International Journal of Mathematics and Computer Research

ISSN: 2320-7167

Volume 12 Issue 11 November 2024, Page no. –4562-4565

Index Copernicus ICV: 57.55, Impact Factor: 8.316

DOI: 10.47191/ijmcr/v12i11.02

Transient Phenomena of Inviscid Accretion Gasradiation Slim Disc in A Gravitational Potential after Adiabatic Perturbations of the Velocities

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ARTICLE INFO	ABSTRACT
Published Online: 08 November 2024	The formalisms for the analysis of the dynamics of slim inviscid accretion discs in a (also, non-Newtonian) gravitational potential are here developped; the role of the horizontal pressure and that of the entropy gradients are to be characterised. The adiabatic perturbations of the velocity are studied: the paradigms of the Lagrangian perturbations of the velocity are implemented.
Corresponding Author: **Orchidea Maria Lecian Sapienza**	The perturbations of the slim disc is studied in the Lagrangean formalism of adiabatic perturbations of the velocities. The new conditions on the Eulerian variation of the gravitational potential are outlined. The equations of the transient phenomena are written.

KEYWORDS: Slim accretion disc; inviscid fluids; viscous fluids; Lagrangian adiabatic perturbations; Euler variations; gravitational potentials; transient phenomena.

1 INTRODUCTION

The slim accretion disc model [1] is investigated.

The slim disc is made of a mixture of gas and radiation, and it stays in an (also, non-Newtonian) gravitational potential. The dynamics of the disc is ruled after the gradient of the entropy.

The aim of the present paper is to analyse the known transient phenomena and to write the new ones.

From [2], the inviscid accretion discs are studied in a gravitational potential.

The case of ’close binary’ potential is also considered.

The analysis is further brought to viscous accretion discs.

The viscous fluid is apt to e studied according to the protocol of Lagrangean perturbations [5]. The known paradigms to outline the transient phenomena are demonstrated from [6]: the trnasient-growth regime and the decay are described.

The viscous accretion disc in the Kerr potential is depicted from [8].

The equations of motion of the inviscid perturbed fluid are taken from [9].

In the present paper, the mechanisms outlined in [9] are further implemented and developped. The case of the perturbed inviscid fluid in a gravitational potential is considered; the Lagrangean formulation of the adiabatic perturbations of the velocities is made use of. Within the framework of a General-Relativistic potential, the equations of motions are newly split according to the order of the Eulerian perturbations and the constraints are found; in particular, the perturbations are written on a gravitational potential Φ as well, as specified at the proper orders. The condition on the Eulerian components of the variations of $\delta\Phi$ are newly found.

The paper is organised as follows.

In Section 1, the main motivations of the study are indicated.

In Section 2, the slim accretion disc is reviewed.

In Section 3, the features of the inviscid accretion discs are recalled.

In Section 4, the properties of the viscous accretion discs are summarised. In Section 5, the the formalism of Lagrangean perturbations of the velocities in a viscous fluid is recapitulated.

In Section 6, some of the paradigms to outline transient phenomena in the known cases are revised.

In Section 7, the viscous accretion disc in the Kerr potential are described. In Section 8, the perturbed inviscid fluid is newly analysed. The Lagrangean perturbations are used. The equations of motions are split after the adiabatic perturbations of the velocities. The new conditions found on the Eulerian variation of the gravitational field are specified.

The prospective investigations and methodologies are exposed in Section 9.

The stress-energy tensor of the viscous slim disc is reported in Appendix A.

2 THE SLIM ACCRETION DISC

In [1], the slim accretion disc made of a mixture of perfect gas and radiation in a pseudo-Newtonian potential generated after the mass $\bar{M}$ is analysed. It is described within the General-Relativistic framework, endowed with a pseudoNewtonian potential $\Phi_A(r,z)$ as

$$\Phi_A \equiv -\frac{G\bar{M}}{R-R_G}, \quad (1)$$

being $R = (r^2 + z^2)^{1/2}$ and R_G a gravitational radius.

