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The Unsteady MHD Free Convective Two Immiscible Fluid Flows in a Horizontal Channel with Heat and Mass Transfer

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Abstract
The unsteady MHD free convective two immiscible fluids flowing in a horizontal channel with heat and mass transfer, with the assumptions that the upper channel and lower channel are porous and non-porous respectively have been studied. The governing equations of the flow were transformed to ordinary differential equations by a regular perturbation method, and the expression for the velocity, temperature and concentration for each fluid flow were obtained. The effects of various governing parameters like Grashof numbers for heat and mass transfer, Prandtl number, Viscosity ratio, conductivity ratio, Radiative parameter, Schmidt number etc. on the velocity, temperature and concentration fields have been presented graphically and discussed quantitatively. Also, the coefficient of skin friction, Nusselt number and Sherwood number have been calculated and tabulated.

Keywords: MHD, Unsteady flow, Mass transfer, Radiation, Immiscible fluid.

List of Symbols

U, V	Velocity components
t	Time
P	Pressure
B_0	Coefficient of electromagnetic field
F	Thermal Radiation parameter
θ	Dimensionless temperature
K	Permeability of porous medium
ϑ	Kinematic viscosity
μ	Fluid viscosity
A	Real positive constant
g	Acceleration due to gravity
C_p	Specific heat at constant pressure
T_{w1}	Fluid temperature at upper wall
T_{w2}	Fluid temperature at lower wall
C_{w1}	Fluid concentration at upper wall
C_{w2}	Fluid concentration at lower wall
Gr	Grashof number
Re	Reynolds number
M^2	Hartmann number
Pr	Prandtl number
Sc	Schmidt number
α	Ratio of viscosity
β	Ratio of thermal conductivity
γ	Ratio of thermal diffusivity
ω	Frequency parameter
ε	Coefficient of periodic parameter
ωt	Periodic frequency parameter

Subscripts 1, 2: Region I & II Respectively.

INTRODUCTION

Magneto hydrodynamic (MHD) is the science of motion of electrically conducting fluid in presence of magnetic field. Some of these fluids include liquid metals (such as mercury, molten iron) and ionized gases known by Physicist as Plasma, an example being the solar atmosphere. The dynamo and motor is a classical example of MHD principle. The unsteady Magnetohy drodynamics (MHD) free convective flows in a horizontal channel have over the years been subjected to numerous studies. These scientific investigations are based on the fact that the studies of such flows have numerous applications in different fields that are scientifically motivated. These applications include MHD power generators, MHD pumps, liquid metal cooling of reactors, Magnetic drug targeting etc. Several Scholars and Authors have contributed their quota since the study of MHD was first initiated by the Swedish electrical engineer Hannes Alfven (1942). Shercliff (1956), Sparrow and Cess (1961), Singh and Ram (1978), Abdulla (1986), Singh (1993) among others have studied several motions of these electrically conducting fluids.

Such flows with heat and mass transfer have a wide variety of applications in engineering and geophysical processes such as geothermal reservoirs, underground energy transport, enhanced oil recovery, packet-bed reactors, etc. Among several studies, Chamkha (2003) studied the Unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Seethamahalakshmi et al (2011) studied the unsteady MHD free convection flow and mass transfer near a moving vertical plate in the presence of thermal radiation. Kumar and Jain (2013) investigated the influence of mass transfer and thermal radiation on unsteady free convective flow through porous media sandwiched between viscous fluids. Joseph et al (2014) then studied the slip effect on MHD oscillatory flow of fluid in a porous medium with heat and mass transfer and chemical reaction.

