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Neighborhood Kepler Banhatti and Modified Neighborhood Kepler Banhatti Indices of Certain Dendrimers

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ABSTRACT

In this paper, we introduce the neighborhood Kepler Banhatti index, modified neighborhood Kepler Banhatti index and their corresponding exponentials of a graph. Also we compute these neighborhood Kepler Banhatti indices of certain dendrimers. Furthermore, we establish some properties of newly defined the neighborhood Kepler Banhatti index.

KEYWORDS: neighborhood Kepler Banhatti index, modified neighborhood Kepler Banhatti index, dendrimer.

1. INTRODUCTION

Let *G* be a finite, simple, connected graph with vertex set *V*(*G*) and edge set *E*(*G*). The degree $d_G(u)$ of a vertex *u* is the number of vertices adjacent to *u*. Let $S_G(u)$ denote the sum of the degrees of all vertices adjacent to a vertex *u*. We refer [1] for undefined notations and terminologies.

A graph index is a numerical parameter mathematically derived from the graph structure. Many graph indices have been considered in Theoretical Chemistry and several graph indices were defined by using vertex degree concept [2]. The Zagreb, Gourava, Nirmala, Sombor, Revan, E-Banhatti indices are the most degree based graph indices in Chemical Graph Theory, see [3-39]. Graph indices have their applications in various disciplines in Science and Technology [40-41].

The Kepler Banhatti index [42] of a graph *G* is defined as

$$KB(G)=\sum_{uv\in E(G)}[\left(d_G(u)+d_G(v)\right)+\sqrt{d_G(u)^2+d_G(v)^2}].$$

Recently, some Kepler Banhatti indices were studied in [43-46].

The neighborhood Kepler Banhatti index of a graph *G* is defined as

$$NKB(G)=\sum_{uv\in E(G)}\left(\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}\right).$$

Considering the neighborhood Kepler Banhatti index, we introduce the neighborhood Kepler Banhatti exponential of a graph *G* and defined it as

$$NKB(G,x)=\sum_{uv\in E(G)}x^{\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}}.$$

We define the modified neighborhood Kepler Banhatti index of a graph *G* as

$$^{m}NKB(G)=\sum_{uv\in E(G)}\frac{1}{\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}}.$$

Considering the modified neighborhood Kepler Banhatti index, we introduce the modified neighborhood Kepler Banhatti exponential of a graph *G* and defined it as

$$^{m}NKB(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}}}$$

Recently, some neighborhood indices were studied in [47-53].

In 2011 [54], Graovac et al. introduced the fifth *M*-Zagreb indices (now we call the first and second neighborhood indices) defined as

$$NM_1(G)=\sum_{uv\in E(G)}\left[S_G(u)+S_G(v)\right],$$

$$NM_2(G) = \sum_{uv \in E(G)} S_G(u) S_G(v)$$

The neighborhood Sombor index [55] of a molecular graph G is defined as

$$NSO(G) = \sum_{uv \in E(G)} \sqrt{S_G(u)^2 + S_G(v)^2}.$$

In this paper, we compute the neighborhood Kepler Banhatti index, modified neighborhood Kepler Banhatti index and their corresponding exponentials of certain families of dendrimers.

2. RESULYS FOR PAMAM DENDRIMER $PD_1[n]$

We consider the PAMAM dendrimers with n growth stages, denoted by $PD_1[n]$ for every n □□0, see Figure 1.

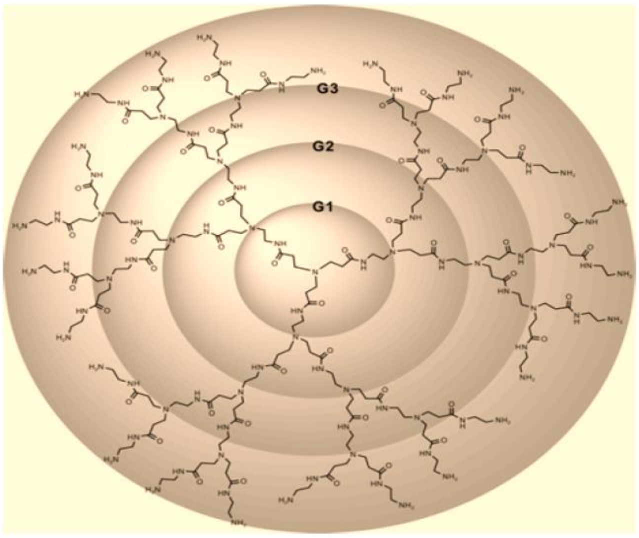

Figure 1. PAMAM dendrimer $PD_1[n]$

Let G be the graph of PAMAM dendrimer $PD_1[n]$. By calculation, we see that G has $12 \times 2^{n+2} - 23$ vertices and $12 \times 2^{n+2} - 24$ edges. Also the edge partition of the form (2,3), (3,4), (3,5), (4,5), (5,5), (5,6) for PAMAM dendrimer $PD_1[n]$ based on the degree sum of neighbors of end vertices of each edge is obtained, as given in Table 1.

Table 1

$S_G(u), S_G(v) \backslash uv$□$E(G)$	Number of edges
(2, 3)	3×2^n
(3, 4)	3×2^n
(3, 5)	$6 \times 2^n - 3$
(4, 5)	$9 \times 2^n - 6$
(5, 5)	$18 \times 2^n - 9$
(5, 6)	$9 \times 2^n - 6$

Theorem 1. The neighborhood Kepler Banhatti index of a PAMAM dendrimer $PD_1[n]$ is

$$NKB(G) = $$
$$= \left(459 + 3\sqrt{13} + 6\sqrt{34} + 9\sqrt{41} + 90\sqrt{2} + 6\sqrt{61}\right)2^n$$
$$-235 - 3\sqrt{34} - 6\sqrt{41} - 45\sqrt{2} - 6\sqrt{61}.$$

