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Numerical Simulation of Convection flow in Warm Bath and Its Possible mixing Behaviour

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ABSTRACT
We have carried out an investigation of the mixing behaviour of fluid with different densities, were density was taken as a quadratic function of temperature and all flow parameters were kept fixed. The result showed that mixed dense fluid that have attained T_m or a temperature close to it was descending from the contact layer in the form of an inverted mushroom like structure towards the floor of the container. Most of the convection flow configurations considered in the past are such that the isothermal walls with heat generation are mostly the vertical walls or both the vertical and the horizontal walls. However, results by Cianfrini et al. 2015 with similar configuration appears similar as compared to ours as the authors also recorded the inverted mushroom like structure behaviour descending towards the floor of the container. Both vertical and horizontal velocity profiles were also considered at some point below the contact layer and plotted against the x-coordinate. The results as presented here are very good as they give insight into the mixing behaviour of a possible warm bath. Thus, it is true that whenever water of different densities come in contact, mixing will occur without any external mixing or perturbation: and any part of the fluid that have mixed up to the T_m will descend to the floor of the container if only either of the temperature is above the temperature of maximum density until the entire ambient fluid is induced through convection flow.

KEYWORDS: Convection flow, Ambient fluid, Temperature of maximum density T_m, Density.

1 INTRODUCTION

Natural convection flows are phenomenal behaviours that occur in our environment. This is a kind of heat distribution or transfer where fluid motion occurs naturally as a result of density difference in the fluid owing to temperature gradient and not influenced or perturbed by external force or sources (Ezan and Kalfa, 2017). It is believed that a fluid around any heat source will certainly gain heat and in turn become less dense and begin to rise forming buoyancy flow (i.e, the heat source in this case can be positioned at any point but not at the surface part of the ambient fluid). In fact, the ambient cold fluid will move to replace the initial fluid that have risen and get hot again and this process will continue until the entire ambient fluid will get warm depending on the degree of heat applied (Kane, 2017; Nayak et al. 2018). With this, it is true that one of the paramount forces for natural convection flow in a fluid is the buoyancy force as a result of the differences in the fluid density (Nayak et al. 2018). For natural convection flows, It is also known that the fluid velocity is likely to be small: thus, the flow parameters into consideration for numerical results are also key. In such flows, rate of heat distribution is also dependent on the geometry and as well the variation of temperature in the ambient fluid. The analysis of natural convection flows are of great importance in many scientific and engineering applications. These flows are evident in our everyday lives and can also be observed through: ventilation of buildings, solar ponds, Cooling of power plants and electronic equipment, cooking, over-head water tanks, etc., (Radhwan & Zaki 2000; Djoubeir et al. 2014; Khurshid, & Silaipillayarputhur, 2018). Because of its importance, this area of research have also received great attention in the literature numerically, theoretically and experimentally due to its wide applications. The following authors have considered some of these flows with different configuration and can be studied for more insight (Ezan & Kalfa, 2017; Hossain & Rees, 2005; Li et al. 2011; Rahman et al. 2010; Kane, 2017; Hidayathulla Khan et

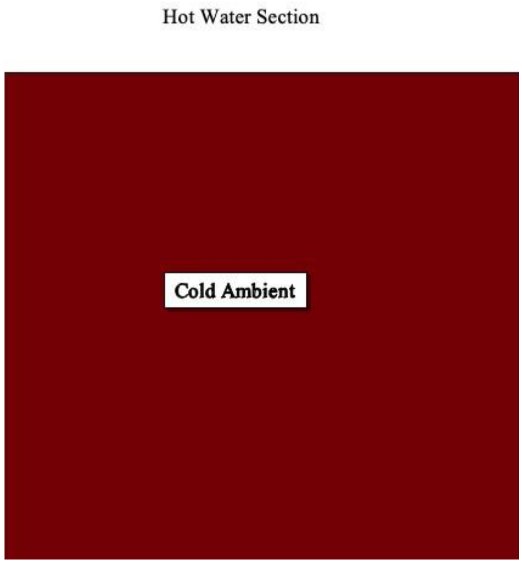

Fig. 1: Schematic presentation of the model studied with the different water temperatures (hot upper section of about 10°*C* and ambient temperature of 0°*C*) at initial time.

al. 2018; El Moutaouakil et al. 2020; Cianfrini et al. 2015; Hasnaoui et al. 1992; Lee & Ha, 2006; Zheng et al. 2021; etc.).