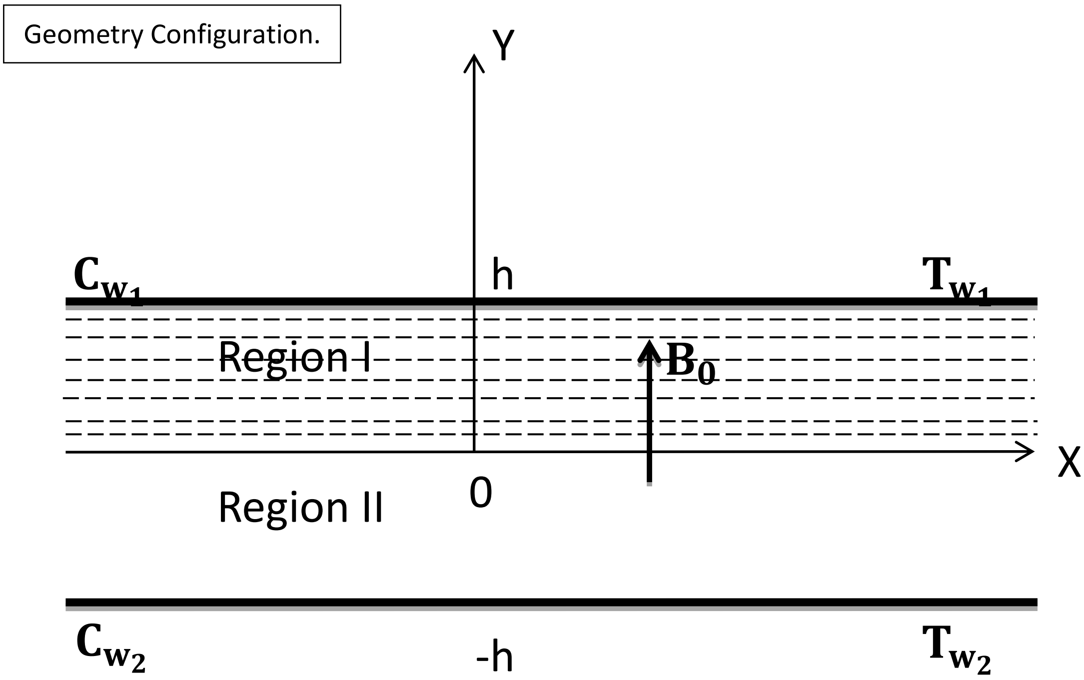
In the petroleum industry, plasma physics, magneto-fluid dynamics, most of the flows that occur are multi fluid flows. When these fluids are immiscible, the significance of their study becomes exceptional in petroleum extraction and transport. Hence among other scholars and Authors, Chamkha (2000) considered the flow of two-immiscible fluids in porous and non-porous channels. Chamkha et al (2004) investigated the oscillatory flow and heat transfer of two immiscible fluids. Umavathi et al (2008) investigated the unsteady magnetohydrodynamic two fluid flows and heat transfer in a horizontal channel. Umavathi et al (2010) also considered the unsteady flow and heat transfer of porous media sandwiched between viscous fluids. Sivaraj (2012) analyzed the MHD mixed convective flow of viscoelastic and viscous fluids in a vertical porous channel. Simon and Shagaiya (2013) studied the convective flow of two immiscible fluids and heat transfer with porous along an inclined channel with pressure gradient.

The aim of this present study is to investigate the effect of unsteady MHD free convective flow of two immiscible fluids, where these two immiscible fluids flow in a horizontal channel with heat and mass transfer.

FORMULATION OF THE PROBLEM

The geometry considered here consists of two immiscible fluids having specific heat at constant pressure C_p with porous upper channel and non-porous lower channel bounded by

two infinite horizontal parallel plates extending in the X- and Z- directions, with the Y-direction normal to the plates. The regions $0 \le y \le h$ and $-h \le y \le 0$ are denoted as Region-I and Region-II respectively. The fluid flowing through Region-I is having density ρ_1, dynamic viscosity μ_1, thermal conductivity k_1, thermal diffusivity D_1, while that flowing through Region-II is having density ρ_2, dynamic viscosity μ_2, thermal conductivity k_2, thermal diffusivity D_2.
All the variables are functions of y' and t' only, due to the bounding surface being infinitely long along the x'-axis. The flow is assumed to be fully developed and that all fluid properties are constants. The magnetic field Reynolds number is assumed very small. Hence the governing equations of the fluid flow for the two different regions are:



REGION-I:
Porous Region

$$\frac{\partial V_1'}{\partial y'} = 0 \tag{1}$$

$$\rho_1\left(\frac{\partial U_1'}{\partial t'} + V_1'\frac{\partial U_1'}{\partial y'}\right) = \mu_1\frac{\partial^2 U_1'}{\partial y'^2} - \frac{\partial P'}{\partial x'} - \sigma B_0^2 U_1' + \rho_1 g\beta_{f_1}\left(T_1' - T_{w_1}'\right) + \rho_1 g\beta_{c_1}^*\left(C_1' - C_{w_1}'\right) \tag{2}$$

$$\rho_1 C_p\left(\frac{\partial T_1'}{\partial t'} + V_1'\frac{\partial T_1'}{\partial y'}\right) = k_1\frac{\partial^2 T_1'}{\partial y'^2} - \frac{\partial q_r}{\partial y} \tag{3}$$

$$\frac{\partial C_1'}{\partial t'} + V_1'\frac{\partial C_1'}{\partial y'} = D_1\frac{\partial^2 C_1'}{\partial y'^2} \tag{4}$$