Proof: Let $G = PD_1[n]$. We have

$$NKB(G)$$
$$= \sum_{uv \in E(G)} \left(\left(S_G(u) + S_G(v)\right) + \sqrt{S_G(u)^2 + S_G(v)^2} \right)$$
$$= 3 \times 2^n \left((2+3) + \sqrt{2^2 + 3^2} \right) + 3 \times 2^n \left((3+4) + \sqrt{3^2 + 4^2} \right)$$
$$+ (6 \times 2^n - 3)\left((3+5) + \sqrt{3^2 + 5^2} \right)$$
$$+ (9 \times 2^n - 6)\left((4+5) + \sqrt{4^2 + 5^2} \right)$$
$$+ (18 \times 2^n - 9)\left((5+5) + \sqrt{5^2 + 5^2} \right)$$
$$+ (9 \times 2^n - 6)\left((5+6) + \sqrt{5^2 + 6^2} \right)$$
$$= \left(459 + 3\sqrt{13} + 6\sqrt{34} + 9\sqrt{41} + 90\sqrt{2} + 6\sqrt{61}\right)2^n$$
$$-235 - 3\sqrt{34} - 6\sqrt{41} - 45\sqrt{2} - 6\sqrt{61}.$$

Theorem 2. The neighborhood Kepler Banhatti exponential of a PAMAM dendrimer $PD_1[n]$ is

$$NKB(G,x) = 3 \times 2^n x^{5+\sqrt{13}} + 3 \times 2^n x^{12}$$
$$+ (6 \times 2^n - 3) x^{8+\sqrt{34}} + (9 \times 2^n - 6) x^{9+\sqrt{41}}$$
$$+ (18 \times 2^n - 9) x^{10+5\sqrt{2}} + (9 \times 2^n - 6) x^{11+\sqrt{61}}.$$

Proof: Let $G = PD_1[n]$. We have

$$NKB(G,x) = \sum_{uv \in E(G)} x^{\left(S_G(u) + S_G(v)\right) + \sqrt{S_G(u)^2 + S_G(v)^2}}$$
$$= 3 \times 2^n x^{(2+3) + \sqrt{2^2+3^2}} + 3 \times 2^n x^{(3+4) + \sqrt{3^2+4^2}}$$
$$+ (6 \times 2^n - 3) x^{(3+5) + \sqrt{3^2+5^2}} + (9 \times 2^n - 6) x^{(4+5) + \sqrt{4^2+5^2}}$$
$$+ (18 \times 2^n - 9) x^{(5+5) + \sqrt{5^2+5^2}} + (9 \times 2^n - 6) x^{(5+6) + \sqrt{5^2+6^2}}.$$

By simplifying the above equation, we obtain the desired result.

Theorem 3. The modified neighborhood Kepler Banhatti index of a PAMAM dendrimer $PD_1[n]$ is

$$^{m}NKB(G)$$
$$= \left(\frac{3}{5+\sqrt{13}} + \frac{3}{12} + \frac{6}{8+\sqrt{34}} + \frac{9}{9+\sqrt{41}} + \frac{18}{10+5\sqrt{2}} \right) 2^n$$
$$+ \left(\frac{9}{11+\sqrt{61}} \right) 2^n - \frac{3}{8+\sqrt{34}} - \frac{6}{9+\sqrt{41}} - \frac{9}{10+5\sqrt{2}}$$
$$- \frac{6}{11+\sqrt{61}}.$$

Proof: Let $G = PD_1[n]$. We have

$$^{m}NKB(G)$$

$$= \sum_{uv\in E(G)} \frac{1}{\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}}$$

$$= \frac{3\times 2^n}{(2+3)+\sqrt{2^2+3^2}} + \frac{3\times 2^n}{(3+4)+\sqrt{3^2+4^2}} + \frac{6\times 2^n-3}{(3+5)+\sqrt{3^2+5^2}} + \frac{9\times 2^n-6}{(4+5)+\sqrt{4^2+5^2}} + \frac{18\times 2^n-9}{(5+5)+\sqrt{5^2+5^2}} + \frac{9\times 2^n-6}{(5+6)+\sqrt{5^2+6^2}}.$$

By simplifying the above equation, we obtain the desired result.

Theorem 4. The modified neighborhood Kepler Banhatti exponential of a PAMAM dendrimer $PD_1[n]$ is

$$^{m}NKB(G)$$

$$= 3\times 2^n x^{\frac{1}{5+\sqrt{13}}} + 3\times 2^n x^{\frac{1}{12}} + \left(6\times 2^n-3\right)x^{\frac{1}{8+\sqrt{34}}} + \left(9\times 2^n-6\right)x^{\frac{1}{9+\sqrt{41}}} + \left(18\times 2^n-9\right)x^{\frac{1}{10+5\sqrt{2}}} + \left(9\times 2^n-6\right)x^{\frac{1}{11+\sqrt{61}}}.$$

Proof: Let $G = PD_1[n]$. We have

$$^{m}NKB(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}}}$$

$$= 3\times 2^n x^{\frac{1}{(2+3)+\sqrt{2^2+3^2}}} + 3\times 2^n x^{\frac{1}{(3+4)+\sqrt{3^2+4^2}}} + \left(6\times 2^n-3\right)x^{\frac{1}{(3+5)+\sqrt{3^2+5^2}}} + \left(9\times 2^n-6\right)x^{\frac{1}{(4+5)+\sqrt{4^2+5^2}}} + \left(18\times 2^n-9\right)x^{\frac{1}{(5+5)+\sqrt{5^2+5^2}}} + \left(9\times 2^n-6\right)x^{\frac{1}{(5+6)+\sqrt{5^2+6^2}}}.$$

By simplifying the above equation, we obtain the desired result.

3. RESULYS FOR POPAM DENDRIMERS $POD_2[n]$

In this section, we focus on the molecular graph of POPAM dendrimers. This family of dendrimers is denoted by $POD_2[n]$, where n is the steps of growth in this type of dendrimers. The molecular graph of $POD_2[2]$ is shown in Figure 2.

