 and Theorem 2.1, we have

$$X_A^C(\alpha) = (A - B)^{-1}\left(D^C(\alpha) - C^C(\alpha)\right), \qquad \forall \alpha \in [0,1],$$
$$X_E^R(\alpha) = |(A - B)^{-1}|\left(D^R(\alpha) + C^R(\alpha)\right), \qquad \forall \alpha \in [0,1],$$

use the following theorem, we continue our investigation into the connection between the algebraic solution $\tilde{X}_A$ and the extended solution $\tilde{X}_E$

Theorem 2.2 [21] Suppose for the dual fuzzy complex linear system (2.1) or (2.2), both the extended and algebraic solutions exist. Then, $X_A^C(\alpha) = X_E^R(\alpha)$.

Proof. Since $\tilde{X}_A$ is an algebraic solution, we have:

$$A\tilde{X}_A + \tilde{C} = B\tilde{X}_A + \tilde{D},$$

Now, with Renmark 2.6.5 and Theorem 2.1, we conclude:

$$A \cdot X_A^C(\alpha) + C^C(\alpha) = B \cdot X_A^C(\alpha) + D^C(\alpha). \qquad (2.4)$$

Conversely, since there is an extended solution $\tilde{X}_E, det\ (A - B) \neq 0$. Furthermore, as the α-center of any fuzzy number is a continuous function with respect to α, then from

$$X_A^C(\alpha) = (A - B)^{-1}\left(D^C(\alpha) - C^C(\alpha)\right) = X_E^R(\alpha).$$

Theorem 2.3. [21] The dual fuzzy complex linear system (2.2) has a unique algebraic solution if and only if both matrices $(A - B)$ and $|A| - |B|$ are nonsingular, and also the family of sets

$$\left[\underline{X_E} + F(\alpha), \overline{X}_E(\alpha) - F(\alpha)\right], \qquad \forall_\alpha \in [0,1],$$

forms the α-level of a vector-valued fuzzy complex number, where$\left[\underline{X_E} + F(\alpha), \overline{X}_E(\alpha) - F(\alpha)\right]$, Represents the α-level of the extended solution from system (2.2), and

$$F(\alpha) = X_E^R(\alpha) + (|B| - |A|)^{-1}\left(D^R(\alpha) - C^R(\alpha)\right). \qquad (2.5)$$

Actually, system (2.2) has a unique algebraic solution with the following α-level:

$$[\tilde{X}_A]_\alpha = \left[\underline{X_E} + F(\alpha), \overline{X}_E(\alpha) - F(\alpha)\right], \ \forall_\alpha \in [0,1]. \qquad (2.6)$$

Further explanation of the proof of Theorem 2.3 is discussed in [23, page 7]

Theorem 2.4 [21] If in the dual fuzzy complex linear system (2.2), both matrices $A - B$ and $|B| - |A|$ invertible and also the vectors $\tilde{C}$ and $\tilde{D}$ are crisp-valued vectors, then for any $\alpha \in [0,1], F(\alpha) = 0$ and consequently, the system has a unique algebraic solution as follows:

$$\tilde{x}_A = \tilde{X}_E = (A - B)^{-1}(\tilde{C} - \tilde{D})$$

Proof. Given that the vectors $\tilde{C}$ and $\tilde{D}$ have crips, then $C^R(\alpha) = 0$ and $C^R(\alpha) = 0$ and also

$$X_E^R(\alpha) = (A - B)^{-1}\left(D^R(\alpha) + C^R(\alpha)\right) = 0,$$

this implies that

$$F(\alpha) = X_E^R(\alpha) + (|B| - |A|)^{-1}\left(D^R(\alpha) - C^R(\alpha)\right) = 0,$$

Consequently, the unique algebraic solution for the dual fuzzy complex linear system (2.2) can be deduced from Theorem 2.4 and Equation (2.6) as follows:

$$\tilde{X}_A = \tilde{X}_E = (A - B)^{-1}(\tilde{D} - \tilde{C}) \blacksquare$$

III. RESULTS AND DISCUSSION

Here is the proposed solution steps for solving the dual fuzzy complex linear equation system $A\tilde{X} + \tilde{C} = B\tilde{X} + \tilde{D}$ where A, B are crisp coefficient matrices and $\tilde{C}, \tilde{D}$ are matrices of fuzzy complex numbers as follows:

1. Given a system of dual fuzzy complex linear form $A\tilde{X}+\tilde{C}=B\tilde{X}+\tilde{D}$ where A,B are crisp coefficient matrices and $\tilde{C},\tilde{D}$ are matrices of fuzzy complex numbers.
2. Next, the second step is transformed into a system of dual fuzzy complex linear equations in the form $(A-B)\tilde{X}=(\tilde{D}-\tilde{C})$ to $det(A-B)\neq 0$ and $det(|B|-|A|)$ hence,the equation has a solution.
3. Based on definition 2.11, the next step is to determine the value of $\tilde{X}$ which can be written as $\tilde{X}_E=(A-B)^{-1}(\tilde{D}-\tilde{C})$.
4. Determining the algebraic solution of fuzzy complex linear equations based on theorem 2.4 using $[\tilde{X}_A]_\alpha=[\underline{X}_E+F(\alpha),\overline{X}_E(\alpha)-F(\alpha)],\ \forall_\alpha\in[0,1]$.
5. Referring to consequence [6] the solution in step five is simplified further.

Based on the solution of the system above, we formulate it into the following theorem.

Theorem 3.1 Given a system of dual fuzzy complex matrix equations $A\tilde{X}+\tilde{C}=B\tilde{X}+\tilde{D}$ with $[\tilde{X}]_\alpha=[\underline{x}(\alpha),\overline{x}(\alpha)]$, $[\tilde{C}]_\alpha=[\underline{c}(\alpha),\overline{c}(\alpha)]$ and $[\tilde{D}]_\alpha=[\underline{d}(\alpha),\overline{d}(\alpha)]$ representing sets of fuzzy complex equations, where A,B are crisp coefficient matrices and $\tilde{C},\tilde{D}$ are matrices of fuzzy complex numbers, then the algebraic solution exist if $(A-B)^{-1}$ and $(|B|-|A|)^{-1}$ exist.

