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Numerical Solution to System of Six Coupled Nonlinear ODEs by Adomian Decomposition Method

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ABSTRACT

In this paper, we have proposed the numerical solutions of the system of six coupled nonlinear Ordinary Differential Equations (ODEs), which are obtained by reducing stratified Boussinesq Equations. We have obtained the numerical solutions on unstable and stable manifolds by Adomian Decomposition Method (ADM). The minimum error in the solution is of the order 10^{-6}. This error can be reduced by reducing size of interval. We have used MATHEMATICA 9 for programming and calculations. We have compared the results with Euler Modified Method (EMM also referred as Modified Euler Method (MEM)) and Runge-Kutta Fourth Order (RK4) Method.

Keywords: Stratified Boussinesq equations, Adomian Polynomials, Coupled Differential Equations, Integrable systems.

TITLE: Numerical solution to system of six coupled nonlinear ODEs by Adomian Decomposition Method.

1 INTRODUCTION

The stratified Boussinesq equations form a system of Partial Differential Equations modeling the movements of planetary atmospheres. The literature also refers Boussinesq approximation as Oberbeck-

Boussinesq approximation [1]. In this view Desale [2] has given the complete analysis of an ideal rotating stratified system of ODEs. Further, in extension of this work Desale and Sharma [3] have given the special solutions of rotating stratified Boussinesq equations. On the other hand Desale and Dasre [4, 5] have given the numerical solutions to the system (1) through the deployment of Euler Modified Method and Runge-Kutta fourth order method. In this paper we have deployed the Adomian Decomposition Method (ADM) to find the numerical solution of system (1) with initial values on the stable and unstable manifolds. We have discussed the implementation of this method in the section 4.1.

2 PRELIMINARY NOTES

In their paper, Srinivasan et. al. [6] have discussed the complete integrability system (1). Also, the system (1) have been tested for integrability via Painleve` test by the authors Desale and Srinivasan [7]. The following is the system of six coupled nonlinear ODEs, which is aroused in the reduction of stratified Boussinesq equations.

$$\left.\begin{aligned} \dot{\mathbf{w}} &= \frac{g}{\rho_b}\hat{\mathbf{e}}_3 \times \mathbf{b}, \\ \dot{\mathbf{b}} &= \frac{1}{2}\mathbf{w} \times \mathbf{b}, \end{aligned}\right\} \tag{1}$$

where $\mathbf{w} = (w_1,w_2,w_3)^T$ is the velocity vector, $\mathbf{b} = (b_1, b_2, b_3)^T$ is the density gradient and $\frac{g}{\rho_b}$ is a non-dimensional constant as mentioned by Desale [8] in his thesis. The above system can be written as component wise as below

$$\left.\begin{aligned} \dot{w}_1 &= -\frac{g}{\rho_b}b_2, \\ \dot{w}_2 &= \frac{g}{\rho_b}b_1, \\ \dot{w}_3 &= 0, \\ \dot{b}_1 &= \frac{1}{2}(w_2b_3 - w_3b_2), \\ \dot{b}_2 &= \frac{1}{2}(w_3b_1 - w_1b_3), \\ \dot{b}_3 &= \frac{1}{2}(w_1b_2 - w_2b_1). \end{aligned}\right\} \tag{2}$$

More detail mathematical analysis of system (1) can be obtained from Desale [8]. The above system (1) is completely integrable and flow of vector field is complete, that is to say, all solutions exists on an invariant surface given by

$$|\mathbf{b}|^2 = c_1\,, \qquad \mathbf{w}\cdot\mathbf{b} = c_2\,, \hat{e}_3\cdot\mathbf{w} = c_3 \text{ and } |\mathbf{w}|^2 + \frac{4g}{\rho_b}\hat{e}_3\cdot\mathbf{b} = c_4. \tag{3}$$

This invariant surface is made up by three pieces named as stable, unstable and center manifolds. These manifolds glue together and forms a two dimensional torus pinched at critical point. In a particular case $c_1 = 1$, $c_2 = 1$, $c_3 = 1$ and $c_4 = \frac{1}{2} + \frac{2g}{\rho_b}$. A critical point $(\hat{e}_3, \hat{e}_3)$ lies on invariant surface. Hence, we have

$$\left.\begin{aligned} w_1 &= \frac{-b_2k}{1-b_3} + \frac{b_1}{1+b_3}, \\ w_2 &= \frac{b_1k}{1-b_3} + \frac{b_2}{1+b_3}, \\ w_3 &= 1. \end{aligned}\right\} \tag{4}$$

