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Numerical Simulation of Infectious Disease Spread Using the SIR Model: an Application of Euler's Method

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ABSTRACT

The spread of infectious diseases is a very complex phenomenon and requires mathematical modeling to understand the dynamics of the spread. One of the common approaches used to model the spread of diseases is the SIR (Susceptible-Infected-Recovered) model. This study aims to implement numerical methods, especially the Euler method, to simulate the spread of infectious diseases using the SIR model. This study focuses on the application of Euler's Method to solve differential equations that describe the changes in infected, susceptible, and recovered populations in a finite system. The Euler method is used to calculate the numerical solution of the system with a small time step. The simulation results show how the spread of the disease can be predicted in various scenarios, with sensitivity analysis to model parameters such as transmission rate and recovery rate. These simulations provide insight into the dynamics of the disease and help in designing more effective public health policies. In conclusion, the Euler's method has proven to be a useful tool for modeling and predicting the spread of diseases, although the accuracy of the results is highly dependent on the choice of time step. Further research can examine the application of other numerical methods and their comparison with analytical models to improve prediction and accuracy in real applications.

KEYWORDS: numerical simulation, SIR model, Euler's method, spread of disease , system of differential equation

I. INTRODUCTION

The spread of infectious diseases, especially in the form of epidemics or pandemics, has become a global issue that has attracted attention from various disciplines, including epidemiology and mathematics (Takács et al., 2020). The Susceptible-Infected-Recovered (SIR) model is one of the important mathematical models in predicting the dynamics of the spread of infectious diseases (Mungkasi, 2021). The SIR model divides the population into three main groups—susceptible, infected, and recovered—to understand the development of the disease over time. According to Kermack and McKendrick, who first introduced this model, the SIR approach can provide important insights into the rate of infection and the chances of recovery in a population (Aziean Mohd Idris et al., 2022; Barro et al., 2018; Khoa et al., 2023). Through numerical simulations, the SIR model serves as an important reference for health authorities in planning effective interventions to control the spread of diseases.

In practice, analytical solutions to the SIR model are often difficult to obtain, especially when faced with complex scenarios that require a dynamic variable approach. Therefore, numerical methods such as the Euler's Method are needed to solve the differential equations that describe population changes in each category of the SIR model. The Euler's method offers a simple and computationally efficient approach in generating approximate solutions to the differential equations that form this model (Pratiwi & Mungkasi, 2021; Triatmodjo, 2002; Yau & Abdullahi Yau, 2011). Based on previous studies, the Euler's method has been applied in several epidemiological simulations to produce predictions of disease progression. This study focuses on the effectiveness of the Euler's method in modeling the spread of infectious diseases based on the SIR model, especially in obtaining accurate and efficient simulation results.

A number of previous studies have explored various numerical methods for simulating epidemiological models. For example, the Runge-Kutta method is widely used because of its high level of accuracy, but this method requires more computation than the Euler's method (Ian Setiawan et al., 2021; Iskandar & Chee Tiong, 2022; Kosasih, 2006; Pudjaprasetya, 2018; Side et al., 2018a). Although the Euler's method is known to be simple and more computationally efficient, studies on the effectiveness of this method in the context of the SIR model are still limited. Several studies have used the Euler’s method for epidemiological simulations, but without an in-depth evaluation of performance or direct comparison with more complex methods such as Runge-Kutta (Ghoreishi et al., 2023; Side et al., 2018b; Suryaningrat et al., 2020). Therefore, this study focuses on the evaluation of the Euler’s method in modeling the dynamics of the spread of infectious diseases, especially in the context of SIR.

The lack of systematic review of the performance of the Euler’s Method in applications to SIR models creates a significant research gap. The Euler’s Method may be considered simple compared to sophisticated methods such as Runge-Kutta, but this study aims to fill this gap. A comprehensive evaluation of the Euler’s Method in SIR simulations is needed to obtain an overview of its accuracy and effectiveness in disease spread scenarios (Rudhito & Putra, 2021). Thus, this study attempts to present a systematic evaluation of the effectiveness of the Euler’s Method, which will provide important insights into the usefulness of this method in simple yet reliable epidemiological simulations.