Proof. Based on Definition 2.5, Definition 2.8, Statement 2.10.1, and Theorem 2.4, we have:

For $[\tilde{X}_E]_\alpha=[\underline{x}_E(\alpha),\overline{x}_E(\alpha)]$

$$\underline{x}_E(\alpha)=x_E^C(\alpha)-x_E^R(\alpha)$$

$$=(A-B)^{-1}(D^C(\alpha)-C^C(\alpha))-|(A-B)|^{-1}(D^R(\alpha)+C^R(\alpha)),$$

$$=(A-B)^{-1}\left(\left(\frac{\underline{m}(\alpha)+\overline{m}(\alpha)}{2}+i\frac{\underline{n}(\alpha)+\overline{n}(\alpha)}{2}\right)-\left(\frac{\underline{g}(\alpha)+\overline{g}(\alpha)}{2}+i\frac{\underline{h}(\alpha)+\overline{h}(\alpha)}{2}\right)\right)-|(A-B)|^{-1}\left(\left(\frac{\overline{m}(\alpha)-\underline{m}(\alpha)}{2}+i\frac{\overline{n}(\alpha)-\underline{n}(\alpha)}{2}\right)+\left(\frac{\overline{g}(\alpha)-\underline{g}(\alpha)}{2}+i\frac{\overline{h}(\alpha)-\underline{h}(\alpha)}{2}\right)\right),$$

$$=(A-B)^{-1}\frac{1}{2}\left(\underline{m}(\alpha)+\overline{m}(\alpha)+i\left(\underline{n}(\alpha)+\overline{n}(\alpha)\right)-\underline{g}(\alpha)+\overline{g}(\alpha)-i\left(\underline{h}(\alpha)+\overline{h}(\alpha)\right)\right)-|(A-B)|^{-1}\frac{1}{2}\left(\overline{m}(\alpha)-\underline{m}(\alpha)+i\left(\overline{n}(\alpha)-\underline{n}(\alpha)\right)+\overline{g}(\alpha)-\underline{g}(\alpha)+i\left(\overline{h}(\alpha)-\underline{h}(\alpha)\right)\right),$$

$$=(a_{ij}-b_{ij})^{-1}\frac{1}{2}\left(\underline{m}_{jE}(\alpha)+\overline{m}_{jE}(\alpha)+i\left(\underline{n}_{jE}(\alpha)+\overline{n}_{jE}(\alpha)\right)-\underline{g}_{jE}(\alpha)+\overline{g}_{jE}(\alpha)-i\left(\underline{h}_{jE}(\alpha)+\overline{h}_{jE}(\alpha)\right)\right)-|(a_{ij}-b_{ij})|^{-1}\frac{1}{2}\left(\overline{m}_{jE}(\alpha)-\underline{m}_{jE}(\alpha)+i\left(\overline{n}_{jE}(\alpha)-\underline{n}_{jE}(\alpha)\right)+\overline{g}_{jE}(\alpha)-\underline{g}_{jE}(\alpha)+i\left(\overline{h}_{jE}(\alpha)-\underline{h}_{jE}(\alpha)\right)\right),$$

$$=(a_{ij}-b_{ij})^{-1}\frac{1}{2}\left(\left(\underline{m}_{jE}(\alpha)+i\underline{n}_{jE}(\alpha)\right)+\left(\overline{m}_{jE}(\alpha)+i\overline{n}_{jE}(\alpha)\right)-\left(\underline{g}_{jE}(\alpha)+i\underline{h}_{jE}(\alpha)\right)-\left(\overline{g}_{jE}(\alpha)+i\overline{h}_{jE}(\alpha)\right)\right)-|(a_{ij}-b_{ij})|^{-1}\frac{1}{2}\left(\left(\overline{m}_{jE}(\alpha)-i\overline{n}_{jE}(\alpha)\right)+\left(\underline{m}_{jE}(\alpha)-i\underline{n}_{jE}(\alpha)\right)+\left(\overline{g}_{jE}(\alpha)-i\overline{h}_{jE}(\alpha)\right)+\left(\underline{g}_{jE}(\alpha)-i\underline{h}_{jE}(\alpha)\right)\right),$$

$$=(A-B)^{-1}\frac{1}{2}\left(\underline{d}(\alpha)+\overline{d}(\alpha)-\underline{c}(\alpha)-\overline{c}(\alpha)\right)-|(A-B)|^{-1}\frac{1}{2}\left(\overline{d}(\alpha)-\underline{d}(\alpha)+\overline{c}(\alpha)-\underline{c}(\alpha)\right),$$

$$\overline{x}_E(\alpha)=x_E^C(\alpha)+x_E^R(\alpha)$$

$$=(A-B)^{-1}(D^C(\alpha)-C^C(\alpha))+|(A-B)|^{-1}(D^R(\alpha)+C^R(\alpha)),$$

$$=(A-B)^{-1}\left(\left(\frac{\underline{m}(\alpha)+\overline{m}(\alpha)}{2}+i\frac{\underline{n}(\alpha)+\overline{n}(\alpha)}{2}\right)-\left(\frac{\underline{g}(\alpha)+\overline{g}(\alpha)}{2}+i\frac{\underline{h}(\alpha)+\overline{h}(\alpha)}{2}\right)\right)+|(A-B)|^{-1}\left(\left(\frac{\overline{m}(\alpha)-\underline{m}(\alpha)}{2}+i\frac{\overline{n}(\alpha)-\underline{n}(\alpha)}{2}\right)+\left(\frac{\overline{g}(\alpha)-\underline{g}(\alpha)}{2}+i\frac{\overline{h}(\alpha)-\underline{h}(\alpha)}{2}\right)\right),$$

$$=(A-B)^{-1}\frac{1}{2}\left(\underline{m}(\alpha)+\overline{m}(\alpha)+i\left(\underline{n}(\alpha)+\overline{n}(\alpha)\right)-\underline{g}(\alpha)+\overline{g}(\alpha)-i\left(\underline{h}(\alpha)+\overline{h}(\alpha)\right)\right)+|(A-B)|^{-1}\frac{1}{2}\left(\overline{m}(\alpha)-\underline{m}(\alpha)+i\left(\overline{n}(\alpha)-\underline{n}(\alpha)\right)+\overline{g}(\alpha)-\underline{g}(\alpha)+i\left(\overline{h}(\alpha)-\underline{h}(\alpha)\right)\right),$$