REGION-II:
Clear Region

$$\frac{\partial V_2'}{\partial y'} = 0 \tag{5}$$

$$\rho_2\left(\frac{\partial U_2'}{\partial t'} + V_2'\frac{\partial U_2'}{\partial y'}\right) = \mu_2\frac{\partial^2 U_2'}{\partial y'^2} - \frac{\partial P'}{\partial x'} - \sigma B_0^2 U_2' - \frac{\mu_2}{K'}U_2' + \rho_2 g\beta_{f_2}\left(T_2' - T_{w_2}'\right) + \rho_2 g\beta_{c_2}^*\left(C_2' - C_{w_2}'\right) \tag{6}$$

$$\rho_2 C_p\left(\frac{\partial T_2'}{\partial t'} + V_2'\frac{\partial T_2'}{\partial y'}\right) = k_2\frac{\partial^2 T_2'}{\partial y'^2} - \frac{\partial q_r}{\partial y} \tag{7}$$

$$\frac{\partial C_2'}{\partial t'} + V_2'\frac{\partial C_2'}{\partial y'} = D_2\frac{\partial^2 C_2'}{\partial y'^2} \tag{8}$$

Assuming that the boundary and interface conditions on velocity are no slip, given that at the boundary and interface, the fluid particles are at rest, prompting the x'- component of the velocity to varnish at the wall.
Therefore, the boundary and interface conditions on the velocity for both fluids are:

$$\left.\begin{aligned} U_1'(h) &= 0 \\ U_2'(-h) &= 0 \\ U_1'(0) &= U_2'(0) \\ \mu_1\frac{\partial U_1'}{\partial y'} &= \mu_2\frac{\partial U_2'}{\partial y'}\ at\ y' = 0 \end{aligned}\right\} \tag{9}$$

The boundary and interface conditions on the temperature field for both fluids are:

$$\left.\begin{aligned} T_1'(h) &= T_{w_1}' \\ T_2'(-h) &= T_{w_2}' \\ T_1'(0) &= T_2'(0) \\ k_1\frac{\partial T_1'}{\partial y'} &= k_2\frac{\partial T_2'}{\partial y'}\ at\ y' = 0 \end{aligned}\right\} \tag{10}$$

The boundary and interface conditions on the concentration field for both fluids are:

$$\left.\begin{aligned} C_1'(h) &= C_{w_1}' \\ C_2'(-h) &= C_{w_2}' \\ C_1'(0) &= C_2'(0) \\ D_1\frac{\partial C_1'}{\partial y'} &= D_2\frac{\partial C_2'}{\partial y'}\ at\ y' = 0 \end{aligned}\right\} \tag{11}$$

The continuity equations (1) and (5) implies that V_1' and V_2' are independent of y', they can be at most a function of time alone. Hence we can write

$$V' = V_0\left(1 + \varepsilon A e^{i\omega t}\right) \tag{12}$$

Assuming that $V_1' = V_2' = V'$.
ε is a very small positive quantity such that $\varepsilon A \ll 1$. Here, it is assumed that the transpiration velocity V' varies periodically with time about a non-zero constant mean velocity, V_0.
By using the following dimensionless quantities:

$$U_i = \frac{U_i'}{u}, \quad y = \frac{y'}{h}, \quad t = \frac{t'\vartheta_1}{h^2}, \quad V = \frac{h}{\vartheta_1}V_1' = \frac{V}{V_0}, \quad P = \frac{-h^2}{\mu_1 u}\left(\frac{\partial P'}{\partial x'}\right), \quad \theta_i = \frac{T_i' - T_{w_1}'}{T_{w_2}' - T_{w_1}'},$$

$$Pr = \frac{\mu_1 C_p}{k_1}, \quad \alpha_1 = \frac{\mu_2}{\mu_1}, \quad \beta_1 = \frac{k_2}{k_1}, \quad \tau_1 = \frac{\rho_2}{\rho_1}, \quad \gamma_1 = \frac{D_2}{D_1}, \quad m_1 = \frac{\beta_{f_2}}{\beta_{f_1}}, \quad \eta_1 = \frac{\beta_{c_2}^*}{\beta_{c_1}^*},$$

