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On Fixed Points of Expansion Mappings in Menger Spaces

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ARTICLE INFO	ABSTRACT
Published Online: 20 March 2025 **Corresponding Author:** A.S.Saluja	The aim of this paper is to establish a common fixed point theorem for expansion mappings involving six mappings in a Menger space, utilizing the concepts of semi-compatibility and weak compatibility while considering the continuity of the mappings. **2020 Mathehematics Subject Classification**: Primary: 54H25; Secondary: 54E50, 47H10.

KEYWORDS: Compatible maps, Semi compatible maps, Weak compatible maps, Expansion mappings, t-norm.

1. INTRODUCTION

Metric spaces have undergone numerous generalizations over time. One such generalization is the Menger space, introduced by Menger in 1942 [12], where distribution functions replace nonnegative real numbers as metric values. Schweizer and Sklar [22] further explored this concept, and significant developments in Menger space theory were later contributed by Sehgal and Bharucha-Reid [18].

Sessa [19] introduced the concept of weakly commuting maps in metric spaces, which was later extended by Jungck [8] to include compatible maps. Mishra [13] further expanded this idea in Menger spaces by incorporating the notions of weak compatibility and compatibility of self-maps.

In this paper, we establish a common fixed point theorem for expansion mappings involving six mappings in Menger spaces, considering the conditions of weak compatibility and semi-compatibility.

2. PRELIMINARY

Definition 2.1- A triangular norm $*$ (shortly t-norm) is a binary operation on the unit interval $[0,1]$ such that for all $a,b,c,d \in [0,1]$ the following conditions are satisfied.

(a) $a*1=a$

(b) $a*b=b*a$

(c) $a*b \le c*d$ whenever $a \le c$ & $b \le d$

(d) $a*(b*c)=(a*b)*c$

Examples of t-norm are $a*b=\max\{a+b-1,0\}$ and $a*b=\min\{a,b\}$.

Definition 2.2- A distribution function is a function $F:[-\infty,\infty]\to[0,1]$ which is left continuous on R, non- decreasing and $F(-\infty)=0, F(\infty)=1$.

We will denote by Δ the family of all distribution functions on $[-\infty,\infty]$. H is special element of Δ defined by

$$H(t)=\begin{cases}0 & if \quad t\le 0\\ 1 & if \quad t>0\end{cases}$$

If X is nonempty set, $F:X\times X\to\Delta$ is called a probabilistic distance on X and $F(x,y)$ is usually denoted by F_{xy}.

Definition 2.3 (Schweizer and Sklar [16])-The ordered pair (X,F) is called a probabilistic metric space (shortly PM space) if X is nonempty set and F is a probabilistic distance satisfying the following conditions: for all $x,y,z\in X$ and $t,s>0$,

(FM0) $F_{xy}(t)=1$ iff $x=y$

(FM1) $F_{xy}(0)=0$

(FM2) $F_{xy}(t)=F_{yx}(t)$

(FM3) $F_{xz}(t)=1, F_{zy}(s)=1 \Rightarrow F_{xy}(t+s)=1$

The ordered triplet $(X,F,*)$ is called Menger space if (X,F) is a PM-space, $*$ is a t-norm and the following condition is also satisfies: for all $x,y,z\in X$ and $t,s>0$,

$(FM\ 4)$ $F_{xy}(t+s)\ge F_{xz}(t)*F_{zy}(s)$.

Proposition 2.4 (Sehgal and Bharucha-Reid [13]) - Let (X,d) be a metric space. Then the metric d induced a distribution function F defined by $F_{xy}(t)=H(t-d(x,y))$ for all $x,y\in X$ and $t>0$. If t-norm $*$ is $a*b=\min\{a,b\}$ for all $a,b\in[0,1]$ then $(X,F,*)$ is a Menger space. Further, $(X,F,*)$ is complete Menger space if (X,d) is complete.

Definition 2.5 (Mishra [14]) -Let $(X,F,*)$ be a Menger space and $*$ be a continuous t-norm.