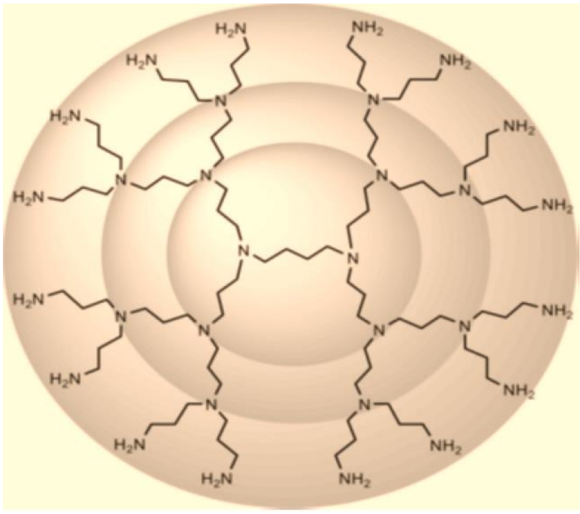

Figure 2. The molecular graph of $POD_2[n]$

Let G be the molecular graph of POPAM dendrimers $POD_2[n]$. By algebraic method, we obtain that $|V(POD_2[n])| = 2^{n+5} - 10$ and $|E(POD_2[n])| = 2^{n+5} - 11$. The edge partition of $POD_2[n]$ based on the degree sum of neighbors of end vertices of each edge is obtained as given in Table 2.

Table 2. Edge partition of $POD_2[n]$ based on $S_G(u)$ and $S_G(v)$

$S_G(u), S_G(v)\backslash uv\in E(G)$	Number of edges
(2, 3)	2^{n+2}
(3, 4)	2^{n+2}
(4, 4)	1
(4, 5)	$3\times 2^n - 6$
(5, 6)	$3\times 2^n - 6$

Theorem 5. The neighborhood Kepler Banhatti index of a POPAM dendrimer $POD_2[n]$ is

$$NKB(G)$$

$$= \left(12+\sqrt{13}\right)\times 2^{n+2} + 60\times 2^n + 8 + 4\sqrt{2} - 6\sqrt{41} - 6\sqrt{61}.$$

Proof: Let $G = POD_2[n]$. We have

$$NKB(G)$$

$$= \sum_{uv\in E(G)} \left(\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}\right)$$

$$= 2^{n+2}\left((2+3)+\sqrt{2^2+3^2}\right) + 2^{n+2}\left((3+4)+\sqrt{3^2+4^2}\right) + 1\left((4+4)+\sqrt{4^2+4^2}\right) + \left(3\times 2^n-6\right)\left((4+5)+\sqrt{4^2+5^2}\right)$$

$$+\left(3\times 2^{n}-6\right)\left((5+6)+\sqrt{5^{2}+6^{2}}\right)$$
$$=\left(12+\sqrt{13}\right)2^{n+2}+60\times 2^{n}+8+4\sqrt{2}-6\sqrt{41}-6\sqrt{61}.$$

Theorem 6. The neighborhood Kepler Banhatti exponential of a POPAM dendrimer $POD_2[n]$ is

$$NKB(G,x)=2^{n+2}x^{5+\sqrt{13}}+2^{n+2}x^{12}$$
$$+1x^{8+4\sqrt{2}}+\left(3\times 2^{n}-6\right)x^{9+\sqrt{41}}+\left(3\times 2^{n}-6\right)x^{11+\sqrt{61}}.$$

Proof: Let $G = POD_2[n]$. We have

$$NKB(G,x)=\sum_{uv\in E(G)}x^{\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}}$$
$$=2^{n+2}x^{(2+3)+\sqrt{2^2+3^2}}+2^{n+2}x^{(3+4)+\sqrt{3^2+4^2}}$$
$$+1x^{(4+4)+\sqrt{4^2+4^2}}+\left(3\times 2^{n}-6\right)x^{(4+5)+\sqrt{4^2+5^2}}$$
$$+\left(3\times 2^{n}-6\right)x^{(5+6)+\sqrt{5^2+6^2}}.$$

By simplifying the above equation, we obtain the desired result.

Theorem 7. The modified neighborhood Kepler Banhatti index of a POPAM dendrimer $POD_2[n]$ is

$${}^{m}NKB(G)$$
$$=\left(\frac{1}{5+\sqrt{13}}+\frac{1}{12}\right)2^{n+2}+\left(\frac{3}{9+\sqrt{41}}+\frac{3}{11+\sqrt{61}}\right)2^{n}$$
$$+\frac{1}{8+4\sqrt{2}}-\frac{6}{9+\sqrt{41}}-\frac{6}{11+\sqrt{61}}.$$

Proof: Let $G = PD_1[n]$. We have

$${}^{m}NKB(G)$$
$$=\sum_{uv\in E(G)}\frac{1}{\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}}$$
$$=\frac{2^{n+2}}{(2+3)+\sqrt{2^2+3^2}}+\frac{2^{n+2}}{(3+4)+\sqrt{3^2+4^2}}$$
$$+\frac{1}{(4+4)+\sqrt{4^2+4^2}}+\frac{3\times 2^{n}-6}{(4+5)+\sqrt{4^2+5^2}}$$
$$+\frac{3\times 2^{n}-6}{(5+6)+\sqrt{5^2+6^2}}.$$

By simplifying the above equation, we obtain the desired result.

Theorem 8. The modified neighborhood Kepler Banhatti exponential of a POPAM dendrimer $POD_2[n]$ is

$${}^{m}NKB(G)$$
$$=2^{n+2}x^{\frac{1}{5+\sqrt{13}}}+2^{n+2}x^{\frac{1}{12}}+1x^{\frac{1}{8+4\sqrt{2}}}$$
$$+\left(3\times 2^{n}-6\right)x^{\frac{1}{9+\sqrt{41}}}+\left(3\times 2^{n}-6\right)x^{\frac{1}{11+\sqrt{61}}}.$$

Proof: Let $G = PD_1[n]$. We have

$${}^{m}NKB(G,x)=\sum_{uv\in E(G)}x^{\frac{1}{\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}}}$$
$$=2^{n+2}x^{\frac{1}{(2+3)+\sqrt{2^2+3^2}}}+2^{n+2}x^{\frac{1}{(3+4)+\sqrt{3^2+4^2}}}$$
$$+1x^{\frac{1}{(4+4)+\sqrt{4^2+4^2}}}+\left(3\times 2^{n}-6\right)x^{\frac{1}{(4+5)+\sqrt{4^2+5^2}}}$$
$$+\left(3\times 2^{n}-6\right)x^{\frac{1}{(5+6)+\sqrt{5^2+6^2}}}.$$

By simplifying the above equation, we obtain the desired result.

