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Temperature Elliptic Sombor and Modified Temperature Elliptic Sombor Indices

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ARTICLE INFO	ABSTRACT
Published Online: 08 March 2025 Corresponding Author: **V.R.Kulli**	In this paper, we introduce the temperature elliptic Sombor index, modified temperature elliptic Sombor index and their corresponding exponentials of a graph. Also we compute these temperature indices for some standard graphs and HC_5C_7 [p, q] nanotubes. Furthermore, we establish some properties of newly defined temperature elliptic Sombor index.

KEYWORDS: temperature elliptic Somber index, modified temperature elliptic Sombor index, graph, nanotubes.

I. INTRODUCTION

In this paper, we consider only finite, simple, connected graphs. Let G be such a graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. For basic notations and terminologies, we refer [1].

In [2], Fajtlowicz defined the temperature of a vertex u of a graph G as

$$T(u) = \frac{d_G(u)}{n - d_G(u)}, \qquad \text{where } |V(G)| = n.$$

The first temperature index of a graph was introduced by Kisori et al in [3] and it is defined as

$$T_1(G) = \sum_{uv \in E(G)} [T(u) + T(v)].$$

The second temperature index of a graph was introduced by Kulli in [4] and it is defined as

$$T_2(G) = \sum_{uv \in E(G)} T(u)T(v).$$

The first hyper temperature index of a graph was introduced by Kulli in [5] and it is defined as

$$HT_1(G) = \sum_{uv \in E(G)} [T(u) + T(v)]^2.$$

The F-temperature index of a graph was introduced by Kulli in [5] and it is defined as

$$FT(G) = \sum_{uv \in E(G)} \left[T(u)^2 + T(v)^2\right].$$

Recently, some temperature indices were studied in [6, 7, 8, 9, 10, 11, 12, 13, 14].

The elliptic Sombor index was introduced by Gutman et al. in [15] and it is defined as

$$ESO(G) = \sum_{uv \in E(G)} \left(d_G(u) + d_G(v)\right)\sqrt{d_G(u)^2 + d_G(v)^2}$$

Ref. [15] was soon followed by a series of publications [16, 17, 18, 19, 20, 21].

We define the temperature elliptic Sombor index of a graph as

$$TESO(G) = \sum_{uv \in E(G)} \left(T(u) + T(v)\right)\sqrt{T(u)^2 + T(v)^2}.$$

Considering temperature elliptic Sombor index of a graph, we define temperature elliptic Sombor exponential of a graph as

$$TESO(G, x) = \sum_{uv \in E(G)} x^{(T(u)+T(v))\sqrt{T(u)^2+T(v)^2}}$$

Also we introduce the modified temperature elliptic Sombor index of a graph as

$$^{m}TESO(G) = \sum_{uv \in E(G)} \frac{1}{\left(T(u) + T(v)\right)\sqrt{T(u)^2 + T(v)^2}}.$$

Considering modified temperature elliptic Sombor index of a graph, we define modified temperature elliptic Sombor exponential of a graph as

$$^{m}TESO(G,x)=\sum_{uv\in E(G)} x^{\frac{1}{(T(u)+T(v))\sqrt{T(u)^2+T(v)^2}}}.$$

. In this paper, the temperature elliptic Sombor index and modified temperature elliptic Sombor index for some standard graphs and HC_5C_7 [p, q] nanotubes are determined. Also some properties of newly defined temperature elliptic Sombor index are established.

II. RESULTS FOR SOME STANDARD GRAPHS

Proposition 1. Let G be r-regular with n vertices and $r\geq 2$. Then

$$TESO(G)=\frac{\sqrt{2}nr^3}{(n-r)^2}.$$

Proof: Let G be an r-regular graph with n vertices and $r\geq 2$ and $\frac{nr}{2}$ edges. Then $T(u)=\frac{r}{n-r}$

$$TESO(G)=\sum_{uv\in E(G)}(T(u)+T(v))\sqrt{T(u)^2+T(v)^2}$$

$$=\frac{nr}{2}\left(\frac{r}{n-r}+\frac{r}{n-r}\right)\sqrt{\left(\frac{r}{n-r}\right)^2+\left(\frac{r}{n-r}\right)^2}$$

$$=\frac{\sqrt{2}nr^3}{(n-r)^2}.$$

Corollary 1.1. Let C_n be a cycle with $n\geq 3$ vertices. Then

$$TESO(C_n)=\frac{8\sqrt{2}n}{(n-2)^2}.$$

Corollary 1.2. Let K_n be a complete graph with $n\geq 3$ vertices. Then

$$TESO(K_n)=\sqrt{2}n(n-1)^3.$$

Proposition 2. Let G be r-regular with n vertices and $r\geq 2$. Then

$$^{m}TESO(G)=\frac{n(n-r)^2}{4\sqrt{2}r}.$$

Proof: Let G be an r-regular graph with n vertices and $r\geq 2$ and $\frac{nr}{2}$ edges. Then $T(u)=\frac{r}{n-r}$