In above equations k is a function of b_3, and it can be expressed as,

$$k^2 = \frac{(1-\mathrm{b}_3)^2}{(1+\mathrm{b}_3)^2}\left[\frac{4\mathrm{g}(1+\mathrm{b}_3)-\rho_\mathrm{b}}{\rho_\mathrm{b}}\right]. \qquad (5)$$

Since, $|\mathbf{b}|^2 = 1$, which would enable to introduce the angular coordinates θ and ϕ given by

$$\left.\begin{array}{c} \mathrm{b}_1 = \cos\theta.\sin\phi, \quad \mathrm{b}_1 = \sin\theta.\sin\phi, \quad \mathrm{b}_1 = \cos\phi, \\ \text{where} \\ \phi = \phi(t), \quad \theta = \theta(t), \quad 0 \le \phi \le \pi. \end{array}\right\} \qquad (6)$$

Because of these angular coordinates, we have

$$k^2 = tan^4\left(\frac{\emptyset}{2}\right)\left[\frac{8\mathrm{g}}{\rho_\mathrm{b}}\cos^2\left(\frac{\emptyset}{2}\right) - 1\right]. \qquad (7)$$

Since k is real, we have the constraint on ϕ as $0 \le \emptyset \le \cos^{-1}\left(\sqrt{\frac{\rho_\mathrm{b}}{8\mathrm{g}}}\right)$ given by Desale [8]. Consequently we have

$$k = \pm tan^2\left(\frac{\emptyset}{2}\right)\sqrt{\frac{8\mathrm{g}}{\rho_\mathrm{b}}\cos^2\left(\frac{\emptyset}{2}\right) - 1}\ . \qquad (8)$$

Furthermore, $k = 0$ gives us the central manifold, $k > 0$ results into an unstable manifold and $k < 0$ results into the stable manifold. One may concern Srinivasan et. al. [6] for more detail analysis of these manifolds.

3 ADOMIAN DECOMPOSITION METHOD (ADM)

In the 1980's, George Adomian [9] introduced a new powerful method for solving nonlinear functional equations. Since then, this method has been recognized as the Adomian Decomposition Method (ADM). The technique in this method is based on a decomposition of the solution of a nonlinear operator equation in a series of functions. Each term of the series is then obtained from a polynomial generated by an expansion of an analytic function into a power series. The details for this method can be referred from [9–15].

J. Biazar et. al. [16] have stated the ADM for solving a system of ordinary differential equations as below. A system of ordinary differential equations of the first order can be considered as:

$$\left.\begin{array}{l} y_1{}' = f_1(x, y_1, \ldots, y_n), \\ y_2{}' = f_2(x, y_1, \ldots, y_n), \\ \vdots \\ y_n{}' = f_n(x, y_1, \ldots, y_n). \end{array}\right\} \qquad (9)$$

Each equation in above system represents the first derivative of the unknown functions $f_1, \ldots, f_n$ in which x is the independent variable.

Since every ordinary differential equation of order n can be written as a system consisting of n ordinary differential equation of order one, so they restricted their study to a system of differential equations of the first order.

Now, we look into systematic implementation of ADM with the reference of J. Biazar et. al. [16]. They have presented the system (9) in the compact form as:

$$Ly_i = f_1(x, y_1, ..., y_n), \quad i = 1, 2, ..., n \tag{10}$$

where L is the linear operator $\frac{d}{dx}$ with the inverse $L^{-1} = \int_0^x (\cdot) dx$. Applying the inverse operator on both sides of (10), we get the following canonical form which is computationally comfort for deployment of ADM.

$$y_i = y_i(0) + \int_0^x f_i(x, y_1, ..., y_n) dx, \quad i = 1, 2, ..., n. \tag{11}$$

As usual in ADM the solution of (11) is considered to be the sum of the series:

$$y_i = \sum_{j=0}^{\infty} f_{i,j}, \quad i = 1, 2, \ldots, n \tag{12}$$

and the integrand in (11) is the sum of the following series:

$$f_i(x, y_1, ..., y_n) = \sum_{j=0}^{\infty} A_{i,j}(f_{i,0}, f_{i,1}, ..., f_{i,j}), \quad i = 1, 2, \ldots, n \tag{13}$$

where $A_{i,j}(f_{i,0}, f_{i,1}, \ldots, f_{i,n})$ are called as Adomian polynomials. Substituting (12) and (13) into (11), which will result into the following equations.

$$\left.\begin{aligned} \textstyle\sum_{j=0}^{\infty} f_{i,j} &= y_i(0) + \textstyle\int_0^x \sum_{j=0}^{\infty} A_{i,j}(f_{i,0}, f_{i,1}, ..., f_{i,j}) dx, \\ &= y_i(0) + \textstyle\sum_{j=0}^{\infty} \int_0^x A_{i,j}(f_{i,0}, f_{i,1}, ..., f_{i,j}) dx. \end{aligned}\right\} \tag{14}$$

The above equations enables to define:

$$\left.\begin{aligned} f_{i,0} &= y_i(0), \\ f_{i,n+1} &= \textstyle\int_0^x A_{i,n}(f_{i,0}, f_{i,1}, ..., f_{i,n}) dx, \quad i = 0, 1, 2, ... \end{aligned}\right\} \tag{15}$$

In the following section we implement this method systematically to the six coupled ODEs (2) and obtained the numerical solutions on stable and unstable manifolds as described the previous section.