4. RESULTS FOR TETRATHIAFULVALENE DENDRIMERS $TD_2[n]$

In this section, we focus on the molecular graph of a tetrathiafulvalene dendrimer. This family of tetrathiafulvalene dendrimers is denoted by $TD_2[n]$, where n is the steps of growth in this type of dendrimers for n□0. The molecular graph of $TD_2[2]$ is shown in Figure 3.

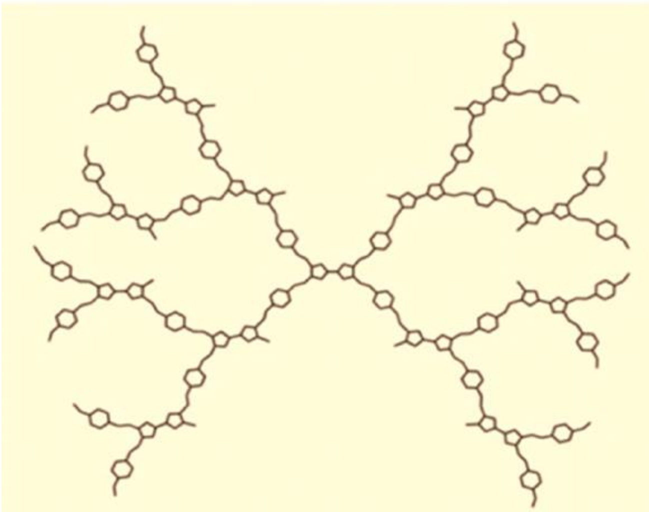

Figure 3. The molecular graph of $TD_2[2]$

Let G be the molecular graph of tetrathiafulvalene dendrimers $TD_2[n]$. By algebraic method, we obtain that $|V(G)|=31\times 2^{n+2}-74$ and $|E(G)|=35\times 2^{n+2}-85$. Also the edge partition of $TD_2[n]$ based on the degree sum of neighbors of end vertices of each edge is obtained as given in Table 3

Table 3. Edge partition of $TD_2[n]$ based on $S_G(u)$ and $S_G(v)$

$S_G(u)$, $S_G(v)$\uv□$E(G)$	Number of edges
(2, 4)	2^{n+2}

(3, 6)	$2^{n+2} - 4$
(4, 6)	2^{n+2}
(5, 5)	$7\times 2^{n+2} - 16$
(5, 6)	$11\times 2^{n+2} - 24$
(5, 7)	$3\times 2^{n+2} - 8$
(6, 6)	$2^{n+2} - 4$
(6, 7)	$8\times 2^{n+2} - 24$
(7, 7)	$2\times 2^{n+2} - 5$

Theorem 9. The neighborhood Kepler Banhatti index of a dendrimer $TD_2[n]$ is

$$NKB(G) = \left(132+5\sqrt{5}+55\sqrt{2}+2\sqrt{13}+11\sqrt{61}+3\sqrt{74}+8\sqrt{85}\right)2^{n+2} - 986-12\sqrt{5}-139\sqrt{2}-24\sqrt{61}-8\sqrt{74}-24\sqrt{85}.$$

Proof: Let $G = TD_2[n]$. We have

$$\begin{aligned}NKB(G) &= \sum_{uv\in E(G)}\left(\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}\right)\\
&= 2^{n+2}\left((2+4)+\sqrt{2^2+4^2}\right)\\
&+\left(2^{n+2}-4\right)\left((3+6)+\sqrt{3^2+6^2}\right)\\
&+2^{n+2}\left((4+6)+\sqrt{4^2+6^2}\right)\\
&+\left(7\times 2^{n+2}-16\right)\left((5+5)+\sqrt{5^2+5^2}\right)\\
&+\left(11\times 2^{n+2}-24\right)\left((5+6)+\sqrt{5^2+6^2}\right)\\
&+\left(3\times 2^{n+2}-8\right)\left((5+7)+\sqrt{5^2+7^2}\right)\\
&+\left(2^{n+2}-4\right)\left((6+6)+\sqrt{6^2+6^2}\right)\\
&+\left(8\times 2^{n+2}-24\right)\left((6+7)+\sqrt{6^2+7^2}\right)\\
&+\left(2\times 2^{n+2}-5\right)\left((7+7)+\sqrt{7^2+7^2}\right)\end{aligned}$$

By simplifying the above equation, we obtain the desired result.

Theorem 10. The neighborhood Kepler Banhatti exponential of a dendrimer $TD_2[n]$ is

$$\begin{aligned}NKB(G,x) &= 2^{n+2}x^{6+2\sqrt{5}}+\left(2^{n+2}-4\right)x^{9+3\sqrt{5}}\\
&+2^{n+2}x^{10+2\sqrt{13}}+\left(7\times 2^{n+2}-16\right)x^{10+5\sqrt{2}}\\
&+\left(11\times 2^{n+2}-24\right)x^{11+\sqrt{61}}+\left(3\times 2^{n+2}-8\right)x^{12+\sqrt{74}}\\
&+\left(2^{n+2}-4\right)x^{12+6\sqrt{2}}+\left(8\times 2^{n+2}-24\right)x^{13+\sqrt{85}}\\
&+\left(2\times 2^{n+2}-5\right)x^{14+7\sqrt{2}}.\end{aligned}$$

Proof: Let $G = TD_2[n]$. We have

$$\begin{aligned}NKB(G,x) &= \sum_{uv\in E(G)} x^{\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}}\\
&= 2^{n+2}x^{(2+4)+\sqrt{2^2+4^2}}+\left(2^{n+2}-4\right)x^{(3+6)+\sqrt{3^2+6^2}}\\
&+2^{n+2}x^{(4+6)+\sqrt{4^2+6^2}}+\left(7\times 2^{n+2}-16\right)x^{(5+5)+\sqrt{5^2+5^2}}\\
&+\left(11\times 2^{n+2}-24\right)x^{(5+6)+\sqrt{5^2+6^2}}\\
&+\left(3\times 2^{n+2}-8\right)x^{(5+7)+\sqrt{5^2+7^2}}+\left(2^{n+2}-4\right)x^{(6+6)+\sqrt{6^2+6^2}}\\
&+\left(8\times 2^{n+2}-24\right)x^{(6+7)+\sqrt{6^2+7^2}}\\
&+\left(2\times 2^{n+2}-5\right)x^{(7+7)+\sqrt{7^2+7^2}}.\end{aligned}$$

By simplifying the above equation, we obtain the desired result.