$$=(a_{ij}-b_{ij})^{-1}\frac{1}{2}\left(\underline{m}_{jE}(\alpha)+\overline{m}_{jE}(\alpha)+i\left(\underline{n}_{jE}(\alpha)+\overline{n}_{jE}(\alpha)\right)-\underline{g}_{jE}(\alpha)+\overline{g}_{jE}(\alpha)-i\left(\underline{h}_{jE}(\alpha)+\overline{h}_{jE}(\alpha)\right)\right)+|(a_{ij}-b_{ij})|^{-1}\frac{1}{2}\left(\overline{m}_{jE}(\alpha)-\underline{m}_{jE}(\alpha)+i\left(\overline{n}_{jE}(\alpha)-\underline{n}_{jE}(\alpha)\right)+\overline{g}_{jE}(\alpha)-\underline{g}_{jE}(\alpha)+i\left(\overline{h}_{jE}(\alpha)-\underline{h}_{jE}(\alpha)\right)\right),$$

$$=(a_{ij}-b_{ij})^{-1}\frac{1}{2}\left(\left(\underline{m}_{jE}(\alpha)+i\underline{n}_{jE}(\alpha)\right)+\left(\overline{m}_{jE}(\alpha)+i\overline{n}_{jE}(\alpha)\right)-\left(\underline{g}_{jE}(\alpha)+i\underline{h}_{jE}(\alpha)\right)-\left(\overline{g}_{jE}(\alpha)+i\overline{h}_{jE}(\alpha)\right)\right)+|(a_{ij}-b_{ij})|^{-1}\frac{1}{2}\left(\left(\overline{m}_{jE}(\alpha)-i\overline{n}_{jE}(\alpha)\right)+\left(\underline{m}_{jE}(\alpha)-i\underline{n}_{jE}(\alpha)\right)+\left(\overline{g}_{jE}(\alpha)-i\overline{h}_{jE}(\alpha)\right)+\left(\underline{g}_{jE}(\alpha)-i\underline{h}_{jE}(\alpha)\right)\right),$$

$$=(A-B)^{-1}\frac{1}{2}\left(\underline{d}(\alpha)+\overline{d}(\alpha)-\underline{c}(\alpha)-\overline{c}(\alpha)\right)+|(A-B)|^{-1}\frac{1}{2}\left(\overline{d}(\alpha)-\underline{d}(\alpha)+\overline{c}(\alpha)-\underline{c}(\alpha)\right),$$

For $[\tilde{X}_A]_\alpha=[\underline{X}_A(\alpha),\overline{X}_A(\alpha)]$

$$\underline{X}_A(\alpha)=\underline{X}_E(\alpha)+F(\alpha)$$

$$=x_E^C(\alpha)-x_E^R(\alpha)+X_E^R(\alpha)+(|B|-|A|)^{-1}(D^R(\alpha)-C^R(\alpha)),$$

$$=x_E^C(\alpha)+(|B|-|A|)^{-1}(D^R(\alpha)-C^R(\alpha)),$$

$$=(A-B)^{-1}(D^C(\alpha)-C^C(\alpha))+(|B|-|A|)^{-1}(D^R(\alpha)-C^R(\alpha)),$$

$= (A-B)^{-1}\left(\left(\frac{\underline{m}(\alpha)+\overline{m}(\alpha)}{2}+i\frac{\underline{n}(\alpha)+\overline{n}(\alpha)}{2}\right)-\left(\frac{\underline{g}(\alpha)+\overline{g}(\alpha)}{2}+i\frac{\underline{h}(\alpha)+\overline{h}(\alpha)}{2}\right)\right)+(|B|-|A|)^{-1}\left(\left(\frac{\overline{m}(\alpha)-\underline{m}(\alpha)}{2}+i\frac{\overline{n}(\alpha)-\underline{n}(\alpha)}{2}\right)-\left(\frac{\overline{g}(\alpha)-\underline{g}(\alpha)}{2}+i\frac{\overline{h}(\alpha)-\underline{h}(\alpha)}{2}\right)\right),$

$= (A-B)^{-1}\frac{1}{2}\left(\left(\underline{m}(\alpha)+\overline{m}(\alpha)\right)+i\left(\underline{n}(\alpha)+\overline{n}(\alpha)\right)-\left(\underline{g}(\alpha)+\overline{g}(\alpha)\right)-i\left(\underline{h}(\alpha)+\overline{h}(\alpha)\right)\right)+(|B|-|A|)^{-1}\frac{1}{2}\left(\left(\overline{m}(\alpha)-\underline{m}(\alpha)\right)+i\left(\overline{n}(\alpha)-\underline{n}(\alpha)\right)-\left(\overline{g}(\alpha)-\underline{g}(\alpha)\right)-i\left(\overline{h}(\alpha)-\underline{h}(\alpha)\right)\right),$

$= \left(a_{ij}-b_{ij}\right)^{-1}\frac{1}{2}\left(\left(\underline{m}_{jE}(\alpha)+\overline{m}_{jE}(\alpha)\right)+i\left(\underline{n}_{jE}(\alpha)+\overline{n}_{jE}(\alpha)\right)-\left(\underline{g}_{jE}(\alpha)+\overline{g}_{jE}(\alpha)\right)-i\left(\underline{h}_{jE}(\alpha)+\overline{h}_{jE}(\alpha)\right)\right)+\left(|b_{ij}|-|a_{ij}|\right)^{-1}\frac{1}{2}\left(\left(\overline{m}_{jE}(\alpha)-\underline{m}_{jE}(\alpha)\right)+i\left(\overline{n}_{jE}(\alpha)-\underline{n}_{jE}(\alpha)\right)-\left(\overline{g}_{jE}(\alpha)-\underline{g}_{jE}(\alpha)\right)-i\left(\overline{h}_{jE}(\alpha)-\underline{h}_{jE}(\alpha)\right)\right),$

$=\left(a_{ij}-b_{ij}\right)^{-1}\frac{1}{2}\left(\left(\underline{m}_{jE}(\alpha)+i\underline{n}_{jE}(\alpha)\right)+\left(\overline{m}_{jE}(\alpha)+i\overline{n}_{jE}(\alpha)\right)-\left(\underline{g}_{jE}(\alpha)+i\underline{h}_{jE}(\alpha)\right)-\left(\overline{g}_{jE}(\alpha)+i\overline{h}_{jE}(\alpha)\right)\right)+\left(|b_{ij}|-|a_{ij}|\right)^{-1}\frac{1}{2}\left(\left(\overline{m}_{jE}(\alpha)+i\overline{n}_{jE}(\alpha)\right)+\left(-\underline{m}_{jE}(\alpha)-i\underline{n}_{jE}(\alpha)\right)-\left(\overline{g}_{jE}(\alpha)+i\overline{h}_{jE}(\alpha)\right)-\left(-\underline{g}_{jE}(\alpha)-i\underline{h}_{jE}(\alpha)\right)\right),$