With these few sources as mentioned, it is certain that whenever fluid of different densities come in contact, convection flow will definitely occur as the fluid mixes further. Thus, convection flow may also be evident in our daily warm bath. For instance, if we put into consideration a container filled with water of about 0°*C*, assuming that this water (ambient water) is quiescent and uniform at initial time. Then, If a hot plate is placed at the surface of this ambient water or introducing hot water at the upper part of the container at initial time such that there is a thin layer/barrier between the hot and the cold water (see figure 1). As time progresses on the hot plate that is in contact with the ambient water at a temperature above the temperature of maximum density T_m which is about 4°*C* for most numerical calculation: or removing the thin layer/barrier that separates the cold and the hot water above T_m such that they come in contact. It is obvious that convection flow will occur as hot water and cold water mixes further. There is also the possibility that the entire water in the container will get mixed without any stirring or perturbation. Convection flow using this configuration have received less attention. Thus, the present investigation is to carryout a numerical simulation with such configuration with the assumption that density is taken as a quadratic function of temperature as also recorded in the literature by George & Osaisai,(2022). This investigation will enable us to properly fathom the mixing behaviour of both the hot and cold water as they come in contact. After both hot and cold water come in contact, it is expected that mixing will occur without any external force or perturbation: and any part of the fluid that have mixed up to the T_m will descend to the floor of the container. This process will continue until the entire ambient fluid is induced through convection flow.

2 MODEL FORMULATION AND GOVERNING EQUATIONS

The mixing behaviour of warm bath as denser fluid undergo a descending plume due to the nonlinear relation between density ρ and temperature T is of interest. Thus, the proposal of a quadratic dependence relation assumption is appropriate for this investigation,

$$\rho = \rho_m - \beta(T - T_m)^2. \tag{1}$$

The quadratic dependence relation assumption have shown to gives a good correlation to the experimentally determined density in fresh water at temperatures below 10°*C*, taken $T_m = 3.98°C$, $\rho_m = 1.000 \times 10^3\, kg.m^{-3}$ and $\beta = 8.0 \times 10^{-3}\, kg.m^{-3}(°C)^{-2}$ (Moore & Weiss, 1973; Oosthuizen &Paul, 1996) and all other fluid properties such as viscosity, thermal diffusivity are assumed constant. We assume that the flow is two dimensional and time dependent with liquid property being constant except for the water density, which changes with temperature. We can non-dimensionalise the coordinates x, y, velocity components u, v, time t, pressure p and temperature T by

$$U = \frac{u}{U_*} \quad V = \frac{v}{U_*} \quad X = \frac{x}{H} \quad Y = \frac{y}{H} \quad \tau = \frac{t}{\frac{H}{U_*}} \quad P = \frac{p}{\rho U_*^2}$$

$$\phi = \frac{T - T_\infty}{T_m - T_\infty}, \tag{2}$$

where x and u are horizontal, y and v are vertical; $U_* = \sqrt{\frac{\rho_\infty - \rho)}{\rho} H}$ is the relative frontal velocity and domain height H. We also define dimensionless parameters, the Reynolds *Re*, Prandtl *Pr* and Froude *Fr* numbers, by

$$\nu = \frac{\mu}{\rho} \quad \alpha = \frac{k}{\rho c_p} \quad Re = \frac{U_* H}{\nu} \quad Pr = \frac{\nu}{\alpha} \quad Fr^2 = \frac{\rho_m U_*^2}{g\beta(T_m - T_\infty)^2 H}, \tag{3}$$

where ν and α are the respective diffusivities of momentum and heat, and μ is viscosity, k is thermal conductivity and c_p is specific heat capacity. With the dimensionless variables and parameters, the continuity equation, horizontal and vertical momentum equations and thermal energy equation are given as