Theorem 11. The modified neighborhood Kepler Banhatti index of a dendrimer $TD_2[n]$ is

$$\begin{aligned}{}^{m}NKB(G) &= \left(\frac{1}{6+2\sqrt{5}}+\frac{1}{9+3\sqrt{5}}+\frac{1}{10+2\sqrt{13}}+\frac{7}{10+5\sqrt{2}}\right)2^{n+2}\\
&+\left(\frac{11}{11+\sqrt{61}}+\frac{3}{12+\sqrt{74}}+\frac{1}{12+6\sqrt{2}}+\frac{8}{13+\sqrt{85}}\right)2^{n+2}\\
&+\left(\frac{2}{14+7\sqrt{2}}\right)2^{n+2}-\frac{4}{9+3\sqrt{5}}-\frac{16}{10+5\sqrt{2}}-\frac{24}{11+\sqrt{61}}\\
&-\frac{8}{12+\sqrt{74}}-\frac{4}{12+6\sqrt{2}}-\frac{24}{13+\sqrt{85}}-\frac{5}{14+7\sqrt{2}}.\end{aligned}$$

Proof: Let $G = TD_2[n]$. We have

$$\begin{aligned}{}^{m}NKB(G) &= \sum_{uv\in E(G)}\frac{1}{\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}}\\
&= \frac{2^{n+2}}{(2+4)+\sqrt{2^2+4^2}}+\frac{2^{n+2}-4}{(3+6)+\sqrt{3^2+6^2}}\\
&+\frac{2^{n+2}}{(4+6)+\sqrt{4^2+6^2}}+\frac{7\times 2^{n+2}-16}{(5+5)+\sqrt{5^2+5^2}}\\
&+\frac{11\times 2^{n+2}-24}{(5+6)+\sqrt{5^2+6^2}}+\frac{3\times 2^{n+2}-8}{(5+7)+\sqrt{5^2+7^2}}\\
&+\frac{2^{n+2}-4}{(6+6)+\sqrt{6^2+6^2}}+\frac{8\times 2^{n+2}-24}{(6+7)+\sqrt{6^2+7^2}}\\
&+\frac{2\times 2^{n+2}-5}{(7+7)+\sqrt{7^2+7^2}}\end{aligned}$$

By simplifying the above equation, we obtain the desired result.

Theorem 12. The modified neighborhood Kepler Banhatti exponential of a dendrimer $TD_2[n]$ is

$$^{m}NKB(G)$$

$$= 2^{n+2}x^{\frac{1}{6+2\sqrt{5}}} + \left(2^{n+2}-4\right)2^{n+2}x^{\frac{1}{9+3\sqrt{5}}} + 2^{n+2}x^{\frac{1}{10+2\sqrt{13}}}$$

$$+\left(7\times 2^{n+2}-16\right)x^{\frac{1}{10+5\sqrt{2}}} + \left(11\times 2^{n+2}-24\right)x^{\frac{1}{11+\sqrt{61}}}$$

$$+\left(3\times 2^{n+2}-8\right)x^{\frac{1}{12+\sqrt{74}}} + \left(2^{n+2}-4\right)x^{\frac{1}{12+6\sqrt{2}}}$$

$$+\left(8\times 2^{n+2}-24\right)x^{\frac{1}{13+\sqrt{85}}} + \left(2\times 2^{n+2}-5\right)x^{\frac{1}{14+7\sqrt{2}}}.$$

Proof: Let $G = TD_2[n]$. We have

$$^{m}NKB(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}}}$$

$$= 2^{n+2}x^{\frac{1}{(2+4)+\sqrt{2^2+4^2}}} + \left(2^{n+2}-4\right)2^{n+2}x^{\frac{1}{(3+6)+\sqrt{3^2+6^2}}}$$

$$+2^{n+2}x^{\frac{1}{(4+6)+\sqrt{4^2+6^2}}} + \left(7\times 2^{n+2}-16\right)x^{\frac{1}{(5+5)+\sqrt{5^2+5^2}}}$$

$$+\left(11\times 2^{n+2}-24\right)x^{\frac{1}{(5+6)+\sqrt{5^2+6^2}}} + \left(3\times 2^{n+2}-8\right)x^{\frac{1}{(5+7)+\sqrt{5^2+7^2}}}$$

$$+\left(2^{n+2}-4\right)x^{\frac{1}{(6+6)+\sqrt{6^2+6^2}}} + \left(8\times 2^{n+2}-24\right)x^{\frac{1}{(6+7)+\sqrt{6^2+7^2}}}$$

$$+\left(2\times 2^{n+2}-5\right)x^{\frac{1}{(7+7)+\sqrt{7^2+7^2}}}.$$

By simplifying the above equation, we obtain the desired result.

5. RESULTS FOR $NS_2[n]$ DENDRIMERS

In this section, we focus on the class of $NS_2[n]$ dendrimers with $n \geq 1$. The graph of $NS_2[2]$ is shown in Figure 4.

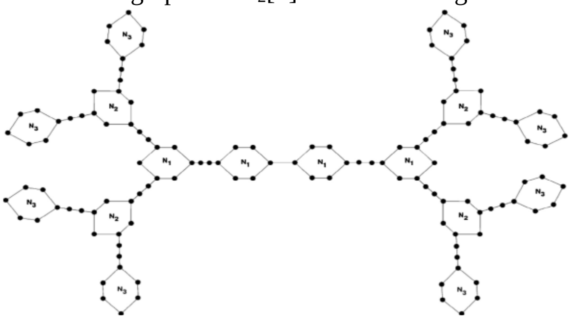

Figure 4. The graph of $NS_2[2]$

Let G be the graph of $NS_2[n]$ dendrimers. By algebraic method, we obtain that $|V(G)|=16\times 2^n - 4$ and $|E(G)|=18\times 2^n - 5$. Also the edge partition of $NS_2[n]$ based on the degree sum of neighbors of end vertices of each edge is obtained as given in Table 4.