$=\frac{1}{2}(A-B)^{-1}\left(\underline{d}(\alpha)+\overline{d}(\alpha)-\underline{c}(\alpha)-\overline{c}(\alpha)\right)+\frac{1}{2}(|B|-|A|)^{-1}\left(\overline{d}(\alpha)-\underline{d}(\alpha)-\overline{c}(\alpha)+\underline{c}(\alpha)\right),$

$\overline{X}_A=\overline{X}_E-F(\alpha)$
$=x_E^C(\alpha)+x_E^R(\alpha)-X_E^R(\alpha)-(|B|-|A|)^{-1}\left(D^R(\alpha)-C^R(\alpha)\right),$
$=x_E^C(\alpha)-(|B|-|A|)^{-1}\left(D^R(\alpha)-C^R(\alpha)\right),$
$=(A-B)^{-1}\left(D^C(\alpha)-C^C(\alpha)\right)-(|B|-|A|)^{-1}\left(D^R(\alpha)-C^R(\alpha)\right),$

$= (A-B)^{-1}\left(\left(\frac{\underline{m}(\alpha)+\overline{m}(\alpha)}{2}+i\frac{\underline{n}(\alpha)+\overline{n}(\alpha)}{2}\right)-\left(\frac{\underline{g}(\alpha)+\overline{g}(\alpha)}{2}+i\frac{\underline{h}(\alpha)+\overline{h}(\alpha)}{2}\right)\right)-(|B|-|A|)^{-1}\left(\left(\frac{\overline{m}(\alpha)-\underline{m}(\alpha)}{2}+i\frac{\overline{n}(\alpha)-\underline{n}(\alpha)}{2}\right)-\left(\frac{\overline{g}(\alpha)-\underline{g}(\alpha)}{2}+i\frac{\overline{h}(\alpha)-\underline{h}(\alpha)}{2}\right)\right),$

$= (A-B)^{-1}\frac{1}{2}\left(\left(\underline{m}(\alpha)+\overline{m}(\alpha)\right)+i\left(\underline{n}(\alpha)+\overline{n}(\alpha)\right)-\left(\underline{g}(\alpha)+\overline{g}(\alpha)\right)-i\left(\underline{h}(\alpha)+\overline{h}(\alpha)\right)\right)-(|B|-|A|)^{-1}\frac{1}{2}\left(\left(\overline{m}(\alpha)-\underline{m}(\alpha)\right)+i\left(\overline{n}(\alpha)-\underline{n}(\alpha)\right)-\left(\overline{g}(\alpha)-\underline{g}(\alpha)\right)-i\left(\overline{h}(\alpha)-\underline{h}(\alpha)\right)\right),$

$= \left(a_{ij}-b_{ij}\right)^{-1}\frac{1}{2}\left(\left(\underline{m}_{jE}(\alpha)+\overline{m}_{jE}(\alpha)\right)+i\left(\underline{n}_{jE}(\alpha)+\overline{n}_{jE}(\alpha)\right)-\left(\underline{g}_{jE}(\alpha)+\overline{g}_{jE}(\alpha)\right)-i\left(\underline{h}_{jE}(\alpha)+\overline{h}_{jE}(\alpha)\right)\right)-\left(|b_{ij}|-|a_{ij}|\right)^{-1}\frac{1}{2}\left(\left(\overline{m}_{jE}(\alpha)-\underline{m}_{jE}(\alpha)\right)+i\left(\overline{n}_{jE}(\alpha)-\underline{n}_{jE}(\alpha)\right)-\left(\overline{g}_{jE}(\alpha)-\underline{g}_{jE}(\alpha)\right)-i\left(\overline{h}_{jE}(\alpha)-\underline{h}_{jE}(\alpha)\right)\right),$

$=\left(a_{ij}-b_{ij}\right)^{-1}\frac{1}{2}\left(\left(\underline{m}_{jE}(\alpha)+i\underline{n}_{jE}(\alpha)\right)+\left(\overline{m}_{jE}(\alpha)+i\overline{n}_{jE}(\alpha)\right)-\left(\underline{g}_{jE}(\alpha)+i\underline{h}_{jE}(\alpha)\right)-\left(\overline{g}_{jE}(\alpha)+i\overline{h}_{jE}(\alpha)\right)\right)-\left(|b_{ij}|-|a_{ij}|\right)^{-1}\frac{1}{2}\left(\left(\overline{m}_{jE}(\alpha)+i\overline{n}_{jE}(\alpha)\right)+\left(-\underline{m}_{jE}(\alpha)-i\underline{n}_{jE}(\alpha)\right)-\left(\overline{g}_{jE}(\alpha)+i\overline{h}_{jE}(\alpha)\right)-\left(-\underline{g}_{jE}(\alpha)-i\underline{h}_{jE}(\alpha)\right)\right),$

$$=\frac{1}{2}(A-B)^{-1}\left(\underline{d}(\alpha)+\overline{d}(\alpha)-\underline{c}(\alpha)-\overline{c}(\alpha)\right)-\frac{1}{2}(|B|-|A|)^{-1}\left(\overline{d}(\alpha)-\underline{d}(\alpha)-\overline{c}(\alpha)+\underline{c}(\alpha)\right)$$

So, the solution of $A\tilde{X}+\tilde{C}=B\tilde{X}+\tilde{D}$ is

$[\tilde{X}_A]_\alpha=\left[\underline{X}_A(\alpha),\overline{X}_A(\alpha)\right],$

$\underline{X}_A(\alpha)=(A-B)^{-1}\frac{1}{2}\left(\left(\underline{d}(\alpha)+\overline{d}(\alpha)\right)-\left(\underline{c}(\alpha)+\overline{c}(\alpha)\right)\right)+(|B|-|A|)^{-1}\frac{1}{2}\left(\left(\underline{d}(\alpha)-\overline{d}(\alpha)\right)-\left(\underline{c}(\alpha)-\overline{c}(\alpha)\right)\right),$