(a) A sequence $\{x_n\}$ in X is said to be convergent to some point x in X iff for every $\varepsilon>0$ and $\lambda\in(0,1)$ there exist an integer $n_0=n_0(\varepsilon,\lambda)$ such that $F_{x_n x}(\varepsilon)>1-\lambda$ for all $n\geq n_0$.

(b) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if for $\varepsilon>0$ and $\lambda\in(0,1)$ There exist an integer $n_0=n_0(\varepsilon,\lambda)$ such that $F_{x_n x_{n+p}}(\varepsilon)>1-\lambda$ for all $n\geq n_0$ and $p>0$.

(c) A Menger space in which every Cauchy sequence is convergent is said to be complete.

Remark 2.6- If $*$ is continuous t-norm, it follows from (FM 4) that the limit of sequence in Menger space is uniquely determined.

Definition 2.7 (Singh and Jain [21]) - Self maps A and B of Menger space $(X,F,*)$ are said to be weakly compatible if they commute at their coincidence points, that is If $Ax=Bx\Rightarrow ABx=BAx$ for some $x\in X$.

Definition 2.8 (Mishra [14]) - Self maps A and B of Menger space $(X,F,*)$ are said to be compatible if $F_{ABx_n,BAx_n}(t)\to 1$ for all $t>0$, whenever $\{x_n\}$ is a sequence in X such that $Ax_n,Bx_n\to x$ for some x in X as $n\to\infty$.

Definition 2.9- Self maps A and B of Menger space $(X,F,*)$ are said to be a semi compatible if $\lim_{n\to\infty}F_{ABx_n,Bx}(t)\to 1$ for all $t>0$, whenever $\{x_n\}$ is a sequence in X such that $\lim Ax_n\to x$, $\lim Bx_n\to x$ as $n\to\infty$ for some x in X.

Lemma 2.10- Let $\{x_n\}$ be a sequence in a Menger space $(X,F,*)$ with continuous t-norm $*$ and $t*t\leq t$. If there exist a constant $k>1$ such that $F_{x_{n-1}x_n}(kt)\leq F_{x_n x_{n+1}}(t)$ For all $t>0$ an $n=1,2,\ldots$, then $\{x_n\}$ is a Cauchy sequence in X.

Lemma 2.11- Let $(X,F,*)$ be a Menger space. If there exist $k>1$ such that $F_{xy}(kt)\leq F_{xy}(t)$ For all $x,y\in X$ and $t>0$, then $x=y$.

3. MAIN RESULTS

Theorem 3.1- Let A,B,S,T,L and M be self maps on a complete Menger space $(X,F,*)$ with $t*t\leq t$ for all $t\in[0,1]$ satisfying,

(a) $ST(X)\subseteq L(X)$, $AB(X)\subseteq M(X)$

(b) For all $x,y\in X$ and $t>0$ there exist a constant $k>1$ such that

$$F^2_{LxMy}(kt)\leq \max\left\{\begin{array}{l}F_{ABxLx}(t/2)*F_{LxSTy}(t/2)*F_{MySTy}(t)\\ F_{LxSTy}(kt/2)*F_{MySTy}(kt/2)\end{array}\right\}$$

(c) Either AB or L is continuous.

(d) $AB=BA, ST=TS, LB=BL$ and $MT=TM$.

(e) Pair (L,AB) is semi compatible and (M,ST) is weak compatible.

Then A,B,S,T,L and M have unique common fixed point in X.

Proof: - Let $x_0\in X$ is any arbitrary point. As $ST(X)\subseteq L(X)$ and $AB(X)\subseteq M(X)$, there exist $x_1\in X$, $x_2\in X$ such that $STx_0=Lx_1=y_1$ and $ABx_1=Mx_2=y_2$, inductively we have $STx_{2n-1}=Lx_{2n}=y_{2n}$ and $ABx_{2n}=Mx_{2n+1}=y_{2n+1}$ for $n=1,2,3,\ldots\ldots\ldots$