4 NUMERICAL SOLUTION OF SIX COUPLED ODES

In this paper, we have obtained the numerical solutions of a system (2) with the initial values on stable and unstable manifolds by ADM. We have used alternative algorithm to calculate Adomian Polynomials [17, 18].

4.1 Implementation of ADM

Proceeding towards to determine the numerical solutions we took the help of mathematical software MATHEMATICA 9 for faster calculations and generating the graphs. We have used the alternative algorithm as described in [17,18] to calculate the Adomian Polynomials. Jun-Sheng Duan [19] have obtained the recurrence relations for the simplified index matrices, which provide a convenient algorithm for rapid generation of the multivariable Adomian polynomials. E. Babolian and Sh. Javadi [21] have presented a good scheme of calculation for Adomian polynomials. Also, in this section we have compared results obtained by using this ADM with the earlier results which were obtained by Euler Modified Method and Runge-Kutta Fourth Order Method. Now we deploy Adomian Decomposition Method to the system (2) so that we start with the initial conditions $\mathbf{b}_0=(b_{10}, b_{20}, b_{30})$ and $\mathbf{w}_0 = (w_{10}, w_{20}, w_{30})$ at $t = 0$. Where $\boldsymbol{b}_0$ and $\boldsymbol{w}_0$ satisfy the equation (3) in particular case $c_1 = 1$, $c_2 = 1$, $c_3 = 1$ and $c_4 = \frac{1}{2} + \frac{2g}{\rho_b}$. The values $\boldsymbol{b}_0$ and $\boldsymbol{w}_0$ lie on the invariant surface. Furthermore $k = k(b_3)$, if $k = k(b_3) > 0$ then $\boldsymbol{b}_0$ and $\boldsymbol{w}_0$ lie on the unstable manifold and if $k = k(b_3) < 0$ then $\boldsymbol{b}_0$ and $\boldsymbol{w}_0$ lie on the stable manifold. Accordingly we can find the general solutions on unstable and stable manifold. Now we proceed to general solutions in the form of series as below

$$\left.\begin{aligned}
w_1 &= \sum_{j=0}^{\infty} w_{1,j},\\
w_2 &= \sum_{j=0}^{\infty} w_{2,j},\\
b_1 &= \sum_{j=0}^{\infty} b_{1,j},\\
b_2 &= \sum_{j=0}^{\infty} b_{2,j},\\
b_3 &= \sum_{j=0}^{\infty} b_{3,j}.
\end{aligned}\right\} \qquad (16)$$

Now we determine $w_{1,j}$,$w_{2,j}$, $b_{1,j}$, $b_{2,j}$, $b_{3,j}$ by implementation of ADM. Consider the following iterations. Let $\frac{g}{\rho_b} = G$ be a non dimensional constant and

$$\left.\begin{aligned}
& b_{1,0} = k_1, b_{2,0} = k_2, b_{3,0} = k_3,\\
& w_{1,0} = k_4, w_{2,0} = k_5, w_{3,0} = 1.0,\\
& \tfrac{1}{2}(w_2 b_3 - w_3 b_2) = \tfrac{1}{2}(k_5 k_3 - k_2) = k_6,\\
& \tfrac{1}{2}(w_3 b_1 - w_1 b_3) = \tfrac{1}{2}(k_1 - k_4 k_3) = k_7,\\
& \tfrac{1}{2}(w_1 b_2 - w_2 b_1) = \tfrac{1}{2}(k_4 k_2 - k_5 k_1) = k_8.
\end{aligned}\right\} \qquad (17)$$