$\overline{X}_A(\alpha)=(A-B)^{-1}\frac{1}{2}\left(\left(\underline{d}(\alpha)+\overline{d}(\alpha)\right)-\left(\underline{c}(\alpha)+\overline{c}(\alpha)\right)\right)-(|B|-|A|)^{-1}\frac{1}{2}\left(\left(\underline{d}(\alpha)-\overline{d}(\alpha)\right)-\left(\underline{c}(\alpha)-\overline{c}(\alpha)\right)\right).$ ∎

Example 3.1 Given the system of linear dual fuzzy complex matrices as follows:

$$\begin{pmatrix}1 & -2\\ 1 & 3\end{pmatrix}\begin{pmatrix}\tilde{x}_1\\ \tilde{x}_2\end{pmatrix}+\begin{pmatrix}\tilde{c}_1\\ \tilde{c}_2\end{pmatrix}=\begin{pmatrix}4 & -1\\ 1 & 2\end{pmatrix}\begin{pmatrix}\tilde{x}_1\\ \tilde{x}_2\end{pmatrix}+\begin{pmatrix}\tilde{d}_1\\ \tilde{d}_2\end{pmatrix}$$

with

$$[\tilde{C}]_\alpha=\begin{pmatrix}[\tilde{c}_1]_\alpha\\ [\tilde{c}_2]_\alpha\end{pmatrix}=\begin{pmatrix}\alpha, 2-\alpha\\ \alpha, \frac{5}{2}-\frac{3}{2}\alpha\end{pmatrix}+i\begin{pmatrix}-2+3\alpha, 2-2\alpha\\ 4+\alpha, -1-2\alpha\end{pmatrix}$$

and

$$[\tilde{D}]_\alpha=\begin{pmatrix}[\tilde{d}_1]_\alpha\\ [\tilde{d}_2]_\alpha\end{pmatrix}=\begin{pmatrix}\alpha, 2-\alpha\\ 4+\alpha, 7-2\alpha\end{pmatrix}+i\begin{pmatrix}1+\alpha, 3-\alpha\\ -4+2\alpha, -1-2\alpha\end{pmatrix}$$

Find the solution of the above equation?

For each $\alpha\in[0,1]$ has the following algebraic solution:

$$(A-B)\tilde{X}=\left(\tilde{D}-\tilde{C}\right)$$

$$\begin{pmatrix}-3 & -1\\ 0 & 1\end{pmatrix}\begin{pmatrix}\tilde{x}_1\\ \tilde{x}_2\end{pmatrix}=\begin{pmatrix}-2+2\alpha,2-2\alpha\\ \frac{3}{2}+\frac{5}{2}\alpha,7-3\alpha\end{pmatrix}+i\begin{pmatrix}-1+3\alpha,5-4\alpha\\ -3+4\alpha,-5-3\alpha\end{pmatrix}$$

By $\det(A-B)=-3$, $\quad\det(|B|-|A|)=-3$

Then

$$[\tilde{X}_E]_\alpha=(A-B)^{-1}\left([\tilde{D}]_\alpha-[\tilde{C}]_\alpha\right)$$

$$=\begin{pmatrix}[\tilde{p}_{1_E}]_\alpha+i[\tilde{q}_{1_E}]_\alpha\\ [\tilde{p}_{2_E}]_\alpha+i[\tilde{q}_{2_E}]_\alpha\end{pmatrix}$$

$$=\begin{pmatrix}\left[\frac{1}{6}-\frac{3}{2}\alpha,-3+\frac{5}{3}\alpha\right]+i\left[\frac{4}{3}-\frac{7}{3}\alpha,\frac{7}{3}\alpha\right]\\ \left[\frac{3}{2}+\frac{5}{2}\alpha,7-3\alpha\right]+i[-3+4\alpha,-5-3\alpha]\end{pmatrix}$$

$$[\tilde{X}_A]_\alpha=[\underline{X}_A(\alpha),\overline{X}_A(\alpha)]$$

$$\underline{X}_A(\alpha)=(A-B)^{-1}\tfrac{1}{2}\left(\left(\underline{d}(\alpha)+\overline{d}(\alpha)\right)-\left(\underline{c}(\alpha)+\overline{c}(\alpha)\right)\right)+(|B|-|A|)^{-1}\tfrac{1}{2}\left(\left(\underline{d}(\alpha)-\overline{d}(\alpha)\right)-\left(\underline{c}(\alpha)-\overline{c}(\alpha)\right)\right)$$

$$\underline{X}_A(\alpha)=\begin{pmatrix}\left[-\frac{3}{2}+\frac{1}{6}\alpha\right]+i\left[-1+\frac{2}{3}\alpha\right]\\ [4]+i[[-8+\alpha]]\end{pmatrix}$$

$$\overline{X}_A(\alpha)=(A-B)^{-1}\tfrac{1}{2}\left(\left(\underline{d}(\alpha)+\overline{d}(\alpha)\right)-\left(\underline{c}(\alpha)+\overline{c}(\alpha)\right)\right)-(|B|-|A|)^{-1}\tfrac{1}{2}\left(\left(\underline{d}(\alpha)-\overline{d}(\alpha)\right)-\left(\underline{c}(\alpha)-\overline{c}(\alpha)\right)\right)$$

$$\overline{X}_A(\alpha)=\begin{pmatrix}\left[-\frac{4}{3}\right]+i\left[\frac{7}{3}-\frac{2}{3}\alpha\right]\\ \left[\frac{9}{2}-\frac{1}{2}\alpha\right]+i[0]\end{pmatrix}$$

$$[\tilde{X}_A]_\alpha=\begin{pmatrix}[\tilde{p}_{1_A}]_\alpha+i[\tilde{q}_{1_A}]_\alpha\\ [\tilde{p}_{2_A}]_\alpha+i[\tilde{q}_{2_A}]_\alpha\end{pmatrix}$$

$$=\begin{pmatrix}\left[-\frac{3}{2}+\frac{1}{6}\alpha,-\frac{4}{3}\right]+i\left[-1+\frac{2}{3}\alpha,\frac{7}{3}-\frac{2}{3}\alpha\right]\\ \left[4,\frac{9}{2}-\frac{1}{2}\alpha\right]+i[-8+\alpha]\end{pmatrix}$$

Example 3.2 Given the system of linear dual fuzzy complex matrices as follows:

Find the solution of the above equation?