Step (1) –Using (b) with $x=x_{2n}$ and $y=x_{2n+1}$

$$F^2_{Lx_{2n}Mx_{2n+1}}(kt)\leq \max\left\{\begin{array}{l}F_{ABx_{2n}Lx_{2n}}(t/2)*F_{Lx_{2n}STx_{2n+1}}(t/2)*F_{Mx_{2n+1}STx_{2n+1}}(t)\\ F_{Lx_{2n}STx_{2n+1}}(kt/2)*F_{Mx_{2n+1}STx_{2n+1}}(kt/2)\end{array}\right\}$$

$$F^2_{y_{2n}y_{2n+1}}(kt)\leq \max\left\{\begin{array}{l}F_{y_{2n+1}y_{2n}}(t/2)*F_{y_{2n}y_{2n+2}}(t/2)*F_{y_{2n+1}y_{2n+2}}(t)\\ F_{y_{2n}y_{2n+2}}(kt/2)*F_{y_{2n+1}y_{2n+2}}(kt/2)\end{array}\right\}$$

$$F^2_{y_{2n}y_{2n+1}}(kt) \le F_{y_{2n+1}y_{2n+2}}(t).F_{y_{2n}y_{2n+1}}(kt)$$

$$F_{y_{2n}y_{2n+1}}(kt) \le F_{y_{2n+1}y_{2n+2}}(t)$$

Similarly it can be found that

$$F_{y_{2n+1}y_{2n+2}}(kt) \le F_{y_{2n+2}y_{2n+3}}(t)$$

Therefore for all n even or odd we have $F_{y_n y_{n+1}}(kt) \le F_{y_{n+1}y_{n+2}}$. Thus $\{y_n\}$ is a Cauchy sequence . Since X is complete then $\{y_n\}$ converges to some point z in X . Or all subsequence $\{STx_{2n-1}\}, \{Lx_{2n}\}, \{Mx_{2n+1}\}$ and $\{ABx_{2n}\}$ also converges to z .

Case (a) – When map AB is continuous .

Step (2)– Since $\lim_{n\to\infty} Lx_{2n} = z$ and $\lim_{n\to\infty} ABx_{2n} = z$. Since AB is continuous map then $\lim_{n\to\infty} ABLx_{2n} = ABz$ and $\lim_{n\to\infty} ABABx_{2n} = ABz$

Also (AB, L) is semi compatible map then if $\lim_{n\to\infty} ABx_{2n} = \lim_{n\to\infty} Lx_{2n} = z$

Therefore $\lim_{n\to\infty} LABx_{2n} = ABz$.

Using (b) with $x = ABx_{2n}$, $y = x_{2n+1}$

$$F^2_{LABx_{2n}Mx_{2n+1}}(kt) \le \max \begin{Bmatrix} F_{ABABx_{2n}LABx_{2n}}(t/2) * F_{LABx_{2n}STx_{2n+1}}(t/2) * F_{Mx_{2n+1}ST_{2n+1}}(t) \\ F_{LABx_{2n}STx_{2n+1}}(kt/2) * F_{Mx_{2n+1}STx_{2n+1}}(kt/2) \end{Bmatrix}$$

Limiting $n\to\infty$ We have

$$F^2_{zABz}(kt) \le \max \begin{Bmatrix} F_{ABzABz}(t/2) * F_{zABz}(t/2) * F_{zz}(t) \\ F_{zABz}(kt/2) * F_{zz}(kt/2) \end{Bmatrix}$$

$$F^2_{zABz}(kt) \le F_{zABz}(t/2).F_{zABz}(kt/2)$$

$$F^2_{zABz}(kt) \le F_{zABz}(t).F_{zABz}(kt)$$

$$F_{zABz}(kt) \le F_{zABz}(t)$$

$$ABz = z$$

Step (3)- Using (b) with $x = z$, $y = x_{2n+1}$

$$F^2_{LzMx_{2n+1}}(kt) \le \max \begin{Bmatrix} F_{ABzLz}(t/2) * F_{LzSTx_{2n+1}}(t/2) * F_{Mx_{2n+1}STx_{2n+1}}(t) \\ F_{LzSTx_{2n+1}}(kt/2) * F_{Mx_{2n+1}STx_{2n+1}}(kt/2) \end{Bmatrix}$$