Where $\boldsymbol{b}_0$ and $\boldsymbol{w}_0$ satisfy the conditions given in equation (3) with particular values and lie on invariant surface. Consider the first iteration,

$$\left.\begin{aligned}
w_{1,1} &= w_{1,0} - G\int_0^t b_{2,0}dt,\\
w_{2,1} &= w_{2,0} + G\int_0^t b_{1,0}dt,\\
w_{3,1} &= w_{3,0} = 1.0,\\
b_{1,1} &= b_{1,0} + \tfrac{1}{2}\int_0^t (w_{2,0}b_{3,0} - w_{3,0}b_{2,0})dt,\\
b_{2,1} &= b_{2,0} + \tfrac{1}{2}\int_0^t (w_{3,0}b_{1,0} - w_{1,0}b_{3,0})dt,\\
b_{3,1} &= b_{3,0} + \tfrac{1}{2}\int_0^t (w_{1,0}b_{2,0} - w_{2,0}b_{1,0})dt.
\end{aligned}\right\} \qquad (18)$$

Using the initial conditions given from (17) , we get the first iteration for the solution of the system (1) as

$$\left.\begin{aligned}
w_{1,1} &= k_4 - Gk_2 t,\\
w_{2,1} &= k_5 + Gk_1 t,\\
b_{1,1} &= k_1 + k_6 t,\\
b_{2,1} &= k_2 + k_7 t,\\
b_{3,1} &= k_3 + k_8 t.
\end{aligned}\right\} \qquad (19)$$

Again integrating (19) from $'0'$ to $'t'$, we get 2[nd] iteration to the system (1) as

$$\left.\begin{aligned}
w_{1,2} &= w_{1,1} - G\int_0^t b_{2,1}dt,\\
w_{2,2} &= w_{2,1} + G\int_0^t b_{1,1}dt,\\
w_{3,2} &= w_{3,1} = 1.0,\\
b_{1,2} &= b_{1,1} + \tfrac{1}{2}\int_0^t (w_{2,1}b_{3,1} - w_{3,1}b_{2,1})dt,\\
b_{2,2} &= b_{2,1} + \tfrac{1}{2}\int_0^t (w_{3,1}b_{1,1} - w_{1,1}b_{3,1})dt,\\
b_{3,2} &= b_{3,1} + \tfrac{1}{2}\int_0^t (w_{1,1}b_{2,1} - w_{2,1}b_{1,1})dt.
\end{aligned}\right\} \qquad (20)$$

Using the values from equation (17) we get

$$\left.\begin{aligned} w_{1,2} &= k_4 - Gk_2t - G\int_0^t b_{2,1}dt,\\ w_{2,2} &= k_5 + Gk_1t + G\int_0^t b_{1,1}dt,\\ b_{1,2} &= k_1 + k_6t + \tfrac{1}{2}\int_0^t (w_{2,1}b_{3,1} - w_{3,1}b_{2,1})dt,\\ b_{2,2} &= k_2 + k_7t + \tfrac{1}{2}\int_0^t (w_{3,1}b_{1,1} - w_{1,1}b_{3,1})dt,\\ b_{3,2} &= k_3 + k_8t + \tfrac{1}{2}\int_0^t (w_{1,1}b_{2,1} - w_{2,1}b_{1,1})dt. \end{aligned}\right\} \tag{21}$$

Applying the alternative algorithm [17–21] to compute general integrations by ADM , with the condition that $w_{3,n} = 1$ for all n, we get

$$\left.\begin{aligned} w_{1n+1} &= w_{1n} - \tfrac{g}{\rho_b}\int_0^t b_{2n}dt,\\ w_{2n+1} &= w_{2n} + \tfrac{g}{\rho_b}\int_0^t b_{1n}dt,\\ b_{1n+1} &= b_{1n} + \tfrac{1}{2}\int_0^t (w_{2n}b_{3n} - b_{2n})dt,\\ b_{2n+1} &= b_{2n} + \tfrac{1}{2}\int_0^t (b_{1n} - w_{1n}b_{3n})dt,\\ b_{3n+1} &= b_{3n} + \tfrac{1}{2}\int_0^t (w_{1n}b_{2n} - w_{2n}b_{1n})dt. \end{aligned}\right\} \tag{22}$$

Using these values in equation (16), we obtain the analytic solution in the form of series and the convergence is guaranteed by [22–24].

5 EXPERIMENTAL RESULTS

We have used MATHEMATICA 9 for calculating polynomials and solutions. After calculations, we have verified the results with exact solutions. The following graphs show results obtained by ADM with the initial conditions b_{10} =0.001, b_{20} = 0.0, b_{30} = 1.0, w_{10} = 0.0005, w_{20} = 0.00309, w_{30} = 1.00. For this initial conditions, we have k = 0.00000155 > 0 , hence $\boldsymbol{b}_0$ and $\boldsymbol{w}_0$ lie on the unstable manifold of invariant surface. With the above initial values, we have obtained the following general solutions on unstable manifold as