$$\begin{pmatrix}2 & 2 & 0\\ 3 & 1 & -1\\ 1 & -2 & 0\end{pmatrix}\begin{pmatrix}\tilde{x}_1\\ \tilde{x}_2\\ \tilde{x}_3\end{pmatrix}+\begin{pmatrix}\tilde{c}_1\\ \tilde{c}_2\\ \tilde{c}_3\end{pmatrix}=\begin{pmatrix}1 & 1 & 1\\ 2 & 3 & -2\\ -1 & -3 & 1\end{pmatrix}\begin{pmatrix}\tilde{x}_1\\ \tilde{x}_2\\ \tilde{x}_3\end{pmatrix}+\begin{pmatrix}\tilde{d}_1\\ \tilde{d}_2\\ \tilde{d}_3\end{pmatrix}$$

Then

$$[\tilde{C}]_\alpha=\begin{pmatrix}[\tilde{c}_1]_\alpha\\ [\tilde{c}_2]_\alpha\\ [\tilde{c}_3]_\alpha\end{pmatrix}=\begin{pmatrix}\alpha,2-\alpha\\ 2+\alpha,3\\ -2,-1-\alpha\end{pmatrix}+i\begin{pmatrix}-2+3\alpha,2-2\alpha\\ 4+\alpha,2-2\alpha\\ -1-2\alpha,3-\alpha\end{pmatrix}$$

and

$$[\tilde{D}]_\alpha=\begin{pmatrix}[\tilde{d}_1]_\alpha\\ [\tilde{d}_2]_\alpha\\ [\tilde{d}_3]_\alpha\end{pmatrix}=\begin{pmatrix}\alpha,2-\alpha\\ \alpha,3-2\alpha\\ 2\alpha,2-\alpha\end{pmatrix}+i\begin{pmatrix}-2+3\alpha,2-2\alpha\\ 2+\alpha,-1-2\alpha\\ 1+\alpha,3-\alpha\end{pmatrix}$$

Find the solution of the above equation?

For each α∈[0,1] has the following algebraic solution:

$$(A-B)\tilde{X}=(\tilde{D}-\tilde{C})$$

$$\begin{pmatrix}1 & 1 & -1\\ 1 & -2 & 1\\ 2 & 1 & -1\end{pmatrix}\begin{pmatrix}\tilde{x}_1\\ \tilde{x}_2\\ \tilde{x}_3\end{pmatrix}=\begin{pmatrix}-2+2\alpha,2-2\alpha\\ -3+\alpha,1-3\alpha\\ 1+3\alpha,4-\alpha\end{pmatrix}+i\begin{pmatrix}-4+5\alpha,4-5\alpha\\ 3\alpha,-5-3\alpha\\ -2+2\alpha,4+\alpha\end{pmatrix}$$

By $\det(A-B)=-1$, $\quad\det(|B|-|A|)=-3$

For

$$[\tilde{X}_E]_\alpha=(A-B)^{-1}([\tilde{D}]_\alpha-[\tilde{C}]_\alpha)$$

$$=\begin{pmatrix}[\tilde{p}_{1_E}]_\alpha+i[\tilde{q}_{1_E}]_\alpha\\ [\tilde{p}_{2_E}]_\alpha+i[\tilde{q}_{2_E}]_\alpha\\ [\tilde{p}_{3_E}]_\alpha+i[\tilde{q}_{3_E}]_\alpha\end{pmatrix}$$

$$=\begin{pmatrix}[3+\alpha,2+\alpha]+i[2-3\alpha,6\alpha]\\ [11-\alpha,1+7\alpha]+i[8-14\alpha,1+20\alpha]\\ [16-2\alpha,1+10\alpha]+i[14-22\alpha,-3+31\alpha]\end{pmatrix}$$

$$[\tilde{X}_A]_\alpha=[\underline{X}_A(\alpha),\overline{X}_A(\alpha)]$$

$$\underline{X}_A(\alpha)=(A-B)^{-1}\tfrac{1}{2}\left(\left(\underline{d}(\alpha)+\overline{d}(\alpha)\right)-\left(\underline{c}(\alpha)+\overline{c}(\alpha)\right)\right)+(|B|-|A|)^{-1}\tfrac{1}{2}\left(\left(\underline{d}(\alpha)-\overline{d}(\alpha)\right)-\left(\underline{c}(\alpha)-\overline{c}(\alpha)\right)\right)$$

$$\underline{X}_A(\alpha)=\begin{pmatrix}\left[\frac{7}{3}+\frac{2}{3}\alpha\right]+i\left[\frac{1}{3}\right]\\ \left[\frac{19}{3}+\frac{8}{3}\alpha\right]+i\left[\frac{13}{3}+3\alpha\right]\\ \left[\frac{26}{3}+\frac{10}{3}\alpha\right]+i\left[\frac{14}{3}+3\alpha\right]\end{pmatrix}$$

$$\overline{X}_A(\alpha)=(A-B)^{-1}\tfrac{1}{2}\left(\left(\underline{d}(\alpha)+\overline{d}(\alpha)\right)-\left(\underline{c}(\alpha)+\overline{c}(\alpha)\right)\right)-(|B|-|A|)^{-1}\tfrac{1}{2}\left(\left(\underline{d}(\alpha)-\overline{d}(\alpha)\right)-\left(\underline{c}(\alpha)-\overline{c}(\alpha)\right)\right)$$

$$\overline{X}_A(\alpha)=\begin{pmatrix}\left[\frac{8}{3}+\frac{4}{3}\alpha\right]+i\left[\frac{5}{3}+3\alpha\right]\\ \left[\frac{17}{3}+\frac{10}{3}\alpha\right]+i\left[\frac{14}{3}+3\alpha\right]\\ \left[\frac{25}{3}+\frac{14}{3}\alpha\right]+i\left[\frac{19}{3}+6\alpha\right]\end{pmatrix}$$

$$= \begin{pmatrix} [\tilde{p}_{1_A}]_\alpha + i[\tilde{q}_{1_A}]_\alpha \\ [\tilde{p}_{2_A}]_\alpha + i[\tilde{q}_{2_A}]_\alpha \\ [\tilde{p}_{3_A}]_\alpha + i[\tilde{q}_{3_A}]_\alpha \end{pmatrix}$$

$$= \begin{pmatrix} \left[\frac{7}{3}+\frac{2}{3}\alpha, \frac{8}{3}+\frac{4}{3}\alpha\right] + i\left[\frac{1}{3}, \frac{5}{3}+3\alpha\right] \\ \left[\frac{19}{3}+\frac{8}{3}\alpha, \frac{17}{3}+\frac{10}{3}\alpha\right] + i\left[\frac{13}{3}+3\alpha, \frac{14}{3}+3\alpha\right] \\ \left[\frac{26}{3}+\frac{10}{3}\alpha, \frac{25}{3}+\frac{14}{3}\alpha\right] + i\left[\frac{14}{3}+3\alpha, \frac{19}{3}+6\alpha\right] \end{pmatrix}$$