The The description is taken on the equatorial plane.

The angular velocity Ω_k of Keplerian, circular orbits in the pseudo-Newtonian potential is calculated as

$$\Omega_k = \left(\frac{GM}{R^3}\right)\left[1+\frac{R_G}{R}\right]^{-1}. \quad (2)$$

The self-gravity of the disc is neglected.

The viscosity of the disc is due only to shear viscosity, and the bulk viscosity is considered as negligible. Its dynamics is described after the entropy gradient

$$TdS = \frac{P}{\rho}\left[\left(12-\frac{21}{2}\beta\right)\frac{dT}{T} - (4-\beta)\frac{d\rho}{\rho}\right] \quad (3)$$

being T the temperature, S the entropy, with the pressure P satisfying an equation of state $f(P,T;\rho) = 0$.

The momentum equation in the r direction is written as

$$\frac{1}{\rho}\frac{dp}{d\rho} - (\Omega^2 - \Omega_k^2)r + v_r\frac{dv_r}{dr} = 0. \quad (4)$$

The momentum equation in the φ direction is calculated after the viscous torque w evaluated as an inner boundary condition as

$$\dot{M}\,(l - l_0) = w(r) - w(R_G) = 4\pi r^2 H\alpha P \quad (5)$$

being l_k the specific angular momentum of the radial component of the gravitational force.

The velocity of the sound in the medium is worked out after the derivative

$$\frac{d\ln v_r}{d\ln r}.$$

3 INVISCID ACCRETION DISCS

The dynamics of inviscid accertion discs is here recalled from [2].

The flow in a cold inviscid disc is supersonic.

The equations of motion with a point-mass M potential are written as

$$\frac{\partial}{\partial t}\vec{v} + \nabla\cdot\vec{v} = -\frac{1}{\rho}\nabla P - \frac{GM}{r^2}\hat{r} \quad (6)$$

At a fixed temperature T, the pressure gradient is written as

$$\frac{1}{\rho}\nabla P = \frac{\mathcal{R}}{M}T\nabla lnP \quad (7)$$

The orbital time scale Ω_0 is obtained as

$$\Omega_0^{-1} = (r_0^3/GM)^{1/2} \quad (8)$$

with velocity $r_0\Omega_0$.

3.1 Accretion in the potential of close binary

The case is here studied, of the accretion gas admitting a non-vanishing angular momentum with respect to the accreting object. The phenomenon here considered is accretion in a 'close binary' consisting of a compact object of mass M_1 and a 'main sequence companion' of mass M_2 (i.e. a white dwarf, a neutron star or a blackhole).

The frequency Ω is given as

$$\Omega^2 = G(M_1 + M_2)/a^3. \quad (9)$$

For a non-corotating gas, the Roche potential [4] Chapter 4 ibidem is here used as

$$\Phi_R = -Gm\left(\frac{1}{r_2}+\frac{1}{r_2}\right) - \frac{1}{2}\Omega^2\varsigma^2, \quad (10)$$

where r_i is the distance from the object i, and ς the distance from the rotation axis.

4 VISCOUS ACCRETION DISC

The viscous accretions discs are here revised after [2].

The surface density Σ is defined as

$$\Sigma = \int_{-\infty}^{+\infty}\rho dz \simeq 2H_0\rho_0, \quad (11)$$

being ρ_0 the 'density at the midplane', and H_0 the 'scaleheight at the midplane'. The conservation of mass is written as

$$\frac{\partial}{\partial t}(r\Sigma) + \frac{\partial}{\partial r}(r\Sigma\nu_r) = 0. \quad (12)$$

The equation of motion in the φ azimuthal component is reconducted to

$$\frac{\partial}{\partial t}\nu_\phi + \nu_r\frac{\partial}{\partial r}\nu_\phi + \frac{\nu_r\nu_\phi}{r} = F_\phi \quad (13)$$

being F_φ the azimuthal component of the viscous force.

$$\mathcal{S} = \int_{-\infty}^{+\infty}\rho\nu dz \simeq \Sigma\nu \quad (14)$$

Eq. (14) is consistent if v is independent of z.