Based on this background, the main objective of this study is to conduct a numerical simulation of the spread of infectious diseases based on the SIR model using the Euler’s Method. This study also aims to evaluate the effectiveness of this method in describing the development of infectious diseases and understand its limitations when compared to more complex numerical methods. It is hoped that this simulation can provide an alternative, more efficient method for the development of epidemiological prediction models, especially in applications that require fast simulations with limited computing resources.

This study contributes by presenting a simple simulation approach that is easy to apply in various epidemiological scenarios. By using the Euler’s Method, this study attempts to offer a choice of methods that are computationally efficient and fairly accurate in describing the spread of disease. It is expected that the results of this study can provide benefits for both researchers and practitioners in the field of public health who need fast, simple, and informative solutions to understand and anticipate the spread of disease in a short time (Dayan et al., 2022).

To provide an overview of the discussion flow, this article is divided into several parts. The first part is the Introduction, which includes the background, problem identification, and objectives and contributions of the study. The second part discusses the research method, including the SIR model and the application of the Euler’s Method in numerical simulations. The third part presents the simulation results along with an analysis of the effectiveness of the Euler’s Method in modeling the disease spread. The last section is a conclusion that summarizes the research results and recommendations for further research that may be needed. With this structure, this article is expected to provide a comprehensive overview of the role of the Euler’s Method in numerical simulation of disease spread based on the SIR model.

II. MATERIALS AND METHODS

A. The SIR Model

The SIR model is a compartment-based epidemiological model that describes the dynamics of the spread of an infectious disease in a population. In this model, the population is divided into three main compartments: Susceptible (S), which represents susceptible or uninfected individuals; Infected (I), which represents individuals who are currently infected and can transmit the disease; and Recovered (R), which represents individuals who have recovered and are considered immune. This model helps analyze and predict the course of an epidemic by simulating the transition between these compartments over time.

The SIR model represents the change in the number of individuals in each compartment over time through the following system of differential equations:

$$\frac{dS}{dt} = -\beta SI$$
$$\frac{dI}{dt} = \beta SI - \gamma I$$
$$\frac{dR}{dt} = \gamma I$$

Where β is the transmission rate, which describes how often susceptible individuals become infected when in contact with infected individuals and γ is the recovery rate, which indicates how quickly infected individuals move to the recovery compartment.

The parameters β and γ can be adjusted according to the type of disease or based on historical data to obtain more accurate distribution characteristics. This model also assumes that the total population size N is constant, so that $S(t)+I(t)+R(t)=N$. This assumption helps simplify the model and maintain balance in the dynamics of the population being studied.

B. Euler’s Method

Euler's method is one of the basic numerical methods used to solve differential equations, including the SIR (Susceptible-Infected-Recovered) model in epidemiology. This method is a simple approach based on Taylor series to calculate the solution of differential equations iteratively.

Suppose we have a first-order differential equation:

$$\frac{dy}{dt} = f(t.y)$$

with initial condition $y(t_0)=y_0$. To calculate the value of y at the next point, $y(t+\Delta t)$, we can use the Taylor series approximation around point t:

$$y(t+\Delta t) = y(t) + \Delta t \cdot \frac{dy}{dt} + \frac{(\Delta t)^2}{2!} \cdot \frac{d^2y}{dt^2} + \frac{(\Delta t)^3}{3!} \cdot \frac{d^3y}{dt^3} + \cdots$$

However, since calculating higher derivatives in this series is complex, Euler's method simplifies it by retaining only the first term after the initial value, which is the linear term of the Taylor series. This yields a first-order Taylor approximation:

$$y(t + \Delta t) \approx y(t) + \Delta t \cdot f(t.y)$$

Recursively, for a discrete point t_n, the value of y at the next step, namely y_{n+1}, can be calculated from the previous value y_n with the equation:

$$y_{n+1} = y_n + \Delta t \cdot f(t_n.y_n)$$

This is the basic formula of Euler's method, which provides a numerical approach to the solution of differential equations by calculating the value of y at each discrete point in time t based on its previous values.