Example 3.3 Given the system of linear dual fuzzy complex matrices as follows:

Find the solution of the above equation?

$$\begin{pmatrix} 2 & 3 & 2 & 4 & 0 \\ 2 & 2 & 2 & 0 & 2 \\ 1 & 3 & 3 & 2 & 1 \\ 3 & -2 & 2 & 4 & 3 \\ 2 & 2 & 2 & 3 & 1 \end{pmatrix} \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \\ \tilde{x}_4 \\ \tilde{x}_5 \end{pmatrix} + \begin{pmatrix} \tilde{c}_1 \\ \tilde{c}_2 \\ \tilde{c}_3 \\ \tilde{c}_4 \\ \tilde{c}_5 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 3 & 0 & 2 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 2 & 2 & 2 & 0 & 1 \\ 0 & -1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 3 & 2 \end{pmatrix} \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \\ \tilde{x}_4 \\ \tilde{x}_5 \end{pmatrix} + \begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \\ \tilde{d}_3 \\ \tilde{d}_4 \\ \tilde{d}_5 \end{pmatrix}$$

Then

$$[\tilde{C}]_\alpha = \begin{pmatrix} [\tilde{c}_1]_\alpha \\ [\tilde{c}_2]_\alpha \\ [\tilde{c}_3]_\alpha \\ [\tilde{c}_4]_\alpha \\ [\tilde{c}_5]_\alpha \end{pmatrix} = \begin{pmatrix} 2+2\alpha, 1-2\alpha \\ 2\alpha, 1-\alpha \\ \frac{1}{2}+\alpha, 3+\alpha \\ 1-\alpha, 2-\alpha \\ -1+\alpha, 1 \end{pmatrix} + i\begin{pmatrix} 1-\alpha, 2+\alpha \\ \alpha, 5-\alpha \\ 2+\alpha, 2+\alpha \\ 2-\alpha, \alpha \\ 1-\alpha, 1+\alpha \end{pmatrix}$$

and

$$[\tilde{D}]_\alpha = \begin{pmatrix} [\tilde{d}_1]_\alpha \\ [\tilde{d}_2]_\alpha \\ [\tilde{d}_3]_\alpha \\ [\tilde{d}_4]_\alpha \\ [\tilde{d}_5]_\alpha \end{pmatrix} = \begin{pmatrix} \alpha, 2-\alpha \\ 1+\alpha, -1-\alpha \\ 1+\alpha, 1-2\alpha \\ 2\alpha, 1-2\alpha \\ 1-\alpha, \alpha \end{pmatrix} + i\begin{pmatrix} \alpha, 1-\alpha \\ 1+\alpha, 2+\alpha \\ 2-\alpha, 1+\alpha \\ 2+\alpha, 3-2\alpha \\ 3+2\alpha, \alpha \end{pmatrix}$$

Find the solution of the above equation?

For each $\alpha \in [0,1]$ has the following algebraic solution:

$$(A-B)\tilde{X} = (\tilde{D} - \tilde{C})$$

$$\begin{pmatrix} 1 & 0 & 2 & 2 & -1 \\ 1 & 1 & 1 & 0 & 2 \\ -1 & 1 & 1 & 2 & 0 \\ 3 & -1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 0 & -1 \end{pmatrix} \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \\ \tilde{x}_4 \\ \tilde{x}_5 \end{pmatrix} = \begin{pmatrix} -1+3\alpha, -3\alpha \\ 2\alpha, -1-3\alpha \\ -2, \frac{1}{2}-3\alpha \\ -2+3\alpha, -\alpha \\ -\alpha, 1 \end{pmatrix} + i\begin{pmatrix} -2,0 \\ -4+2\alpha, 2 \\ -2\alpha, -1 \\ 2, 1-\alpha \\ 2+\alpha, -1+2\alpha \end{pmatrix}$$

By $\det(A-B) = -36$, $\det(|B|-|A|) = -48$

For

$$[\tilde{X}_E]_\alpha = (A-B)^{-1}([\tilde{D}]_\alpha - [\tilde{C}]_\alpha)$$

$$= \begin{pmatrix} [\tilde{p}_{1_E}]_\alpha + i[\tilde{q}_{1_E}]_\alpha \\ [\tilde{p}_{2_E}]_\alpha + i[\tilde{q}_{2_E}]_\alpha \\ [\tilde{p}_{3_E}]_\alpha + i[\tilde{q}_{3_E}]_\alpha \\ [\tilde{p}_{4_E}]_\alpha + i[\tilde{q}_{4_E}]_\alpha \\ [\tilde{p}_{5_E}]_\alpha + i[\tilde{q}_{5_E}]_\alpha \end{pmatrix}$$

$$= \begin{pmatrix} \left[-\frac{1}{3}-\alpha, \frac{7}{9}+\frac{4}{3}\alpha\right] + i\left[\frac{10}{3}+\frac{1}{9}\alpha, -\frac{2}{3}+\frac{7}{9}\alpha\right] \\ \left[-\frac{2}{3}-\frac{5}{2}\alpha, \frac{41}{36}+\frac{2}{3}\alpha\right] + i\left[\frac{11}{3}-\frac{4}{9}\alpha, -\frac{4}{3}+\frac{25}{18}\alpha\right] \\ \left[1+\frac{7}{2}\alpha, -\frac{19}{12}-3\alpha\right] + i\left[-7+\frac{5}{3}\alpha, 2-\frac{5}{6}\alpha\right] \\ \left[-\frac{4}{3}-\alpha, \frac{31}{36}+\frac{1}{3}\alpha\right] + i\left[\frac{10}{3}-\frac{14}{9}\alpha, -\frac{7}{6}+\frac{1}{9}\alpha\right] \\ \left[\alpha, -\frac{2}{3}-\alpha\right] + i\left[-2+\frac{1}{3}\alpha, 1-\frac{2}{3}\alpha\right] \end{pmatrix}$$