$$^{m}TESO(G)=\frac{nr}{2}\frac{1}{\left(\frac{r}{n-r}+\frac{r}{n-r}\right)\sqrt{\left(\frac{r}{n-r}\right)^2+\left(\frac{r}{n-r}\right)^2}}$$

$$=\frac{n(n-r)^2}{4\sqrt{2}r}.$$

Corollary 2.1. Let C_n be a cycle with $n\geq 3$ vertices. Then

$$^{m}TESO(C_n)=\frac{n(n-2)^2}{8\sqrt{2}}.$$

Corollary 2.2. Let K_n be a complete graph with $n\geq 3$ vertices. Then

$$^{m}TESO(K_n)=\frac{n}{4\sqrt{2}(n-1)}.$$

III. PROPERTIES OF TEMPERATURE ELLIPTIC SOMBOR INDEX

Theorem 1. Let G be a connected graph. Then

$$\frac{1}{\sqrt{2}}HT_1(G)\leq TESO(G)<HT_1(G).$$

Proof: For any two positive numbers a and b,

$$\frac{1}{\sqrt{2}}(a+b)\leq\sqrt{a^2+b^2}<a+b.$$

For $a=T(u)$ and $b=T(v)$, the above inequality becomes

$$\frac{1}{\sqrt{2}}(T(u)+T(v))\leq\sqrt{\left(T(u)^2+T(v)^2\right)}<(T(u)+T(v)).$$

$$\frac{1}{\sqrt{2}}(T(u)+T(v))^2\leq(T(u)+T(v))\sqrt{\left(T(u)^2+T(v)^2\right)}$$

$$<(T(u)+T(v))^2$$

By the definitions, we have

$$\frac{1}{\sqrt{2}}\sum_{uv\in E(G)}(T(u)+T(v))^2$$

$$\leq\sum_{uv\in E(G)}(T(u)+T(v))\sqrt{T(u)^2+T(v)^2}$$

$$<\sum_{uv\in E(G)}(T(u)+T(v))^2$$

Thus we get the desired result.

Theorem 2. Let G be a connected graph. Then

$$\frac{1}{\sqrt{2}}\left(FT(G)+2T_2(G)\right)\leq TESO(G)$$

$$<FT(G)+2T_2(G).$$

Proof: From Theorem 1, we have

$$\frac{1}{\sqrt{2}}\sum_{uv\in E(G)}(T(u)+T(v))^2$$

$$\leq\sum_{uv\in E(G)}(T(u)+T(v))\sqrt{T(u)^2+T(v)^2}$$

$$< \sum_{uv\in E(G)} \left(T(u)+T(v)\right)^2$$

Thus

$$\frac{1}{\sqrt{2}} \sum_{uv\in E(G)} \left(T(u)^2+T(v)^2+2T(u)T(v)\right)$$

$$\leq \sum_{uv\in E(G)} \left(T(u)+T(v)\right)\sqrt{T(u)^2+T(v)^2}$$

$$< \sum_{uv\in E(G)} \left(T(u)^2+T(v)^2+2T(u)T(v)\right)$$

Thus we get the desired result.

Theorem 3. Let G be a connected graph. Then

$$FT(G) < TESO(G) \leq \sqrt{2}FT(G).$$

Equality holds if and only if G is regular.

Proof: For any two positive numbers a and b,

$$\frac{1}{\sqrt{2}}(a+b) \leq \sqrt{a^2+b^2} < a+b.$$

$$\sqrt{a^2+b^2} < a+b \leq \sqrt{2}\sqrt{a^2+b^2}$$

$$(a^2+b^2) < (a+b)\sqrt{a^2+b^2} \leq \sqrt{2}(a^2+b^2)$$

Using the above inequality and the definition of TESO, we obtain

$$\sum_{uv\in E(G)} \left(T(u)^2+T(v)^2\right)$$

$$< \sum_{uv\in E(G)} \left(T(u)+T(v)\right)\sqrt{T(u)^2+T(v)^2}$$

$$\leq \sqrt{2} \sum_{uv\in E(G)} \left(T(u)^2+T(v)^2\right)$$

Thus we get the desired result.