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{4}$$

$$\frac{\partial U}{\partial \tau} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{Re}(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}) \tag{5}$$

$$\frac{\partial V}{\partial \tau} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{Re}(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}) + \frac{1}{Fr^2}[\phi^2 - 2\phi] \tag{6}$$

$$\frac{\partial \phi}{\partial \tau} + U\frac{\partial \phi}{\partial X} + V\frac{\partial \phi}{\partial Y} = \frac{1}{RePr}(\frac{\partial^2 \phi}{\partial X^2} + \frac{\partial^2 \phi}{\partial Y^2}) \tag{7}$$

Our computational domain is consists of a domain length L of total $L = 70$, i.e., $0 \leq X \leq 70$, and a domain height $H = 90$ i.e., $0 \leq Y \leq 90$. Where the domain length and height of the hot upper section is $L_1 = 70$ and $H_1 = 10$: while, the ambient fluid domain length and height is $L = 70$ and $H = 80$. We assume that all the side walls and the horizontal base/floor of the container are considered insulated with an insignificant heat loss from the surface or an adiabatic surface condition.

Therefore, our initial conditions are an undisturbed, homogeneous medium,

$$U = 0, \quad V = 0, \quad \varphi = 0, \quad \textit{for the cold section} \quad \tau < 0 \tag{8}$$

And

$$U = 0, \quad V = 0, \quad \varphi = 2.5, \quad \textit{for the hot section} \quad \tau < 0 \tag{9}$$

For $\tau \geq 0$ we have boundary conditions as follows. On the side walls:

$$U = 0, \quad V = 0, \quad \frac{\partial \phi}{\partial X} = 0 \tag{10}$$

At the interaction layer (source):

$U = 0$, $V(X,0) = 1$, $\varphi = 2.5$ for L_1 and $\varphi = 0$ for L,

for $X = 70$, *at* $Y = H_1$ *and* H *respectively.* (11)

On the floor of the domain:

$$U = 0, \quad V = 0, \quad \frac{\partial \phi}{\partial Y} = 0 \tag{12}$$

At the top of the domain:

$$\frac{\partial U}{\partial Y} = 0, \quad V = 0, \quad \frac{\partial \phi}{\partial Y} = 0 \tag{13}$$

The Reynolds number $Re = 5$, Froude number $Fr = 2.5$ and Prandtl number $Pr = 9.5$ will be fixed throughout this study. The dimensionless temperature $\varphi = 2.5$ in the L_1 is equivalent to a discharge at $10^{\circ}C$ into an ambient at $0^{\circ}C$. Numerical result of the above equations is by means of COMSOL Multiphysics software. This commercial package uses the finite element solver with discretization by the Galerkin method and stabilisation to prevent spurious oscillations. Note that the exactness of any numerical solution is also linked to the mesh size used (i.e., the smaller the mesh size the more accurate the result becomes). Therefore, the mesh size as used for the computation is 0.05 so that the solution will be independent of the mesh used, if and only if the mesh size is ≤ 0.05. Time stepping is by COMSOL's Backward Differentiation Formulas. More information about the numerical methods is available from the COMSOL Multiphysics website (COMSOL Multiphysics Cyclopedia, 2016). Results will be presentation will by surface temperature plots of dimensionless temperature on a colour scale from dark red for the ambient temperature $\varphi = 0.0$, through yellow to white for the source temperature $\varphi = 2.5$. Note that $\varphi = 1.0$ corresponds to the temperature of maximum density while $\varphi = 2.0$ is the temperature at which warm water has the same density as the ambient cold water.