$$[\tilde{X}_A]_\alpha = [\underline{X}_A(\alpha), \overline{X}_A(\alpha)]$$

$$\underline{X}_A(\alpha) = (A-B)^{-1}\tfrac{1}{2}\Big(\big(\underline{d}(\alpha)+\overline{d}(\alpha)\big) - \big(\underline{c}(\alpha)+\overline{c}(\alpha)\big)\Big) + (|B|-|A|)^{-1}\tfrac{1}{2}\Big(\big(\underline{d}(\alpha)-\overline{d}(\alpha)\big) - \big(\underline{c}(\alpha)-\overline{c}(\alpha)\big)\Big)$$

$$\underline{X}_A(\alpha) = \begin{pmatrix} \left[-\frac{13}{144}+\frac{1}{8}\alpha\right] + i\left[\frac{3}{4}+\frac{72}{125}\alpha\right] \\ \left[\frac{635}{288}-\frac{7}{16}\alpha\right] + i\left[\frac{15}{8}+\frac{53}{144}\alpha\right] \\ \left[-\frac{43}{96}+\frac{48}{65}\alpha\right] + i\left[-\frac{23}{24}-\frac{5}{48}\alpha\right] \\ \left[-\frac{97}{144}+\frac{23}{24}\alpha\right] + i\left[-\frac{1}{12}-\frac{19}{72}\alpha\right] \\ \left[-\frac{1}{3}+\frac{1}{3}\alpha\right] + i\left[-\frac{1}{3}+\alpha\right] \end{pmatrix}$$

$$\overline{X}_A(\alpha) = (A-B)^{-1}\tfrac{1}{2}\Big(\big(\underline{d}(\alpha)+\overline{d}(\alpha)\big) - \big(\underline{c}(\alpha)+\overline{c}(\alpha)\big)\Big) - (|B|-|A|)^{-1}\tfrac{1}{2}\Big(\big(\underline{d}(\alpha)-\overline{d}(\alpha)\big) - \big(\underline{c}(\alpha)-\overline{c}(\alpha)\big)\Big)$$

$$\overline{X}_A(\alpha) = \begin{pmatrix} \left[\frac{77}{144} + \frac{5}{24}\alpha\right] + i\left[\frac{23}{12} - \frac{61}{72}\alpha\right] \\ \left[-\frac{499}{288} - \frac{67}{48}\alpha\right] + i\left[\frac{11}{24} + \frac{83}{144}\alpha\right] \\ \left[-\frac{13}{96} + \frac{89}{48}\alpha\right] + i\left[-\frac{97}{24} + \frac{15}{16}\alpha\right] \\ \left[\frac{29}{144} - \frac{13}{8}\alpha\right] + i\left[\frac{9}{4} - \frac{85}{72}\alpha\right] \\ \left[-\frac{1}{3} - \frac{1}{3}\alpha\right] + i\left[-\frac{2}{3} + \frac{2}{3}\alpha\right] \end{pmatrix}$$

$$= \begin{pmatrix} [\tilde{p}_{1_A}]_\alpha + i[\tilde{q}_{1_A}]_\alpha \\ [\tilde{p}_{2_A}]_\alpha + i[\tilde{q}_{2_A}]_\alpha \\ [\tilde{p}_{3_A}]_\alpha + i[\tilde{q}_{3_A}]_\alpha \\ [\tilde{p}_{4_A}]_\alpha + i[\tilde{q}_{4_A}]_\alpha \\ [\tilde{p}_{5_A}]_\alpha + i[\tilde{q}_{5_A}]_\alpha \end{pmatrix} =$$

$$\begin{pmatrix} \left[-\frac{13}{144} + \frac{1}{8}\alpha, \frac{77}{144} + \frac{5}{24}\alpha\right] + i\left[\frac{3}{4} + \frac{72}{125}\alpha, \frac{23}{12} - \frac{61}{72}\alpha\right] \\ \left[\frac{635}{288} - \frac{7}{16}\alpha, -\frac{499}{288} - \frac{67}{48}\alpha\right] + i\left[\frac{15}{8} + \frac{53}{144}\alpha, \frac{11}{24} + \frac{83}{144}\alpha\right] \\ \left[-\frac{43}{96} + \frac{48}{65}\alpha, -\frac{13}{96} + \frac{89}{48}\alpha\right] + i\left[-\frac{23}{24} - \frac{5}{48}\alpha, -\frac{97}{24} + \frac{15}{16}\alpha\right] \\ \left[-\frac{97}{144} + \frac{23}{24}\alpha, \frac{29}{144} - \frac{13}{8}\alpha\right] + i\left[-\frac{1}{12} - \frac{19}{72}\alpha, \frac{9}{4} - \frac{85}{72}\alpha\right] \\ \left[-\frac{1}{3} + \frac{1}{3}\alpha, -\frac{1}{3} - \frac{1}{3}\alpha\right] + i\left[-\frac{1}{3} + \alpha, -\frac{2}{3} + \frac{2}{3}\alpha\right] \end{pmatrix}$$

IV. CONCLUSIONS

Based on the discussion above, the system of dual fuzzy complex linear equation $A\tilde{X} + \tilde{C} = B\tilde{X} + \widetilde{D}$ has a solution.

$$\underline{X}_A(\alpha) = (A-B)^{-1}\tfrac{1}{2}\Big(\big(\underline{d}(\alpha) + \overline{d}(\alpha)\big) - \big(\underline{c}(\alpha) + \overline{c}(\alpha)\big)\Big) + (|B|-|A|)^{-1}\tfrac{1}{2}\Big(\big(\underline{d}(\alpha) - \overline{d}(\alpha)\big) - \big(\underline{c}(\alpha) - \overline{c}(\alpha)\big)\Big)$$

$$\overline{X}_A(\alpha) = (A-B)^{-1}\tfrac{1}{2}\Big(\big(\underline{d}(\alpha) + \overline{d}(\alpha)\big) - \big(\underline{c}(\alpha) + \overline{c}(\alpha)\big)\Big) - (|B|-|A|)^{-1}\tfrac{1}{2}\Big(\big(\underline{d}(\alpha) - \overline{d}(\alpha)\big) - \big(\underline{c}(\alpha) - \overline{c}(\alpha)\big)\Big).$$

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