The iterative equation of Euler's method for the SIR model can be written as follows:

$$S_{n+1} = S_n + \Delta t \cdot (-\beta \cdot S_n \cdot I_n)$$
$$I_{n+1} = I_n + \Delta t \cdot (\beta \cdot S_n \cdot I_n - \gamma \cdot I_n)$$
$$R_{n+1} = R_n + \Delta t \cdot (\gamma \cdot I_n)$$

The iteration process begins with the initial conditions S_0, I_0 and R_0, which are the initial number of susceptible, infected, and recovered individuals. Based on these conditions, iteration is carried out from time t = 0 to t = T, where T is the desired end time for the simulation.

III. RESULTS AND DISCUSSION

The case simulated in this study is the spread of influenza in a community with a population of 10,000 people, starting from an initial condition with 9,990 susceptible individuals, 10 infected individuals, and no individuals have recovered. The epidemic parameters used in this simulation include a baseline transmission rate (β) of 0.3, a recovery rate (γ) of 0.1, a simulation period of 60 days, and a time step (Δt) of 1 day. These parameters reflect the general situation often encountered in infectious disease outbreaks with moderate spread without specific interventions.

To understand the effect of parameter changes on the dynamics of disease spread, four simulation scenarios were conducted. Scenario 1 uses basic parameters as the baseline condition to describe the pattern of disease spread without environmental changes or interventions. Scenario 2 simulates an increased transmission rate (β = 0.5), which represents a situation with higher social interaction, such as in large crowds or environments with relaxed health protocol policies. Scenario 3 explores the effect of accelerating treatment through increasing the recovery rate (γ = 0.2), so that the disease can be controlled more quickly. Finally, Scenario 4 combines high β (β = 0.5) and high γ (γ = 0.2) to understand how the combination of increased transmission and recovery affects disease dynamics.

The simulation results of each scenario are visualized in a graph showing the changes in the number of susceptible (S), infected (I), and recovered (R) individuals during the simulation period. Comparisons between scenarios provide insight into the impact of changes in epidemic parameters on disease spread. With the simple yet effective Euler method, this analysis can provide an initial overview for public health policy making in controlling infectious diseases.

A. *Scenario 1: Baseline*

In scenario 1, the simulation was conducted with basic parameters to model the spread of influenza in a community with a population of 10,000 people for 60 days. The parameters used were the transmission rate (β=0.3), the recovery rate (γ=0.1), and the time step (Δt=1 day). The initial population condition consisted of 9,990 susceptible individuals (S(0)), 10 infected individuals (I(0)), and no recovered individuals (R(0)=0). The results of this simulation are shown in Figure 1, which illustrates the dynamics of changes in the number of susceptible individuals (S), infected individuals (I), and recovered individuals (R) over time.

The simulation results in Figure 1 show that the number of susceptible individuals decreases exponentially over the simulation period, reflecting the widespread spread of the disease. The curve of infected individuals (I) increases sharply at the beginning, reaching a peak around day 20, when more than half of the population is simultaneously infected. After reaching the peak, the number of infected individuals begins to decline as many recover, as seen from the sharp increase in the curve of recovered individuals (R). By the end of the simulation, almost the entire population has recovered or been exposed, indicating that the outbreak is over.