$$b_1 = 0.001 + 0.00309t + 0.00103749t^2 - 0.000386264t^3 - 0.0000864582t^4 + 6.43756\times10^{-6}t^5 - 2.7517\times10^{-10}t^6 - 1.77324\times10^{-10}t^7 - 9.11841\times10^{-11}t^8 - 1.7302\times10^{-11}t^9 - 1.48059\times10^{-12}t^{10} - 1.42617\times10^{-12}t^{11} - 6.24261\times10^{-13}t^{12} - 8.49761\times10^{-14}t^{13} - 9.16258\times10^{-19}t^{14} - 8.2046\times10^{-19}t^{15} - 2.52515\times10^{-19}t^{16} - 5.01867\times10^{-20}t^{17} - 4.63536\times10^{-21}t^{18} - 5.78547\times10^{-26}t^{19} - 2.54018\times10^{-26}t^{20} - 1.8486\times10^{-27}t^{21} + 3.79518\times10^{-32}t^{22} - 2.61256\times10^{-37}t^{23} + 6.02809\times10^{-43}t^{24} + \dots$$

$$b_2 = 0.0 + 5\times10^{-4}t + 0.00115875t^2 + 3.45829\times10^{-4}t^3 - 9.65659\times10^{-5}t^4 - 0.0000193749t^5 + 4.77509\times10^{-11}t^6 - 3.87138\times10^{-12}t^7 + 3.86758\times10^{-13}t^8 - 9.46058\times10^{-12}t^9 - 8.03788\times10^{-12}t^{10} - 3.07135\times10^{-12}t^{11} - 4.60284\times10^{-13}t^{12} - 8.96157\times10^{-19}t^{13} - 1.74133\times10^{-18}t^{14} - 1.85618\times10^{-19}t^{15} + 2.05084\times10^{-24}t^{16} - 5.25597\times10^{-26}t^{17} - 5.53518\times10^{-27}t^{18} + 3.68283\times10^{-32}t^{19} + \dots$$

$b_3 = 1. - 0.000016995t - 2.38726 \times 10^{-6}t^2 - 1.77675 \times 10^{-6}t^3 - 5.81188 \times 10^{-7}t^4 - 9.26956 \times 10^{-8}t^5 + 9.85997 \times 10^{-8}t^6 + 2.23779 \times 10^{-8}t^7 - 6.16238 \times 10^{-9}t^8 - 1.38577 \times 10^{-9}t^9 + 3.49743 \times 10^{-15}t^{10} - 5.93659 \times 10^{-15}t^{11} - 7.30831 \times 10^{-16}t^{12} + 1.54358 \times 10^{-17}t^{13} - 5.58945 \times 10^{-21}t^{14} - 2.06705 \times 10^{-21}t^{15} - 4.30585 \times 10^{-22}t^{16} - 3.78019 \times 10^{-23}t^{17} + 5.83633 \times 10^{-28}t^{18} - 2.27599 \times 10^{-33}t^{19} + \ldots$

$w_1 = 0.0005 + 0.t - 1.8375 \times 10^{-3}t^2 - 3.78526 \times 10^{-3}t^3 - 8.47282 \times 10^{-4}t^4 + 6.30942 \times 10^{-5}t^5 + 1.9495 \times 10^{-10}t^6 + 7.80998 \times 10^{-11}t^7 - 2.02055 \times 10^{-11}t^8 - 2.61528 \times 10^{-11}t^9 - 4.76506 \times 10^{-12}t^{10} + 3.72678 \times 10^{-17}t^{11} + \ldots$

$w_2 = 0.00309 + 0.0098t + 0.0113558st^2 + 0.00338913t^3 - 0.000946346t^4 - 0.000189875t^5 + 5.99331 \times 10^{-14}t^6 - 4.27014 \times 10^{-10}t^7 - 2.31585 \times 10^{-10}t^8 - 9.70443 \times 10^{-11}t^9 - 1.43407 \times 10^{-11}t^{10} + 2.07682 \times 10^{-16}t^{11} - 7.6088 \times 10^{-22}t^{12} + \ldots$

We have plotted the following graphs using MATHEMATICA 9.

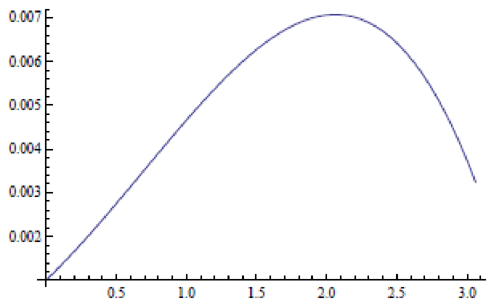

Figure 1: Graph for b_1 on unstable manifold.