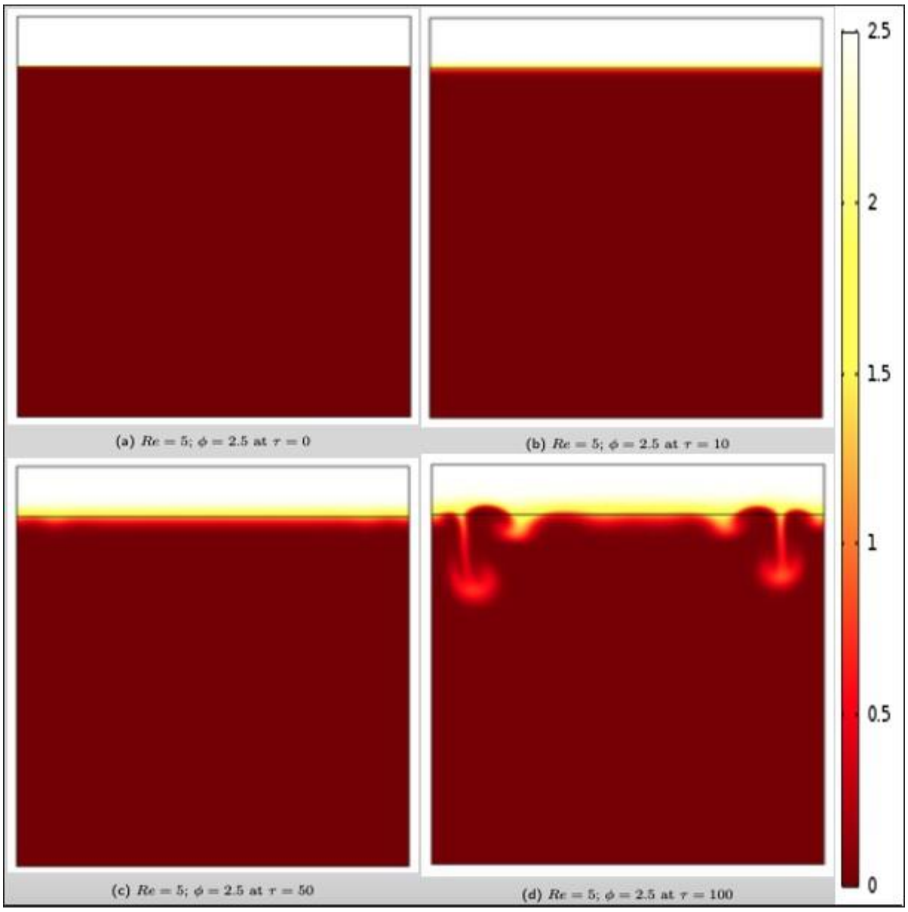

Fig. 2: Evolution of temperature field in convective flow for Re = 5, Fr = 2.5 and Prandtl number Pr = 9.5 and dimensionless temperature φ =2.5 at the upper section within the time range $0 \leq \tau \leq 100$.

3 NUMERICAL RESULTS

An investigation into the mixing behaviour of fluid with different densities had just been conducted with the assumption that density was taken as a quadratic function of temperature. We have fixed the Re = 5, Fr = 2.5 and Pr = 9.5 throughout the investigation for a laminar flow scenario and results are shown below. Figure 2 (a), (b) & (c) shows the evolution of the temperature flied as both hot and cold water come in contact. As time progresses, mixed dense fluid that have attained T_m or a temperature close to it could be seen descending from the mixing/contact layer in the form of an inverted mushroom like structure (descending plume) towards the floor of the container (see Fig. 2(d)). As this process continues, it was also observed that mixing takes place in the entire contact layer where both hot and cold water meet. Having that volume of the hot water into consideration is far less than that of the ambient water, the hot water at the top most part have reduced significantly. Meanwhile, the ambient fluid is gradually being induced by this warm but dense descending water (see Fig. 3 (e), (f), (g) & (h)). it is expected that mixing will continue without any external force or perturbation and any part of the fluid that have mixed up to the T_m will continue to descend to the floor of the container. With time, the entire hot water at the upper most part of the container will get mixed up and the entire fluid in the container will become the same temperature. It is worth mentioning that most of the convection flow configurations considered in the past are such that the isothermal walls with heat generation are mostly the vertical walls or the heat activation point is on both the vertical and the horizontal walls. Thus, the configuration as considered here have received less attention: but then, the results by Cianfrini et al. 2015 with similar configuration appears similar as compared to ours as the authors also recorded the inverted mushroom like structure behaviour descending towards the floor of the container. Though, their results showed that by fixing the temperature of the cooled wall, penetrative convection takes place provided the temperature of the upper heated surface is lower than a limit value that increases with increasing the cavity width. Note, we have not given any discussion on penetrative convection because our domain of configuration and temperature of the fluid in the various sections was kept fixed. Besides, penetrative convection become important when in free convection where the horizontal water layer is differently heated at both the bottom and top boundaries with density inversion in the bulk which leads to the formation of an upper stably stratified fluid region,