Table 4. Edge partition of $NS_2[n]$ based on $S_G(u)$ and $S_G(v)$

$S_G(u), S_G(v)\backslash uv\in E(G)$	Number of edges
(4, 4)	2×2^n
(5, 4)	2×2^n
(5, 5)	$2\times 2^n+2$
(5, 6)	6×2^n
(7, 7)	1
(5, 7)	4
(6, 6)	$6\times 2^n - 12$

Theorem 13. The neighborhood Kepler Banhatti index of a dendrimer $NS_2[n]$ is

$$NKB(G) =$$

$$=\left(192+54\sqrt{2}+2\sqrt{41}+6\sqrt{61}\right)2^n - 69 - 55\sqrt{2} + 4\sqrt{74}.$$

Proof: Let $G = NS_2[n]$. We have

$$NKB(G)$$

$$= \sum_{uv\in E(G)} \left(\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}\right)$$

$$= 2\times 2^n\left((4+4)+\sqrt{4^2+4^2}\right) + 2\times 2^n\left((5+4)+\sqrt{5^2+4^2}\right)$$

$$+\left(2\times 2^n+2\right)\left((5+5)+\sqrt{5^2+5^2}\right)$$

$$+6\times 2^n\left((5+6)+\sqrt{5^2+6^2}\right)$$

$$+1\left((7+7)+\sqrt{7^2+7^2}\right)+4\left((5+7)+\sqrt{5^2+7^2}\right)$$

$$+\left(6\times 2^n-12\right)\left((6+6)+\sqrt{6^2+6^2}\right)$$

By simplifying the above equation, we obtain the desired result

Theorem 14. The neighborhood Kepler Banhatti exponential of a dendrimer $NS_2[n]$ is

$$NKB(G,x) = 2\times 2^n x^{8+4\sqrt{2}} + 2\times 2^n x^{9+\sqrt{41}}$$

$$+\left(2\times 2^n+2\right)x^{10+5\sqrt{2}} + 6\times 2^n x^{11+\sqrt{61}}$$

$$+1x^{14+7\sqrt{2}} + 4x^{12+\sqrt{74}} + \left(6\times 2^n-12\right)x^{12+6\sqrt{2}}$$

Proof: Let $G = NS_2[n]$. We have

$$NKB(G,x) = \sum_{uv\in E(G)} x^{\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}}$$

$$= 2\times 2^n x^{(4+4)+\sqrt{4^2+4^2}} + 2\times 2^n x^{(5+4)+\sqrt{5^2+4^2}}$$

$$+\left(2\times 2^n+2\right)x^{(5+5)+\sqrt{5^2+5^2}} + 6\times 2^n x^{(5+6)+\sqrt{5^2+6^2}}$$

$$+1x^{(7+7)+\sqrt{7^2+7^2}} + 4x^{(5+7)+\sqrt{5^2+7^2}}$$

$$+\left(6\times 2^n-12\right)x^{(6+6)+\sqrt{6^2+6^2}}$$

By simplifying the above equation, we obtain the desired result

Theorem 15. The modified neighborhood Kepler Banhatti index of a POPAM dendrimer $POD_2[n]$ is

$$^{m}NKB(G) = \left(\frac{1}{4+2\sqrt{2}}+\frac{2}{9+\sqrt{41}}+\frac{2}{10+5\sqrt{2}}+\frac{6}{11+\sqrt{61}}\right)2^n + \left(\frac{1}{2+\sqrt{2}}\right)2^n+\frac{1}{7+7\sqrt{2}}+\frac{4}{12+\sqrt{74}}-\frac{2}{2+\sqrt{2}}.$$

Proof: Let $G = NS_2[n]$. We have

$$^{m}NKB(G) = \sum_{uv\in E(G)}\frac{1}{\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}}$$

$$=\frac{2\times 2^n}{(4+4)+\sqrt{4^2+4^2}}+\frac{2\times 2^n}{(5+4)+\sqrt{5^2+4^2}} +\frac{2\times 2^n+2}{(5+5)+\sqrt{5^2+5^2}}+\frac{6\times 2^n}{(5+6)+\sqrt{5^2+6^2}} +\frac{1}{(7+7)+\sqrt{7^2+7^2}}+\frac{4}{(5+7)+\sqrt{5^2+7^2}} +\frac{6\times 2^n-12}{(6+6)+\sqrt{6^2+6^2}}.$$

By simplifying the above equation, we obtain the desired result.

Theorem 16. The modified neighborhood Kepler Banhatti exponential of a POPAM dendrimer $POD_2[n]$ is

$$^{m}NKB(G) = 2\times 2^n x^{\frac{1}{8+4\sqrt{2}}}+2\times 2^n x^{\frac{1}{9+\sqrt{41}}}+\left(2\times 2^n+2\right)x^{\frac{1}{10+5\sqrt{2}}} +\left(2\times 2^n+2\right)x^{\frac{1}{10+5\sqrt{2}}}+6\times 2^n x^{\frac{1}{11+\sqrt{61}}}+1x^{\frac{1}{14+7\sqrt{2}}} +4x^{\frac{1}{12+\sqrt{74}}}+\left(6\times 2^n-12\right)x^{\frac{1}{12+6\sqrt{2}}}.$$

Proof: Let $G = NS_2[n]$. We have

$$^{m}NKB(G,x) = \sum_{uv\in E(G)} x^{\frac{1}{\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}}}$$

$$=2\times 2^n x^{\frac{1}{(4+4)+\sqrt{4^2+4^2}}}+2\times 2^n x^{\frac{1}{(5+4)+\sqrt{5^2+4^2}}} +\left(2\times 2^n+2\right)x^{\frac{1}{(5+5)+\sqrt{5^2+5^2}}}+6\times 2^n x^{\frac{1}{(5+6)+\sqrt{5^2+6^2}}} +1x^{\frac{1}{(7+7)+\sqrt{7^2+7^2}}}+4x^{\frac{1}{(5+7)+\sqrt{5^2+7^2}}} +\left(6\times 2^n-12\right)x^{\frac{1}{(6+6)+\sqrt{6^2+6^2}}}.$$

By simplifying the above equation, we obtain the desired result.