Theorem 4. Let G be a connected graph with m edges. Then

$$TESO(G) \leq \sqrt{HT_1(G)FT(G)}.$$

Proof: Using the Cauchy-Schwarz inequality, we obtain

$$TESO(G) = \sum_{uv\in E(G)} \left(T(u)+T(v)\right)\sqrt{T(u)^2+T(v)^2}$$

$$\leq \sqrt{\sum_{uv\in E(G)} \left(T(u)+T(v)\right)^2 \sum_{uv\in E(G)} \left[\sqrt{T(u)^2+T(v)^2}\right]^2}$$

$$= \sqrt{HT_1(G)FT(G)}.$$

Thus $\qquad TESO(G) \leq \sqrt{HT_1(G)FT(G)}.$

Theorem 5. Let G be a connected graph with m edges. Then

$$TESO(G) \leq \sqrt{\left(FT(G)+2T_2(G)\right)FT(G)}.$$

Proof: From Theorem 4, we have

$$TESO(G)$$

$$\leq \sqrt{\sum_{uv\in E(G)} \left(T(u)+T(v)\right)^2 \sum_{uv\in E(G)} \left[\sqrt{T(u)^2+T(v)^2}\right]^2}$$

We have

$$\sum_{uv\in E(G)} \left(T(u)+T(v)\right)^2$$

$$= \sum_{uv\in E(G)} \left(T(u)^2+T(v)^2+2T(u)T(v)\right)$$

$$= FT(G)+2T_2(G)$$

Thus we conclude that

$$TESO(G) \leq \sqrt{\left(FT(G)+2T_2(G)\right)FT(G)}.$$

IV. RESULTS FOR HC_5C_7 [p, q] NANOTUBES

In this section, we consider $HC_5C_7[p, q]$ nanotubes in which p is the number of heptagons in the first row and q rows of pentagons repeated alternately. The 2-D lattice of HC_5C_7 [8, 4] nanotube is presented in Figure 1.

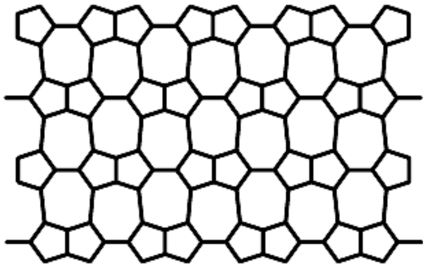

Figure 1. 2-*D* lattice of HC_5C_7 [8, 4] nanotube

Let G be a graph of a nanotube $HC_5C_7[p, q]$. By calculation, G has $4pq$ vertices and $6pq - p$ edges. Clearly, G has two types of edges based on the degree of end vertices of each edge as follows:

$E_1 = \{uv \in E(G) | d_G(u)=2, d_G(v) = 3\}$, $\qquad |E_1| = 4p$

$E_2 = \{uv \in E(G) | d_G(u)= d_G(v) = 3\}$, $\qquad |E_2| = 6pq - 5p$.

Therefore, in $TA[n]$, there are two types of edges based on the temperature of end vertices of each edge as follows:

$TE_1=\{uv\in E(G) \mid T(u)=\frac{2}{4pq-2},\ T(v)=\frac{3}{4pq-3}\}$, $|TE_1| = 4p$.

$TE_2=\{uv\in E(G) \mid T(u)=\frac{3}{4pq-3},\ T(v)=\frac{3}{4pq-3}\}$, $|TE_2| = 6pq - 5p$.

.

Theorem 6. The temperature elliptic Sombor index of a nanotube $HC_5C_7[p, q]$ is

$$TESO(G) =$$

$$= 4p\left(\frac{20pq-12}{(4pq-2)(4pq-3)}\right)\sqrt{\left(\frac{2}{4pq-2}\right)^2+\left(\frac{3}{4pq-3}\right)^2}$$

$$+2\sqrt{2}(6pq-5p)\left(\frac{3}{4pq-3}\right)^2.$$

Proof: Let $G = HC_5C_7[p, q]$. We have

$$TESO(G)=\sum_{uv\in E(G)}\left(T(u)+T(v)\right)\sqrt{T(u)^2+T(v)^2}$$

$$=4p\left(\frac{2}{4pq-2}+\frac{3}{4pq-3}\right)\sqrt{\left(\frac{2}{4pq-2}\right)^2+\left(\frac{3}{4pq-3}\right)^2}$$
$$+(6pq-5p)$$
$$\left(\frac{3}{4pq-3}+\frac{3}{4pq-3}\right)\sqrt{\left(\frac{3}{4pq-3}\right)^2+\left(\frac{3}{4pq-3}\right)^2}$$
$$=4p\left(\frac{20pq-12}{(4pq-2)(4pq-3)}\right)\sqrt{\left(\frac{2}{4pq-2}\right)^2+\left(\frac{3}{4pq-3}\right)^2}$$
$$+2\sqrt{2}(6pq-5p)\left(\frac{3}{4pq-3}\right)^2.$$