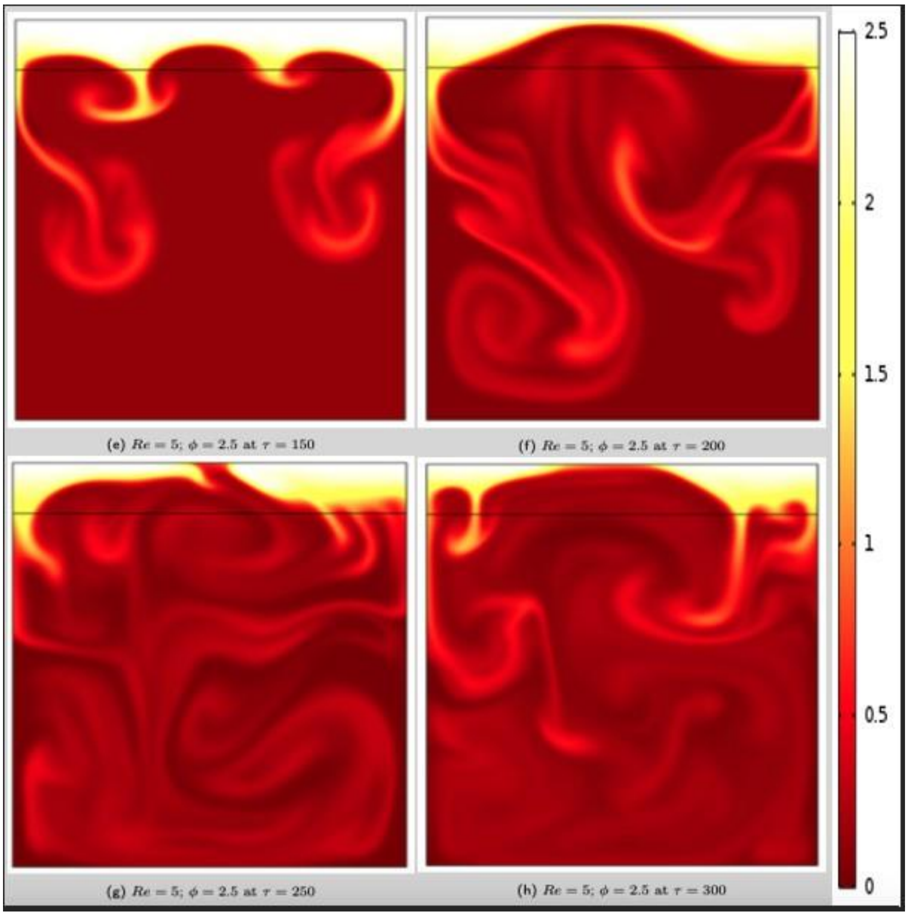

Fig. 3: Evolution of temperature field in convective flow for *Re* = 5, *Fr* = 2.5 and Prandtl number *Pr* = 9.5 and dimensionless temperature φ =2.5 at the upper section within the time range 150≤ τ ≤300.