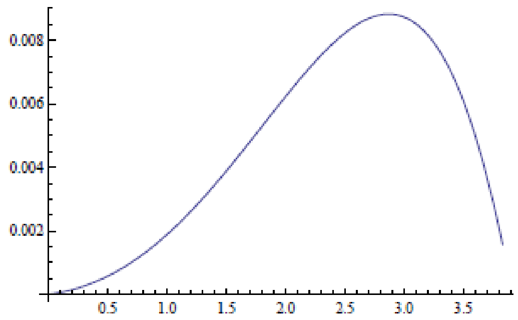

Figure 2: Graph for b_2 on unstable manifold.

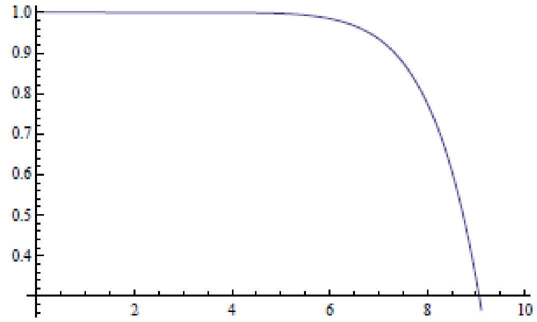

Figure 3: Graph for b_3 on unstable manifold.

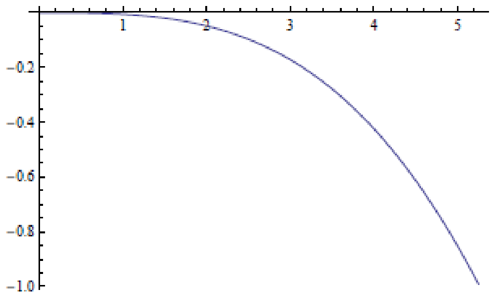

Figure 4: Graph for w_1 on unstable manifold.

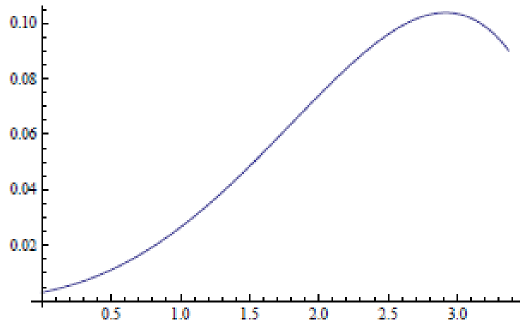

Figure 5: Graph for w_2 on unstable manifold.

5.1 Comparison of results obtained by ADM and Exact Solutions

Here we have the comparison of the values of **b**(b_1, b_2, b_3) and **w**(w_1,w_2,w_3) obtained by ADM and exact solutions. The error decreases after refinement of the intervals. ADM gives more accurate results if implemented to very small intervals.

Table 1: Values of b_1 and error before and after refinement.

b_1					
		ADM			
t	Exact	Before refinement	Error	After refinement	Error
0	0.001	0.001	0	0.001	0
0.0001	0.001	0.001000309	3.09×10^{-7}	0.001000309	3.09×10^{-7}
0.0002	0.001	0.001000618	6.18×10^{-7}	0.001000618	6.18×10^{-7}
0.0003	0.001	0.001000927	9.27×10^{-7}	0.001000927	9.27×10^{-7}
0.0004	0.001001	0.001001236	2.36×10^{-7}	0.001001237	2.37×10^{-7}
0.0005	0.001001	0.001001545	5.45×10^{-7}	0.001001546	5.46×10^{-7}
0.0006	0.001001	0.001001854	8.54×10^{-7}	0.001001856	8.55×10^{-7}
0.0007	0.001001	0.001002164	1.16×10^{-6}	0.001002165	1.16×10^{-6}
0.0008	0.001001	0.001002473	1.47×10^{-6}	0.001002474	1.47×10^{-6}
0.0009	0.001001	0.001002782	1.78×10^{-6}	0.001002784	1.78×10^{-6}

Abbreviations: ADM-Adomian Decomposition Method,

Table 2: Values of b_2 and error before and after refinement.