Now limiting $n\to\infty$ we have

$$F^2_{zLz}(kt) \le \max \begin{Bmatrix} F_{zLz}(t/2) * F_{zLz}(t/2) * F_{zz}(t) \\ F_{zLz}(kt/2) * F_{zz}(kt/2) \end{Bmatrix}$$

$$F^2_{zLz}(kt) \le F_{zLz}(t/2).F_{zLz}(kt/2)$$

$$F^2_{zLz}(kt) \le F_{zLz}(t).F_{zLz}(kt)$$

$$F_{zLz}(kt) \le F_{zLz}(t) \quad \Rightarrow Lz = z$$

Step (4) - Since $LB = BL$ then $LBz = BLz = Bz$

Also $AB = BA \Rightarrow ABBz = BABz = Bz$

By using (b) with $x = Bz, y = x_{2n+1}$

$$F^2_{LBzMx_{2n+1}}(kt) \le \max \begin{Bmatrix} F_{ABBzLBz}(t/2) * F_{LBzSTx_{2n+1}}(t/2) * F_{Mx_{2n+1}ST_{2n+1}}(t) \\ F_{LBzSTx_{2n+1}}(kt/2) * F_{Mx_{2n+1}STx_{2n+1}}(kt/2) \end{Bmatrix}$$

$$F^2_{LBzMx_{2n+1}}(kt) \le \max \begin{Bmatrix} F_{BzBz}(t/2) * F_{zBz}(t/2) * F_{zz}(t) \\ F_{zBz}(kt/2) * F_{zz}(kt/2) \end{Bmatrix}$$

$$F^2_{zBz}(kt) \le F_{zBz}(t/2).F_{zBz}(kt/2)$$

$$F^2_{zBz}(kt) \le F_{zBz}(t).F_{zBz}(kt)$$

$$F_{zBz}(kt) \le F_{zBz}(t) \Rightarrow Bz = z$$

Step (5) - Since $Lz = z$ and $ST(X) \subseteq L(X)$

Then there exists a point $w \in X$ such that $Lz = STw$

By using (b) with $x = x_{2n}$, $y = w$

$$F^2_{Lx_{2n}Mw}(kt) \le \max \begin{Bmatrix} F_{ABx_{2n}Lx_{2n}}(t/2) * F_{Lx_{2n}STw}(t/2) * F_{MwSTw}(t) \\ F_{Lx_{2n}STw}(kt/2) * F_{MwSTw}(kt/2) \end{Bmatrix}$$

Now limiting $n\to\infty$ we have

$$F^2_{zMw}(kt) \le \max \begin{Bmatrix} F_{zz}(t/2) * F_{zLz}(t/2) * F_{MwLz}(t) \\ F_{zLz}(kt/2) * F_{MwLz}(kt/2) \end{Bmatrix}$$

$$F^2_{zMw}(kt) \le F_{zMw}(t).F_{zMw}(kt/2)$$

$$F^2_{zMw}(kt) \le F_{zMw}(t).F_{zMw}(kt)$$

$$F_{zMz}(kt) \le F_{zMw}(t) \Rightarrow Mw = z$$

Therefore $STw == Mw$

Since (ST, M) is weak compatible, then $STMw = MSTw$

$\Rightarrow STz = Mz$

Step (6) – Using (b) with $x = x_{2n}$, $y = z$

$$F^2_{Lx_{2n}Mz}(kt) \le \max \begin{Bmatrix} F_{ABx_{2n}Lx_{2n}}(t/2) * F_{Lx_{2n}STz}(t/2) * F_{MzSTz}(t) \\ F_{Lx_{2n}STz}(kt/2) * F_{MzSTz}(kt/2) \end{Bmatrix}$$

$$F^2_{zMz}(kt) \le \max \begin{Bmatrix} F_{zz}(t/2) * F_{zMz}(t/2) * F_{MzMz}(t) \\ F_{zMz}(kt/2) * F_{MzMz}(kt/2) \end{Bmatrix}$$

$$F^2_{zMz}(kt) \le F_{zMz}(t/2).F_{zMz}(kt/2)$$

$$F^2_{zMz}(kt) \le F_{zMz}(t).F_{zMz}(kt)$$

$$F_{zMz}(kt) \le F_{zMz}(t) \Rightarrow Mz = z .$$

Step (7) – Since $MT = TM$ therefore $MTz = TMz = Tz$

Again Since $ST = TS$ therefore $STTz = TSTz = Tz$

Now using (b) with $x = x_{2n}$, $y = Tz$

$$F^2_{Lx_{2n}MTz}(kt) \le \max \begin{Bmatrix} F_{ABx_{2n}Lx_{2n}}(t/2) * F_{Lx_{2n}STTz}(t/2) * F_{MTzSTTz}(t) \\ F_{Lx_{2n}STTz}(kt/2) * F_{MTzSTTz}(kt/2) \end{Bmatrix}$$