Theorem 7. The temperature elliptic Sombor exponential of a nanotube $HC_5C_7[p, q]$ is

$$TESO(G,x)=4px^{\left(\frac{20pq-12}{(4pq-2)(4pq-3)}\right)\sqrt{\left(\frac{2}{4pq-2}\right)^2+\left(\frac{3}{4pq-3}\right)^2}}$$
$$+(6pq-5p)x^{2\sqrt{2}\left(\frac{3}{4pq-3}\right)^2}.$$

Proof: Let $G = HC_5C_7[p, q]$. We have

$$TESO(G,x)=\sum_{uv\in E(G)}x^{(T(u)+T(v))\sqrt{T(u)^2+T(v)^2}}$$
$$=4px^{\left(\frac{2}{4pq-2}+\frac{3}{4pq-3}\right)\sqrt{\left(\frac{2}{4pq-2}\right)^2+\left(\frac{3}{4pq-3}\right)^2}}$$
$$+(6pq-5p)x^{\left(\frac{3}{4pq-3}+\frac{3}{4pq-3}\right)\sqrt{\left(\frac{3}{4pq-3}\right)^2+\left(\frac{3}{4pq-3}\right)^2}}.$$

By simplifying the above equation, we obtain the desired result.

Theorem 8. The modified temperature elliptic Sombor index of a nanotube $HC_5C_7[p, q]$ is

$$^{m}TESO(G)=$$
$$=\frac{4p}{\left(\frac{20pq-12}{(4pq-2)(4pq-3)}\right)\sqrt{\left(\frac{2}{4pq-2}\right)^2+\left(\frac{3}{4pq-3}\right)^2}}$$
$$+\frac{(6pq-5p)}{2\sqrt{2}\left(\frac{3}{4pq-3}\right)^2}.$$

Proof: Let $G = HC_5C_7[p, q]$. We have

$$^{m}TESO(G)=\sum_{uv\in E(G)}\frac{1}{(T(u)+T(v))\sqrt{T(u)^2+T(v)^2}}$$
$$=\frac{4p}{\left(\frac{2}{4pq-2}+\frac{3}{4pq-3}\right)\sqrt{\left(\frac{2}{4pq-2}\right)^2+\left(\frac{3}{4pq-3}\right)^2}}$$
$$+\frac{(6pq-5p)}{\left(\frac{3}{4pq-3}+\frac{3}{4pq-3}\right)\sqrt{\left(\frac{3}{4pq-3}\right)^2+\left(\frac{3}{4pq-3}\right)^2}}$$
$$=\frac{4p}{\left(\frac{20pq-12}{(4pq-2)(4pq-3)}\right)\sqrt{\left(\frac{2}{4pq-2}\right)^2+\left(\frac{3}{4pq-3}\right)^2}}$$
$$+\frac{(6pq-5p)}{2\sqrt{2}\left(\frac{3}{4pq-3}\right)^2}.$$

Theorem 9. The modified temperature elliptic Sombor exponential of a nanotube $HC_5C_7[p, q]$ is

$$^{m}TESO(G,x)=4px^{\left(\frac{20pq-12}{(4pq-2)(4pq-3)}\right)\sqrt{\left(\frac{2}{4pq-2}\right)^2+\left(\frac{3}{4pq-3}\right)^2}}$$
$$+(6pq-5p)x^{2\sqrt{2}\left(\frac{3}{4pq-3}\right)^2}.$$

Proof: Let $G = HC_5C_7[p, q]$. We have

$$^{m}TESO(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{(T(u)+T(v))\sqrt{T(u)^2+T(v)^2}}}$$
$$=4px^{\frac{1}{\left(\frac{2}{4pq-2}+\frac{3}{4pq-3}\right)\sqrt{\left(\frac{2}{4pq-2}\right)^2+\left(\frac{3}{4pq-3}\right)^2}}}$$
$$+(6pq-5p)x^{\frac{1}{\left(\frac{3}{4pq-3}+\frac{3}{4pq-3}\right)\sqrt{\left(\frac{3}{4pq-3}\right)^2+\left(\frac{3}{4pq-3}\right)^2}}}.$$

By simplifying the above equation, we get the desired result.

V. CONCLUSION

In this paper, we have introduced the temperature elliptic Sombor index, the modified temperature elliptic Sombor index of a graph. We have computed these indices for some standard graphs and $HC_5C_7[p, q]$ nanotubes. Also we have obtained some properties of the temperature elliptic Sombor index.

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