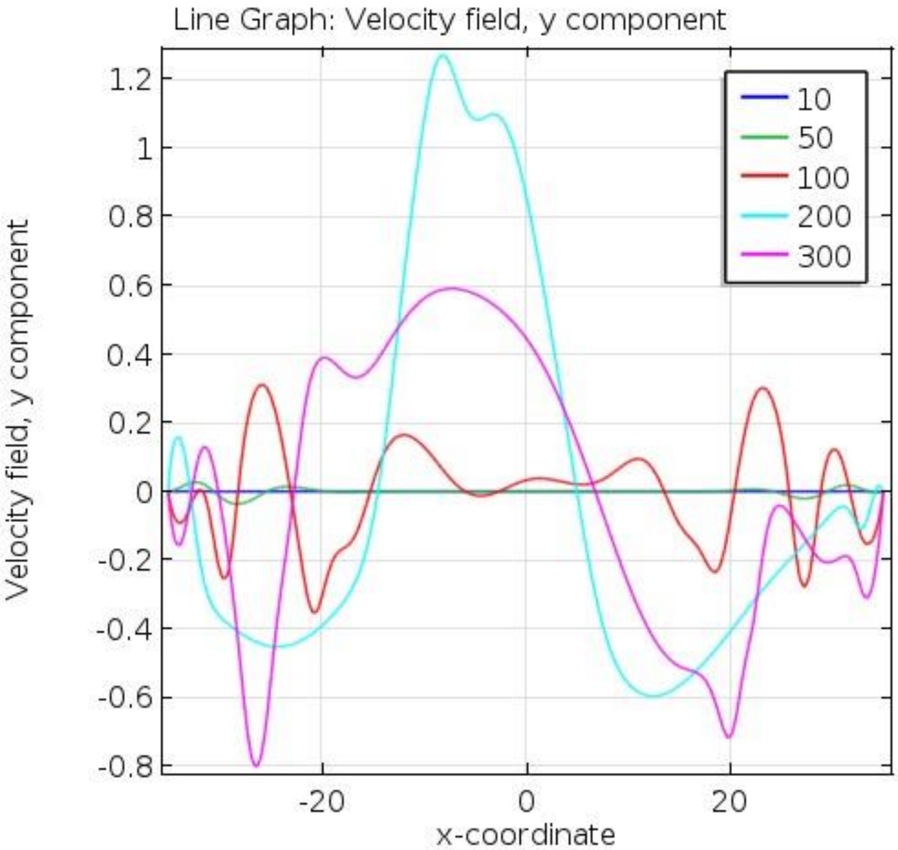

Fig. 4: Dimensionless vertical velocity profiles at some point close to the mixing layer V (X, 69) at time τ = 10,50,100,200,300. and a lower convectively unstable region from which motion can propagate upwards.

We have also considered the velocity field in the y-component at some point below the contact layer and plotted against the x-coordinate (see figure 4). This region indicates that there was a strong interaction between the two fluid which resulted to a fluid motion in the vertical direction. Though, at earlier time interval between τ = 10 & 50, there was no significant mixed descending fluid in the region and this is also evident in figure 2 (b) & (c) at that time. But as time progresses much significant mixed fluid were noticed in the region. The downwards curves here represent descending fluid whereas, the upwards curves represents fluid that were still slightly positively buoyant. Thus, fluid motion were noticed as descending dense fluid continue to interact with the surrounding fluid. The dimensionless horizontal velocity profiles was also considered and plotted against the x-coordinate (see figure 5). The result also indicated that there was a fluid motion from both left to the right and right to the left. This must be true as descending fluid in vortex form continue to descend, interacting with the ambient fluid further downwards. The results as presented here are very good as they give insight into the mixing behaviour of a possible warm bath. Thus, it is true that whenever water of different densities come in contact, mixing will occur without any external mixing or perturbation: and any part of the fluid that have mixed up to the T_m will descend to the floor of the container if only either of the temperature is above the temperature of maximum density until the entire ambient fluid is induced through convection flow.