b_2					
		ADM			
t	Exact	Before refinement	Error	After refinement	Error
0	0	0	0	0	0
0.0001	0	5.00×10^{-8}	5.00×10^{-8}	-7.95×10^{-8}	7.95×10^{-8}
0.0002	0	1.00×10^{-7}	1.00×10^{-7}	-1.59×10^{-7}	1.59×10^{-7}
0.0003	0	1.50×10^{-7}	1.50×10^{-7}	-2.38×10^{-7}	2.38×10^{-7}
0.0004	0	2.00×10^{-7}	2.00×10^{-7}	-3.18×10^{-7}	3.18×10^{-7}
0.0005	0	2.50×10^{-7}	2.50×10^{-7}	-3.97×10^{-7}	3.97×10^{-7}
0.0006	0	3.00×10^{-7}	3.00×10^{-7}	-4.77×10^{-7}	4.77×10^{-7}
0.0007	0	3.50×10^{-7}	3.50×10^{-7}	-5.56×10^{-7}	5.56×10^{-7}
0.0008	0	4.00×10^{-7}	4.00×10^{-7}	-6.36×10^{-7}	6.36×10^{-7}
0.0009	0	4.50×10^{-7}	4.50×10^{-7}	-7.15×10^{-7}	7.15×10^{-7}

Table 3: Values of b_3 and error before and after refinement.

b3					
		ADM			
t	Exact	Before refinement	Error	After refinement	Error
0	1	1	0	1	0
0.0001	0.999999	0.999999998	9.98×10^{-7}	0.999999	3.09×10^{-10}
0.0002	0.999999	0.999999997	9.96×10^{-7}	0.999998899	6.18×10^{-10}
0.0003	0.999999	0.999999995	9.94×10^{-7}	0.999998899	9.27×10^{-10}
0.0004	0.999999	0.999999993	9.93×10^{-7}	0.999998899	1.23×10^{-9}
0.0005	0.999999	0.999999992	9.91×10^{-7}	0.999998898	1.54×10^{-9}
0.0006	0.999999	0.99999999	9.89×10^{-7}	0.999998898	1.85×10^{-9}
0.0007	0.999999	0.999999988	9.88×10^{-7}	0.999998898	2.16×10^{-9}
0.0008	0.999999	0.999999986	9.86×10^{-7}	0.999998898	2.47×10^{-9}
0.0009	0.999999	0.999999985	9.84×10^{-7}	0.999998897	2.78×10^{-9}

Table 4: Values of w_1 and error before and after refinement.

W1					
		ADM			
t	Exact	Before refinement	Error	After refinement	Error
0	0.0005	0.0005	0	0.0005	1.33×10^{-11}
0.0001	0.0005	0.0005	1.83×10^{-11}	0.0005	1.33×10^{-11}
0.0002	0.0005	0.0005	7.35×10^{-11}	0.0005	5.34×10^{-11}
0.0003	0.0005	0.0005	1.65×10^{-10}	0.0005	1.20×10^{-10}
0.0004	0.0005	0.0005	2.94×10^{-10}	0.0005	2.13×10^{-10}
0.0005	0.0005	0.0005	4.59×10^{-10}	0.0005	3.33×10^{-10}
0.0006	0.0005	0.000499999	6.62×10^{-10}	0.0005	4.80×10^{-10}
0.0007	0.0005	0.000499999	9.01×10^{-10}	0.000500001	6.54×10^{-10}
0.0008	0.0005	0.000499999	1.17×10^{-9}	0.000500001	8.54×10^{-10}
0.0009	0.0005	0.000499999	1.49×10^{-9}	0.000500001	1.08×10^{-9}

Table 5: Values of w_2 and error before and after refinement.

W2					
		ADM			
t	Exact	Before refinement	Error	After refinement	Error
0	0.00309	0.00309	0	0.00309	0
0.0001	0.003091	0.00309098	1.98×10^{-8}	0.00309098	1.98×10^{-8}
0.0002	0.003091	0.00309196	9.60×10^{-7}	0.00309196	9.60×10^{-7}
0.0003	0.003092	0.003092941	9.41×10^{-7}	0.003092941	9.41×10^{-7}
0.0004	0.003092	0.003093922	1.92×10^{-6}	0.003095922	3.92×10^{-6}
0.0005	0.003093	0.003094903	1.90×10^{-6}	0.003096903	3.90×10^{-6}
0.0006	0.003093	0.003095884	2.88×10^{-6}	0.003097884	4.88×10^{-6}
0.0007	0.003094	0.003096866	2.86×10^{-6}	0.003098866	4.86×10^{-6}
0.0008	0.003094	0.003097847	3.84×10^{-6}	0.003099847	5.84×10^{-6}
0.0009	0.003095	0.003098829	3.82×10^{-6}	0.003100829	5.82×10^{-6}

5.2 Comparison of results by ADM, EMM and RK4 methods

In this section we have compared the results obtained by ADM with the results obtained by Desale and Dasre [4, 5] using MEM and RK4 methods and exact solutions. The accuracy of the solution almost same. The Euler Modified Method and Runge-Kutta Fourth order methods give more accurate results than ADM on any intervals but after refinement of interval ADM also gives good results.