The angular momentum balance is written from [3], [4] as

$$\frac{\partial}{\partial t}(r\Sigma\Omega r^2) + \frac{\partial}{\partial r}(r\Sigma\nu_r\Omega r^2) = \frac{\partial}{\partial r}(\mathcal{S}r^3\frac{\partial}{\partial r}\Omega); \quad (15)$$

The rhs of Eq. (15) is the divergence of the 'viscous angular momentum flux'. From Eq. (12) and from Eq. (15), the following equality is obtained

$$r\frac{d\Sigma}{dt} = 3\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial}{\partial r}(\nu\Sigma r^{1/2})\right]. \quad (16)$$

Eq. (16) represents the thin disc diffusion equation. Let $\dot{M}$ be the mass flux at any point of the disc, as

$$\dot{M} = -2\pi r\Sigma\nu_r = 6\pi r^{1/2}\frac{\partial}{\partial r}(\nu\Sigma r^{1/2}). \quad (17)$$

All the quantities which affect the behaviour of the disc are encoded in the viscosity.

5 Lagrangian perturbations of the velocities in viscous fluid

From [5], the passage from stable regime to turbulent regime is implemented from the density ρ, pressure P and the kinematic viscosity ν from the definition of the derivatives of the velocity vector ~v as

$$\frac{d}{dt}\vec{v} = -\frac{\nabla P}{\rho} + \nu\nabla^2\vec{v}, \qquad (18)$$

being

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \vec{v}\cdot\vec{v} \qquad (19)$$

with

$\nabla \cdot$~$v = 0$. (20)

5.1 The laminar flow

The laminar flow (plane Chouette flow) is discussed as follows. Perturbations of the viscid flow are written as

$$\vec{r}(R;t) = \sum_{n=0}^{n=\infty} \vec{b}^{(n)}(R;t)\delta^n \qquad (21)$$

being $b^{(n)}$ the n-th order term., and δ the parameter related with the initial velocity perturbation. The initial conditions are given as

~$b^{(0)}(R$~$;0) = R,$~ (22a)

$b^{(n)}(R$~$;0) = 0\ \forall n > 0.$ (22b)

6 THE TRANSIENT PHENOMENA

From [6], the viscous stress tensor σ_{jk} is defined for the 3-velocity ~v as

$$\sigma_{jk} = \eta\left[\frac{dv_j}{dx_k} + \frac{dv_k}{dx_j} - \frac{2}{3}\delta_{ik}\vec{\nabla}\cdot\vec{v}\right] \qquad (23)$$

5

For a potential Φ_U as

$$\Phi_U = \frac{GM}{r^2+z^2} \qquad (24)$$

The dynamics is investigated:

i) the density fluctuations and the vertical-velocity fluctuations do not induce radial motion and do not cause azimuthal motion;

ii) 'driven general acoustics' generalise initial perturbation radial structure; *iii*) the initial perturbations of the radial structure are found via *iiia*) the derivative of the velocity with respect tot he pressure v_p', *iiib*) the derivative of the density with respect to the pressure ρ_p', which give raise to a transient growing periods and a decay.

Transitions are present in the dynamics.

The acoustic energy consists in the kinetic energy in the vertical velocity disturbances.

The compression energy is due to the density disturbances.

Let $E_a(t;\alpha)$ be the total disturbance acoustic energy of the disc.

The ratio $E_a(T)/E_a(0)$ is used to analyze the transient growth.

The role of α is to decrease the magnitude of the transient growth. The time corresponding to the maximum amplitude is studied after the initial conditions on the radial velocity. The initial conditions on the radial velocity are posed as

$A(r) = ei\pi/4e{-}(r{-}r0)2/\Delta f$ (25)

being Δ_f the standard deviation of the Gaussian distribution of the velocities.