4 Discussion/Conclusion

The mixing behaviour of fluid with different densities had just been investigated with the assumption that density was taken as a quadratic function of temperature and all flow parameters were kept fixed. The result showed that mixed dense fluid that have attained T_m or a temperature close was seen descending from the mixing/contact layer in the form of an inverted mushroom like structure (descending plume) towards the floor of the container. Mixing here takes place in the entire contact layer where both hot and cold water meet. Most of the convection flow configurations considered in the past are such that the isothermal walls with heat generation are mostly the vertical walls or the heat activation point is on both the vertical and the horizontal walls. However, results by Cianfrini et al. 2015 with similar configuration appears

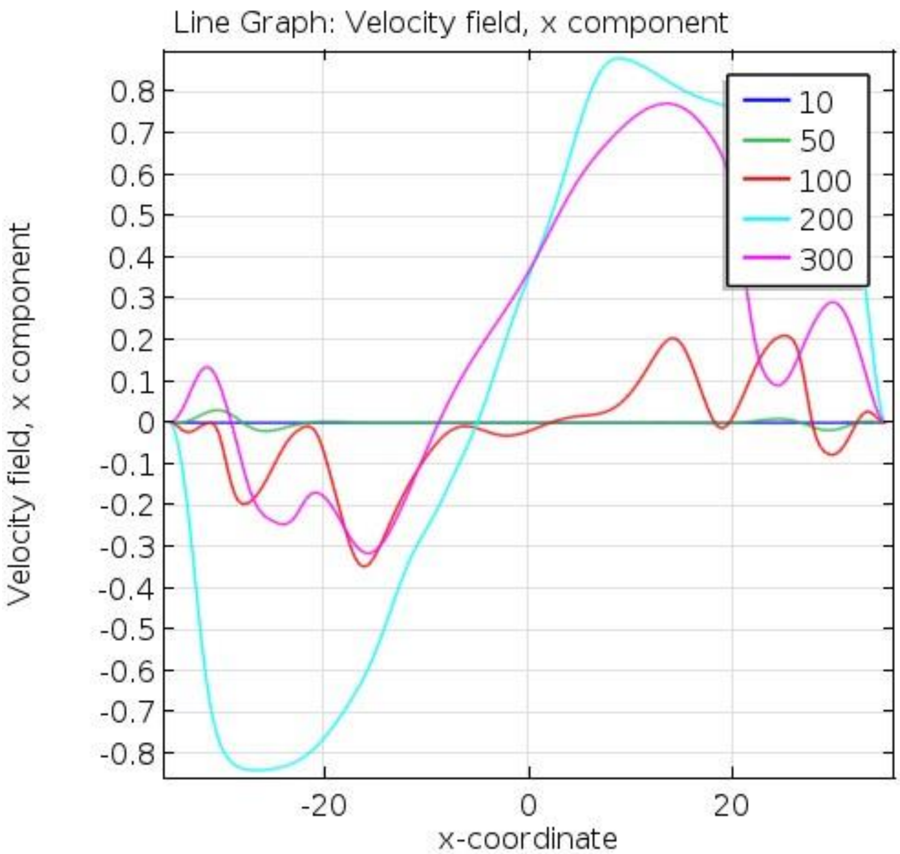

Fig. 5: Dimensionless horizontal velocity profiles at some point close to the mixing layer V (X, 69) at time τ =10,50,100,200,300.

similar as compared to ours as the authors also recorded the inverted mushroom like structure behaviour descending towards the floor of the container. Though, their results showed that by fixing the temperature of the cooled wall, penetrative convection takes place provided the temperature of the upper heated surface is lower than a limit value that increases with increasing the cavity width. But we have not given any discussion on penetrative convection because our domain of configuration and temperature of the fluid in the various sections was kept fixed. Both vertical and horizontal velocity profiles were considered at some point below the contact layer and plotted against the x-coordinate. The results as presented here are very good as they give insight into the mixing behaviour of a possible warm bath. Thus, it is true that whenever water of different densities come in contact, mixing will occur without any external mixing or perturbation: and any part of the fluid that have mixed up to the T_m will descend to the floor of the container if only either of the temperature is above the temperature of maximum density until the entire ambient fluid is induced through convection flow.

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