Table 6: Comparison for values of b_1 obtained by ADM, MEM, RK4 and Exact.

b_1				
t	ADM	MEM	RK4	Exact
0	0.001	0.001	0.001	0.001
0.001	0.001	0.001002	0.001002	0.001002
0.002	0.001001	0.001003	0.001003	0.001003
0.003	0.001001	0.001005	0.001005	0.001005
0.004	0.001001	0.001006	0.001006	0.001006
0.005	0.001002	0.001008	0.001008	0.001008
0.006	0.001002	0.001009	0.001009	0.001009
0.007	0.001002	0.001011	0.001011	0.001011
0.008	0.001002	0.001012	0.001012	0.001012
0.009	0.001003	0.001014	0.001014	0.001014

Abbreviations: ADM-Adomian Decomposition Method, MEM- Modified Euler Method,
RK4-Runge-Kutta Fourth Order Method.

Table 7: Comparison for values of b_2 obtained by ADM, MEM, RK4 and Exact.

b_2				
t	ADM	MEM	RK4	Exact
0	0	0	0	0
0.001	5.00×10^{-8}	0	0	0
0.002	1.00×10^{-7}	0.000001	0.000001	0.000001
0.003	1.5×10^{-7}	0.000001	0.000001	0.000001
0.004	2×10^{-7}	0.000001	0.000001	0.000001
0.005	2.5×10^{-7}	0.000001	0.000001	0.000001
0.006	3×10^{-7}	0.000002	0.000002	0.000002
0.007	3.51×10^{-7}	0.000002	0.000002	0.000002
0.008	4.01×10^{-7}	0.000002	0.000002	0.000002
0.009	4.51×10^{-7}	0.000002	0.000002	0.000002

Table 8: Comparison for values of b_3 obtained by ADM, MEM, RK4 and Exact.

b_3				
t	ADM	MEM	RK4	Exact
0	1	1	1	1
0.001	0.999999	0.999999	0.999999	0.999999
0.002	0.999999	0.999999	0.999999	0.999999
0.003	0.999999	0.999999	0.999999	0.999999
0.004	0.999999	0.999999	0.999999	0.999999
0.005	0.999999	0.999999	0.999999	0.999999
0.006	0.999999	0.999999	0.999999	0.999999
0.007	0.999999	0.999999	0.999999	0.999999
0.008	0.999999	0.999999	0.999999	0.999999
0.009	0.999999	0.999999	0.999999	0.999999

Table 9: Comparison for values of w_1 obtained by ADM, MEM, RK4 and Exact.

w_1				
t	ADM	MEM	RK4	Exact
0	0.0005	0.0005	0.0005	0.0005
0.001	0.0005	0.0005	0.0005	0.0005
0.002	0.0005	0.0005	0.0005	0.0005
0.003	0.0005	0.0005	0.0005	0.0005
0.004	0.0005	0.0005	0.0005	0.0005
0.005	0.0005	0.0005	0.0005	0.0005
0.006	0.0005	0.0005	0.0005	0.0005
0.007	0.0005	0.0005	0.0005	0.0005
0.008	0.0005	0.0005	0.0005	0.0005
0.009	0.0005	0.0005	0.0005	0.0005

Table 10: Comparison for values of w_2 obtained by ADM, MEM, RK4 and Exact.

w_2				
t	ADM	MEM	RK4	Exact
0	0.00309	0.00309	0.00309	0.00309
0.001	0.0030909	0.003095	0.003095	0.003095
0.002	0.0030919	0.0031	0.0031	0.0031
0.003	0.0030929	0.003105	0.003105	0.003105
0.004	0.0030939	0.00311	0.00311	0.00311
0.005	0.0030949	0.003115	0.003115	0.003115
0.006	0.0030958	0.00312	0.00312	0.00312
0.007	0.0030968	0.003125	0.003125	0.003125
0.008	0.0030978	0.00313	0.00313	0.00313
0.009	0.0030988	0.003135	0.003135	0.003135

6 CONCLUSION

Here we have presented the scheme of Adomian Decomposition Method for the numerical solution of the system of six coupled nonlinear ODEs (1). In our calculation initially we have the error of 10^{-6}. This method is very useful for numerical solutions on the small intervals but gives more error on large intervals. The error in all variables decreases after refinement except in b_2. The error in b_1 and b_3 decreases whereas the error in b_2 increases this is due to $|b|^2 = 1$. The convergence of this method is guaranteed and the error is bounded. The error can be made smaller by taking refinement of the interval. This method gives very good results if the large interval is refined into finite number of small intervals and ADM is applied on this small intervals. We have used alternative approach [17–21] to calculate the Adomian polynomials.

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