6. PROPERTIES OF THE TEMPERATURE ELLIPTIC SOMBOR INDEX

Theorem 17. Let G be a connected graph. Then

$$\sqrt{2}NM_1(G)\le TNKB(G)<2NM_1(G)..$$

Proof: For any two positive numbers a and b,

$$\frac{1}{\sqrt{2}}(a+b)\le\sqrt{a^2+b^2}<a+b.$$

$$\sqrt{2}(a+b)\le(a+b)+\sqrt{a^2+b^2}<2(a+b)$$

For $a=S_G(u)$ and $b=S_G(v)$, the above inequality becomes

$$\sqrt{2}\left(S_G(u)+S_G(v)\right) \le\left(S_G(u)+S_G(v)\right)+\sqrt{\left(S_G(u)^2+S_G(v)^2\right)} <2\left(S_G(u)+S_G(v)\right)$$

By the definitions, we have

$$\sqrt{2}\sum_{uv\in E(G)}\left(S_G(u)+S_G(v)\right) \le\sum_{uv\in E(G)}\left(\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}\right) <2\sum_{uv\in E(G)}\left(S_G(u)+S_G(v)\right)$$

Thus we get the desired result.

Theorem 18. Let G be a connected graph. Then

$$NKB(G)=NM_1(G)+NSO(G).$$

Proof: We have

$$\sum_{uv\in E(G)}\left(\left(S_G(u)+S_G(v)\right)+\sqrt{S_G(u)^2+S_G(v)^2}\right) =\sum_{uv\in E(G)}\left(S_G(u)+S_G(v)\right)+\sum_{uv\in E(G)}\sqrt{S_G(u)^2+S_G(v)^2}. =NM_1(G)+NSO(G)$$

.

7. CONCLUSION

In this study, we have introduced the neighborhood Kepler Banhatti index, modified neighborhood Kepler Banhatti index of a graph. We have computed these indices and their corresponding exponentials of certain families of dendrimers.

REFERENCES

1. V.R.Kulli, College Graph Theory, Vishwa International Publications, Gulbarga, India (2012).
2. V.R.Kulli, Graph indices, in *Hand Book of Research on Advanced Applications of Application*

Graph Theory in Modern Society, M. Pal. S. Samanta and A. Pal, (eds) IGI Global, USA (2019) 66-91.

3. I.Gutman and N.Trinajstic, Graph Theory and Molecular Orbitals. Total phi-electron energy of alternant hydrocarbons, *Chemical Physics Letters*, 17(4) (1972) 535-538.
4. M.Aruvi, J.M.Joseph and E.Ramganesh, The second Gourava index of some graph products, *Advances in Mathematics: Scientific Journal*, 9(12) (2020) 10241-10249.
5. V.R.Kulli, Gourava domination indices of graphs, *International Journal of Mathematics and Computer Research*, 11(8) (2023) 3680-3684. DOI:10.47191/ijmcr/v11i8.08.
6. V.R.Kulli, On hyper Gourava domination indices, *International Journal of Engineering Sciences & Research Technology*, 12(10) (2023) 12-20. DOI: 10.5281/zenodo.10036777.
7. I.Gutman and V.R.Kulli, Estimating the Gourava Sombor index, *Ser. A: Appl. Math. Inform. And Mech.* (2024).
8. V.R.Kulli, On alpha and gamma Gourava indices, *International Journal of Mathematics and Computer Research*, 12(4) (2024) 4139-4144. DOI:10.47191/ijmcr/v12i4.04.
9. V.R.Kulli, Nirmala alpha Gourava and modified Nirmala alpha Gourava indices of certain dendrimers, *International Journal of Mathematics and Computer Research*, 12(5) (2024) 4256-4263.
10. R.P.Somani and V.Jethwani, Introducing new exponential Gourava indices for graphs, *International Journal of Mathematics and Statistics Invention*, 11(3) (2023) 1-5.
11. V.R.Kulli, Reverse Nirmala index, *International Journal of Engineering Sciences & Research Technology, 1*1(8) (2022) 12-19. DOI: 10.5281/zenodo.7079293.
12. V.R.Kulli, Multiplicative Nirmala and Banhatti-Nirmala indices of certain nanostar dendrimers, *International Journal of Mathematical Archive*, 13(10) (2022) 8-15.
13. B.K.Majhi, V.R.Kulli ad I.N.Cangul, Nirmala leap indices of some chemical drugs against COVID-19, *Journal of Discrete Mathematics and its Applications*, 10(1) (2023) 13-21. DOI: 10.22061/JDMA.2023.10093.1058.
14. S.Raghu, V.R.Kulli, K.M.Niranjan and V.M.Goudar, Nirmala indices of certain antiviral drugs, *International Journal of Mathematics and Computer Research*, 11(11) (2023) 3862-3866.
15. V.R.Kulli, I.Gutman, B.Furtula and I.Redzepovic, Sombor, Nirmala Dhaewad and F-Sombor indices: A Comparative Study, *SSRG International Journal of Applied Chemistry*, 10(2) (2023) 7-10.
16. V.R.Kulli, Domination Nirmala indices of graphs, *International Journal of Mathematics and Computer Research*, 11(6) (2023) 3497--3502.
17. V.R.Kulli, *ve*-Degree Sombor indices of certain networks, *International Journal of Mathematics Trends and Technology*, 68(9) (2022) 5-10. https://doi.org/10.14445/22315373/IJMTT-V68I9P502.
18. V.R.Kulli, Multiplicative KG Sombor indices of some networks, *International Journal of Mathematics Trends and Technology*, 68(10) (2022) 1-7.
19. I.Gutman, V.R.Kulli and I.Redzepovic, Irregularity Sombor, index, *Bull. Acad. Serbe. Sci. Arts (Cl. Sci. Math. Natur.)* 156 (2023) 31-37.
20. V.R.Kulli, Modified elliptic Sombor index and its exponential of a graph, *International Journal of Mathematics and Computer Research*,12 (1)(2024)3949-3954.
21. V.R.Kulli, Reverse elliptic Sombor and modified reverse elliptic Sombor indices , *International Journal of Mathematical Archive*, 15(1) (2024) 1-7.
22. I.Gutman, Note on the temperature Sombor index, *Vijnotehancki Glasnik/ Military Technical courier*, 71(3) (2023) 507-515..
23. I.Gutman, B.Furtula and M.S.Oz, Geometric approach to vertex degree based topological indices-Elliptic Sombor index theory and application, *International Journal of Quantum Chemistry*, 124(2) (2024) e27151.
24. V.R.Kulli, Multiplicative elliptic Sombor and multiplicative modified elliptic Sombor indices, *Annals of Pure and Applied Mathematics*, 29(1) (2024) 19-23.
25. V.R.Kulli, Status elliptic Sombor and modified status elliptic Sombor indices of graphs, *Journal of Mathematics and Informatics*, 27 (2024) 49-54.
26. R.Aguilar-Sanchez,I.F.Herrera-Gonzalez, J.A.Mendez-Bermudez and J.M.Sigaareta, Revan degree indices on random graphs, arXiv: 2210.04749v1 [math. CO] 10 Oct. 2022.
27. M.Kamran, S.Delen, R.H.Khan, N.Salamat, A.Q.Baig, I.N.Cangul and Md.A.alam, Physico-Chemical Characterization of amylase and amylopectin using Revan topological indices, *Journal of Mathematics*, vol.2022, 12 pages, Article ID 2840217, 2022.
28. V.R.Kulli, Elliptic Revan index and its exponential of certain networks, *International Journal of Mathematics and Computer Research*, 12(2) (2024) 4055-4061.
29. V.R.Kulli, Modified elliptic Revan index of certain two families of nanotubes, *Annals of Pure and Applied Mathematics*, 29(2) (2024) 103-107.