7 VISCOUS ACCRETION DISC IN THE KERR POTENTIAL

The viscous accretion disc in the Kerr potential[1] is here studied after [8].

7.1 Equation of state

The equation of state is given as a polytropic relation between the 'vertically integrated pressure' P and the surface (rest)-mass density Σ_N from [7] as

$$P = K\Sigma_0^{\Gamma} \qquad (26)$$

being $K \equiv K(s)$ the constant that takes into account the entropy of the flow and the polytropic index Γ is one of a two-dimensional flow as

$$N = \frac{1}{\Gamma - 1}. \qquad (27)$$

The adiabatic speed of sound a_{ad} is calculated as

$$c_{ad}^2 = \left(\frac{dP}{d\Sigma}\right)\frac{\Sigma_0}{\Sigma + P} \equiv \frac{\Gamma P}{\Sigma_0}\frac{1}{1+\frac{N\Gamma P}{\Sigma_0}}$$

; (28) it is rewritten as a function of the modified accretion rate M

$c\ ^2\ _{ad} = $ hN + M˙ $^{-1/N}$(r | u |)1/Ni (29)

being

$$\mathcal{M} \equiv \Gamma^N K^N \frac{1}{2\pi}\dot{M} \qquad (30)$$

with M the accretion mass.

For a fixed mass rate $\dot{M}$ and a fixed Γ, the modified mass rate M measures the entropy of the flow.

For a fixed K and a fixed Γ, the modified mass rate takes into account the accretion rate.

The specific enthalpy μ is written as a function of the adiabatic sound speed a_{ad} as

$\mu \equiv (1 - Na^2{}_{ad})^{-1}$. (31)

8 EQUATIONS OF MOTION OF A PERTURBED INVISCID FLUID

The equations of motion of a perturbed inviscid fluid is followed from [9]. The equations of motions of an inviscid fluid are written in the General-Relativistic form with implementing the gradients to their covariant expressions as[2]

[1] The Kerr potential is one obtained from the General-Relativistic spacetime of a rotating blackhole

[2] the Einstein notation of summation over saturated indices is here applied.

$\rho\partial_t^2\xi_\mu+2\rho v^\nu\nabla_\nu\partial_t\xi_\mu+\rho(v^\nu\nabla_\nu)^2\xi_\mu-\nabla_\mu(\gamma\rho\nabla_\nu\xi^\nu)+\nabla_\mu p\nabla_\nu\xi^\nu-\nabla_\nu p\nabla_\mu\xi^\nu+\rho\xi^\nu\nabla_\nu\nabla_\mu\Phi+\rho\nabla_\mu\delta\Phi=0$

32) The equations of motion of the adiabatic perturbations of the velocities are here specified as

$$\partial_t\xi^\mu=0. \quad (33)$$

The perturbations are taken

$$\Delta\vec{V}=0=\vec{0}+\vec{\delta_v}, \quad (34a)$$

$$\Delta\rho=\delta\rho-\nabla_\sigma(\rho\xi^\sigma), \quad (34b)$$

$$\Delta p=\delta p+\xi^\sigma\nabla_\sigma p. \quad (34c)$$

From Eq. (34a), Eq. (refeqy1) is specified for adiabatic perturbations of the velocities as

$\rho(v^\nu\nabla_\nu)^2\xi-\nabla_\mu(\gamma\rho\nabla_\nu\xi^\nu)+\nabla_\mu p\nabla_\nu\xi^\nu-\nabla_\nu p\nabla_\mu\xi^\nu+\rho\xi^\nu\nabla_\nu\nabla_\mu\Phi+\rho\nabla_\mu\delta\Phi=0.$