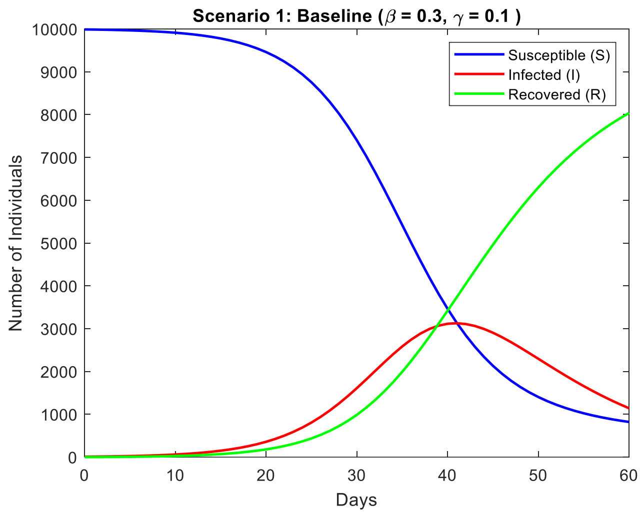

Figure 1 Scenario 1: Baseline

B. *Scenario 2: High Transmission*

In Scenario 2, the simulation explores the impact of increasing the transmission rate (β=0.5) while holding the recovery rate (γ=0.1) constant. This scenario reflects a situation where the disease spreads more aggressively due to high levels of interaction or lack of preventive measures. The simulation runs for 60 days, with an initial population of 10,000 individuals consisting of 9,990 susceptible individuals (S(0)), 10 infected individuals (I(0)), and no individuals who have recovered (R(0)=0).

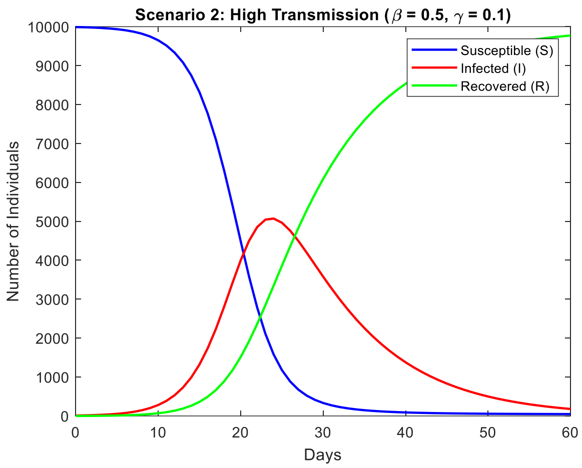

Figure 2 Scenario 2: High Transmission

The resulting dynamics, as seen in Figure 2, show a significant increase in the speed and intensity of the outbreak. The number of susceptible individuals (S) decreases much more rapidly compared to Scenario 1, with a sharp decline occurring during the first 10–15 days. This rapid decline indicates accelerated disease spread throughout the community.

The population of infected individuals (I) grows explosively, peaking around day 15, much earlier than in Scenario 1. At this peak, more than two-thirds of the population is simultaneously infected, reflecting the very high rate of disease transmission under these conditions. This earlier and higher peak suggests that the health system will face extreme stress in the short term. Meanwhile, the recovery curve (R) rises sharply after the peak of infections, as individuals begin to recover in large numbers. By the end of the simulation, most of the population has moved into the recovered category, leaving only a few individuals still susceptible.

C. Scenario 3: High Recovery

In Scenario 3, the simulation is conducted by increasing the recovery rate (γ=0.2) while the transmission rate (β=0.3) remains at its baseline value. This simulation aims to illustrate the impact of accelerating recovery, for example through more effective treatment or rapid medical intervention, in a population of 10,000 individuals. The initial conditions consist of 9,990 susceptible individuals (S(0)), 10 infected individuals (I(0)), and no recovered individuals (R(0)=0). The simulation lasts for 60 days with a time step (Δt=1).