30. B.K.Majhi, V.R.Kulli and I.N.Cangul, Revan indices and their polynomials of square snake graphs, *Montes Taurus J. Pure Appl. Math.* 6(1) (2024) 12-17.
31. V.R.Kulli, Revan Kepler Banhatti and modified Revan Kepler Banhatti indices of certain nanotubes, *Annals of Pure and Applied Mathematics*, 30(2), (2024) 129-136.
32. A.Mahboob, G.Muhiuddin, I.Siddique and S.M.Alam, A view of Banhatti and Revan indices in chemical graphs, *Journal of Mathematics,* vol.2022, 8 pages, Article ID 5680712, 2022.
33. D.Narasmhan, R.Vignesh and K.Desikan, Results on Revan and hyper Revan indices of some HEX derived networks, *Fuzzy Mathematical Analysis and Advances in Computational Mathematics,* Springer Singapore (2022) 209-220.
34. V.R.Kulli, E-Banhatti Sombor indices, *International Journal of Mathematics and Computer Research*, 10(12) (2022) 2986-2994. DOI: https://doi.org/10.47191/ijmcr/v10i12.01.
35. V.R.Kulli, The (a, b)-KA E-Banhatti indices of graphs, *Journal of Mathematics and Informatics,* 23 (2022) 55-60.
DOI: http://dx.doi.org/10.22457/jmi.v23a06214.
36. V.R.Kulli, FE-Banhatti index of certain nanostructures, *International Journal of Mathematical Archive*, 14(1) (2023) 21-29.
37. V.R.Kulli, K.M.Niranjan, V.M.Goudar and S.Raghu, E-Banhatti indices of chain silicate networks, *Commun. Combin. Cryptogr. & Compuer Sci.* 2 (2024) 107-118
38. V.R.Kulli, K.M.Niranjan, V.M.Goudar and S.Raghu, Kepler E-Banhatti and modified Kepler E-Banhatti indices, *Commun. Combin. Cryptogr. & Compuer Sci.* 2 (2024) 220-233.
39. V.R.Kulli, Harmonic E-Banhatti index of graphs, *Annals of Pure and Applied Mathematics,* 30(2) (2024) 79-85.
DOI:http://dx.doi.org/10.22457/apam.v30n2a01950
40. S.Wagner and H.Wang, *Introduction Chemical Graph Theory,* Boca Raton, CRC Press, (2018).
41. M.V.Diudea (ed.) *QSPR/QSAR Studies by Molecular Descriptors,* NOVA New York, (2001).
42. V.R.Kulli, Kepler Banhatti and modified Kepler Banhatti indices, *International Journal of Mathematics and Computer Research,* 12(6) (2024) 4310-4314. DOI: 10-47191/ijmcr/v12i6.08.
43. V.R.Kulli, Reverse Kepler Banhatti and modified reverse Kepler Banhatti indices, *International Journal of Mathematical Archive,* 15(7) (2024) 1-8.
44. V.R.Kulli, Revan Kepler Banhatti and modified Revan Kepler Banhatti indices of certain nanotubes, *Annals of Pure and Applied Mathematics*, 30(2), (2024) 129-136.
45. V.R.Kulli, Domination Kepler Banhatti and modified domination Kepler Banhatti indices of graphs, *Annals of Pure and Applied Mathematics*, 31(1), (2025) 23-29.
46. V.R.Kulli, Delta Kepler Banhatti indices of certain nanostructures, *International Journal of Mathematical Archive,* 16 (2025) to appear.
47. V.R.Kulli, F1-neighborhood and square neighborhood Dakshayani indices of some nanostructures, *International Journal of Engineering Sciences & Research Technology,* 8(8) (2019) 126-138.
48. V.R.Kulli, Computation of distance based connectivity status neighborhood Dakshayani indices, *International Journal of Mathematics Trends and Technology,* 66(6) (2020) 118-128.
49. V.R.Kulli, Multiplicative status neighborhood indices of graphs, *International Journal of Engineering Sciences & Research Technology,* 9(6) (2020) 110-120.
50. V.R.Kulli, Some multiplicative neighborhood topological indices of nanocones and dendrimers, *International Journal of Engineering Sciences & Research Technology*, 9(10) (2020) 119-132.
51. V.R.Kulli, Degree based neighborhood indices of some nanostructures, *SSRG International Journal of Applied Chemistry*, 7(3) (2020) 17-24..
52. V.R.Kulli, Multiplicative connectivity status neighborhood indices of graphs, *International Journal of Engineering Sciences & Research Technology*, 9(12) (2020) 59-68.
53. V.R,Kulli, Neighborhood sum atom bond connectivity indices of some nanostar dendrimers, *International Journal of Mathematics and Computer Research,*11(1) (2023) 3230-3235.
54. Graovac, M. Ghorbani and M.A. Hosseinzadeh, Computing fifth geometric-arithmetic index of nanostar dendrimers, *Journal of Mathematical Nanoscience*, 1(1) (2011) 33-42.
55. V.R.Kulli, Neighborhood Sombor index of some nanostructures, *International Journal of Mathematics Trends and Technology*, 67(5) (2021) 101-108.
https://doi.org/10.22147/22315373/IJMTT.V67I5P512.