35) After implementing Eq. (34b) and Eq. (34c), Eq. (35) is written as containing the different orders as

$$(\delta\rho)(v^\nu\nabla_\nu)^2\xi-\nabla_\mu(\gamma(\delta\rho)\nabla_\nu\xi^\nu)+\nabla_\mu(\delta p)\nabla_\nu\xi^\nu-\nabla_\nu(\delta p)\nabla_\mu\xi^\nu+(\delta\rho)\xi^\nu\nabla_\nu\nabla_\mu\Phi+(\delta\rho)\nabla_\mu\delta\Phi=0 \quad (36)$$

and

$$[-\nabla_\sigma(\rho\xi^\sigma)](v^\nu\nabla_\nu)^2\xi-\nabla_\mu(\gamma[-\nabla_\sigma(\rho\xi^\sigma)]\nabla_\nu\xi^\nu)+\nabla_\mu[\xi^\sigma\nabla_\sigma p]\nabla_\nu\xi^\nu-\nabla_\nu[\xi^\sigma\nabla_\sigma p]\nabla_\mu\xi^\nu+[-\nabla_\sigma(\rho\xi^\sigma)]\xi^\nu\nabla_\nu\nabla_\mu\Phi+[-\nabla_\sigma(\rho\xi^\sigma)]\nabla_\mu\delta\Phi=0.$$

(37)

After considering the different orders in Eq. (36), the different equations are obtained

$$(\delta\rho)(v^\nu\nabla_\nu)^2\xi-\nabla_\mu(\gamma(\delta\rho)\nabla_\nu\xi^\nu)+\nabla_\mu(\delta p)\nabla_\nu\xi^\nu-\nabla_\nu(\delta p)\nabla_\mu\xi^\nu+(\delta\rho)\xi^\nu\nabla_\nu\nabla_\mu\Phi+(\delta\rho)\nabla_\mu\delta\Phi=0$$

(38)

and

$$(\delta\rho)(v^\nu\nabla_\nu)^2\xi\simeq0 \quad (39)$$

and

$$(\delta\rho)\nabla_\mu\delta\Phi\simeq0. \quad (40)$$

In particular, the new (non-Newtonian) condition Eq (40) is found for any (also, non-Newtonian) potentials.

9 DISCUSSION

The time behaviours of thin polytropic accretion discs under particular axisymmetric perturbations are discussed in [10]. More in detail, both time independent perturbations and time-dependent perturbations are considered. Numerical von Neumann methods are recapitulated in [11].

A. The stress-energy tensor of the viscous slim disc

The stress-energy tensor for a single-component fluid including viscosity and heat flux is specified after the entropy flux S, u^μ the position 4-vector of the local rest frame, v^μ the velocity 4-vector and the viscous tensor $\kappa^{\mu\nu}$. The viscous tensor $\tau^{\mu\nu}$ is defined as

$$\kappa^{\mu\nu}=-\zeta\Theta H^\mu\nu-2\eta\pi^{\mu\nu} \quad (41)$$

being $\sigma\mu\nu$ the shear tensor, η the shear viscosity coefficient, ζ the bulk viscosity coefficient, and $\Theta^{\mu\nu}=\nabla^\nu u_\nu$. The tensor $H^{\mu\nu}$ is defined as

$$H\mu\nu=u\mu u\nu+g\mu\nu \quad (42)$$

and represents the ’expansion of the fluid worldlines’. The stress-energy tensor $T^{\mu\nu}{}_d$ of the disc is therefore written as

$$T^{\mu\nu}{}_d=(\rho+p)u^\mu u^\nu+pg^{\mu\nu}+\kappa^{\mu\nu}+q^\mu u^\nu+u^\mu q^\nu \quad (43)$$

It is specified after the choice of the gravitational potential calculated after the metric tensor $g^{\mu\nu}$.

In the analysis of [8], it is specified after the metric tensor of the Kerr blackhole spacetime.

CONFLICTS OF INTEREST

The Author declares no conflicts of interest.

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