Now limiting $n \to \infty$ we have

$$F^2_{zTz}(kt) \le \max \begin{Bmatrix} F_{zz}(t/2) * F_{zTz}(t/2) * F_{TzTz}(t) \\ F_{zTz}(kt/2) * F_{TzTz}(kt/2) \end{Bmatrix}$$

$$F^2(kt) \le F_{zTz}(t/2).F_{zTz}(kt/2)$$

$$F^2_{zTz}(kt) \le F_{zTz}(t).F_{zTz}(kt)$$

$$F_{zTz}(kt) \le F_{zTz}(t) \Rightarrow Tz = z$$

Since $STz = z \Rightarrow Sz = z$

Therefore $Az = Bz = Sz = Tz = Lz = Mz = z$, that is z is the common fixed point of the six maps.

Case (b) –When map L is continuous.

Step (8) – Since $\lim_{n\to\infty} ABx_{2n} = \lim_{n\to\infty} Lx_{2n} = z$ and

$$\lim_{n\to\infty} LABLx_{2n} = \lim_{n\to\infty} LLx_{2n} = Lz$$

Also pair (AB, L) is semi compatible pair since

$$\lim_{n\to\infty} ABx_{2n} = \lim_{n\to\infty} Lx_{2n} = z$$

$$\lim_{n\to\infty} ABLx_{2n} = Lz$$

By using (b) with $x = Lx_{2n}, y = x_{2n+1}$

$$F^2_{LLx_{2n}Mx_{2n+1}}(kt) \le \max \begin{Bmatrix} F_{ABLx_{2n}LLx_{2n}}(t/2) * F_{LLx_{2n}STx_{2n+1}}(t/2) * F_{Mx_{2n+1}STx_{2n+1}}(t) \\ F_{LLx_{2n}STx_{2n+1}}(kt/2) * F_{Mx_{2n+1}STx_{2n+1}}(kt/2) \end{Bmatrix}$$

Now limiting $n \to \infty$ we have

$$F^2_{zLz}(kt) \le \max \begin{Bmatrix} F_{LzLz}(t/2) * F_{zLz}(t/2) * F_{zz}(t) \\ F_{zLz}(kt/2) * F_{zz}(kt/2) \end{Bmatrix}$$

$$F^2_{zLz}(kt) \le F_{zLz}(t/2).F_{zLz}(kt/2)$$

$$F^2_{zLz}(kt) \le F_{zLz}(t).F_{zLz}(kt)$$

$$F_{zLz}(kt) \le F_{zLz}(t) \Rightarrow Lz = z.$$

Step (9) - Again using (b) with $x = z, y = x_{2n+1}$

$$F^2_{LzMx_{2n+1}}(kt) \le \max \begin{Bmatrix} F_{ABzLz}(t/2) * F_{LzSTx_{2n+1}}(t/2) * F_{Mx_{2n+1}STx_{2n+1}}(t) \\ F_{LzSTx_{2n+1}}(kt/2) * F_{Mx_{2n+1}STx_{2n+1}}(kt/2) \end{Bmatrix}$$

Now limiting $n \to \infty$ we have

$$F^2_{zz}(kt) \le \max \begin{Bmatrix} F_{zABz}(t/2) * F_{zz}(t/2) * F_{zz}(t) \\ F_{zLz}(kt/2) * F_{zz}(kt/2) \end{Bmatrix}$$

$$F^2_{zz}(kt) \le F_{zABz}(t/2).F_{zLz}(kt/2)$$

$$F^2_{zz}(kt) \le F_{zABz}(t).F_{zLz}(kt)$$

$$F_{zz}(kt) \le F_{zABz}(t)$$

$$\Rightarrow 1 \le F_{zABz}(t) \Rightarrow ABz = z$$

$$\Rightarrow ABz = Az = z.$$

Rest proof is same as **step-4** onward.