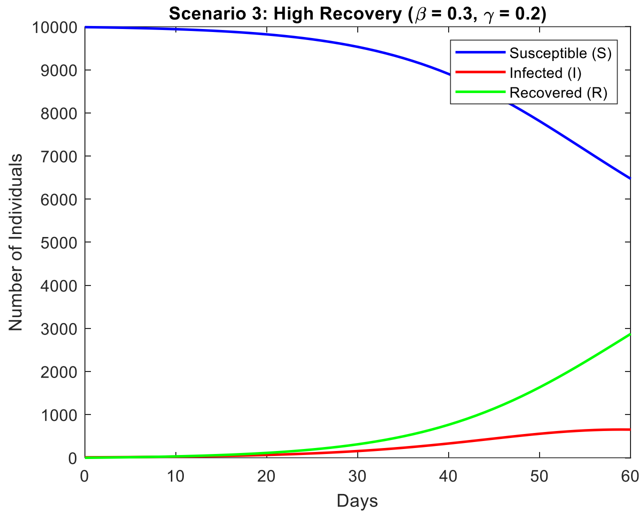

Figure 3 Scenario 3: High Recovery

The simulation results, visualized in Figure 3, show that increasing the recovery rate significantly changes the dynamics of disease spread. The number of susceptible individuals (S) still decreases, but this decrease occurs more slowly compared to the previous scenario. This suggests that accelerating recovery helps reduce the number of infected individuals before they can transmit the disease further.

D. Scenario 4

In Scenario 4, the simulation is performed by combining a high transmission rate (β=0.5) and a high recovery rate (γ=0.2). This combination of parameters represents a situation where the disease spreads faster in the community, but infected individuals also recover at a higher rate. The initial population consists of 9,990 susceptible individuals (S(0)), 10 infected individuals (I(0)), and no recovered individuals (R(0)=0). The simulation is performed for 60 days with a time step (Δt=1).

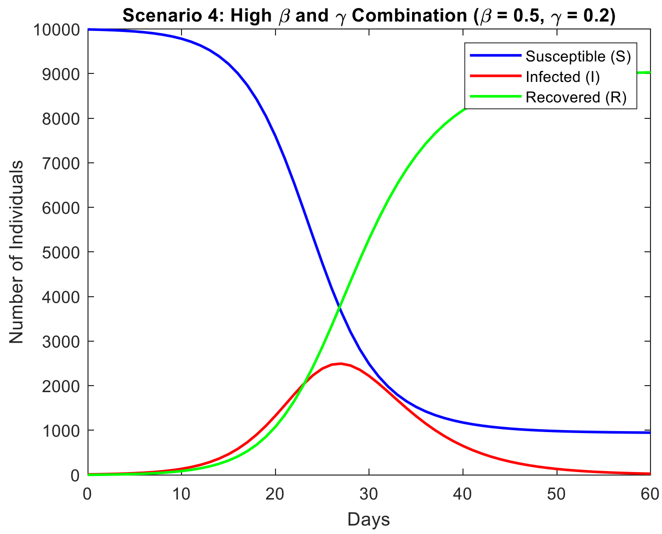

Figure 4

The simulation results graph in Figure 4 shows that the high transmission rate causes the number of susceptible individuals (S) to decrease significantly in a relatively short time. However, the increase in the recovery rate (γ=0.2) mitigates the impact of high transmission. The number of infected individuals (I) increases rapidly at the beginning, reaching a peak around day 12. The peak of infection occurs earlier compared to the previous scenario, but with a lower number of infected individuals compared to Scenario 2 (high transmission only).

IV. CONCLUSSION

Based on numerical simulations of disease spread using the SIR model with the Euler method, the results show that variations in transmission rate (β) and recovery rate (γ) have a significant effect on the dynamics of the outbreak. In the baseline scenario (β=0.3,γ=0.1), the disease spreads moderately with the peak of infection occurring on day 20. Increasing the transmission rate (β=0.5) accelerates the spread of the outbreak, with a higher peak of infection occurring earlier (day 12), reflecting the impact of increased social interaction or lack of preventive measures. In contrast, increasing the recovery rate (γ=0.2) controls the outbreak by reducing the peak of infection and shortening the duration of the epidemic. The combination of high transmission and recovery rates (β=0.5,γ=0.2) results in a faster but less severe outbreak compared to the high transmission scenario alone, indicating the mitigating effect of effective treatment or rapid recovery. This simulation emphasizes the importance of controlling the transmission rate and increasing the recovery rate to effectively manage infectious disease outbreaks.

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