Uniqueness: - Let u be another common fixed point of A, B, S, T, L and M then

$$Au = Bu = Su = Tu = Lu = Mu = u.$$

By using (b) with $x = u, y = x_{2n+1}$

$$F^2_{LuMx_{2n+1}}(kt) \le \max \begin{Bmatrix} F_{ABuLu}(t/2) * F_{LuSTx_{2n+1}}(t/2) * F_{Mx_{2n+1}STx_{2n+1}}(t) \\ F_{LuSTx_{2n+1}}(kt/2) * F_{Mx_{2n+1}STx_{2n+1}}(kt/2) \end{Bmatrix}$$

Now limiting $n \to \infty$ we have

$$F^2_{uz}(kt) \le \max \begin{Bmatrix} F_{uu}(t/2) * F_{uz}(t/2) * F_{zz}(t) \\ F_{uz}(kt/2) * F_{zz}(kt/2) \end{Bmatrix}$$

$$F^2_{uz}(kt) \le F_{uz}(t/2).F_{uz}(kt/2)$$

$$F^2_{uz}(kt) \le F_{uz}(t).F_{uz}(kt)$$

$F_{uz}(kt) \le F_{uz}(t) \Rightarrow u = z$. This completes the proof of the theorem.

Example 3.2: - Let $x \in [0,1]$ with the metric d defined by $d(x,y) = |x - y|$ and defines $F_{xy}(t) = H(t - d(x,y))$ for all $x, y \in X, t > 0$. Clearly $(X, F, *)$ is a complete Menger space where t-norm $*$ is defined by $a * b = \min\{a, b\}$ for all $a, b \in [0,1]$. Let A, B, S, T, L and M be maps from X into itself defined as $A(x) = x/3$, $B(x) = x$, $S(x) = x/2$, $T(x) = x$, $L(x) = x/3$ $M(x) = x/4$ for all $x \in X$. Then $L(X) = \{0, 1/3\} \in \{0, 1/2\} = ST(X)$ and $M(X) = \{0, 1/4\} \in \{0, 1/3\} = AB(X)$. Clearly $AB = BA, MT = TM, ST = TS, LB = BL$ and AB, L are continuous. Moreover $\{x_n\}$ is a sequence such that $\lim_{n\to\infty} \{x_n\} = 0$. Since $\lim_{n\to\infty} ABx_n = \lim_{n\to\infty} Lx_n = 0$ for $0 \in X$, then $\lim_{n\to\infty} ABLx_n = \lim_{n\to\infty} LABx_n$. Therefore (AB, L) is compatible . Also M and ST are weakly compatible at 0. Condition (b) of main theorem satisfy equal condition for $k = 2, t = 1/3$ and $x = 1, y = 1/2$ where $\{1, 1/2\} \in X$ and less than condition for $k = 2$, $t = 1/20$ and $x = 0, y = 1/2$ where $\{0, 1/2\} \in X$. Thus all the condition of main theorem satisfied and 0 is the unique common fixed point of A, B, S, T, L and M.

Corollary: - Let A, B, S, T be self maps on a complete Menger space $(X, F, *)$ with $t * t \leq t$ for all $t \in [0, 1]$ satisfying,

(a) $A(X) \subset T(X)$, $B(X) \subset S(X)$

(b) For all $x, y \in X$ and $t > 0$ there exist a constant $k > 1$ such that

$$F^2_{AxBy}(kt) \leq \max \begin{Bmatrix} F_{SxAx}(t/2) * F_{AxTy}(t/2) * F_{ByTy}(t) \\ F_{AxTy}(kt/2) * F_{ByTy}(kt/2) \end{Bmatrix}$$

(c) Either A or S is continuous .

(d) Pair (A, S) is semi compatible and (B, T) is weak compatible .

Then $A, B, S \,\&\, T$ have unique common fixed point in X .

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