$$K^2 = \frac{h^2}{K'}, \quad Sc = \frac{\vartheta_1}{D_1}, \quad C_i = \frac{C_i' - C_{w_1}'}{C_{w_2}' - C_{w_1}'}, \quad M^2 = \frac{\sigma h^2 B_0^2}{\mu_1}, \quad F = \frac{4I'h_1^2}{k_1}, \quad \frac{\partial q_r}{\partial y} =$$

$$4\left(T_i' - T_{w_1}'\right)I', \quad Gr = \frac{\rho_1 g h^2 \beta_{f_1}(T_{w_2}' - T_{w_1}')}{\mu_1 u}, \quad \xi_1 = \frac{1}{\tau_1} = \frac{\rho_1}{\rho_2},$$

$$Gc = \frac{\rho_1 g h^2 \beta_{c_1}^* (C_{w_2}' - C_{w_1}')}{\mu_1 u}.$$

Equations (2), (3), (4), (6), (7), and (8) becomes

REGION-I

$$\frac{\partial U_1}{\partial t} + \left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial U_1}{\partial y} = \frac{\partial^2 U_1}{\partial y^2} + P - M^2 U_1 + Gr\theta_1 + GcC_1 \quad (13)$$

$$\frac{\partial \theta_1}{\partial t} + \left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial \theta_1}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta_1}{\partial y^2} - \frac{F\theta_1}{Pr} \quad (14)$$

$$\frac{\partial C_1}{\partial t} + \left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial C_1}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C_1}{\partial y^2} \quad (15)$$

REGION-II

$$\frac{\partial U_2}{\partial t} + \left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial U_2}{\partial y} = \alpha_1 \xi_1 \frac{\partial^2 U_2}{\partial y^2} + \xi_1 P - \xi_1 M^2 U_2 - \alpha_1 \xi_1 K^2 U_2 + Grm_1\theta_2 + Gc\eta_1 C_2 \quad (16)$$

$$\frac{\partial \theta_2}{\partial t} + \left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial \theta_2}{\partial y} = \frac{\beta_1 \xi_1}{Pr}\frac{\partial^2 \theta_2}{\partial y^2} - \frac{F\xi_1\theta_2}{Pr} \quad (17)$$

$$\frac{\partial C_2}{\partial t} + \left(1 + \varepsilon e^{i\omega t}\right)\frac{\partial C_2}{\partial y} = \frac{\gamma_1}{Sc}\frac{\partial^2 C_2}{\partial y^2} \quad (18)$$

The boundary and interface conditions in dimensionless form are given as follows:

$$\left.\begin{aligned} U_1(1) &= 0 \\ U_2(-1) &= 0 \\ U_1(0) &= U_2(0) \\ \frac{\partial U_1}{\partial y} &= \alpha_1 \frac{\partial U_2}{\partial y} \; at \; y = 0 \end{aligned}\right\} \tag{19}$$

$$\left.\begin{aligned} \theta_1(1) &= 1 \\ \theta_2(-1) &= 0 \\ \theta_1(0) &= \theta_2(0) \\ \frac{\partial \theta_1}{\partial y} &= \beta_1 \frac{\partial \theta_2}{\partial y} \; at \; y = 0 \end{aligned}\right\} \tag{20}$$

$$\left.\begin{aligned} C_1(1) &= 1 \\ C_2(-1) &= 0 \\ C_1(0) &= C_2(0) \\ \frac{\partial C_1}{\partial y} &= \gamma_1 \frac{\partial C_2}{\partial y} \; at \; y = 0 \end{aligned}\right\} \tag{21}$$

METHOD OF SOLUTION/SOLUTION OF THE PROBLEM

In order to solve the governing equations (13) to (18) under the boundary and interface conditions (19) to (21), we expand $U_1(y,t)$, $\theta_1(y,t)$, $C_1(y,t)$, $U_2(y,t)$, $\theta_2(y,t)$, $C_2(y,t)$ as a power series in the perturbative parameter ε. Here, we assumed small amplitude of oscillation ($\varepsilon \ll 1$), thus,

$$\begin{aligned} U_1(y,t) &= U_{10}(y) + \varepsilon e^{i\omega t} U_{11}(y) \\ \theta_1(y,t) &= \theta_{10}(y) + \varepsilon e^{i\omega t} \theta_{11}(y) \\ C_1(y,t) &= C_{10}(y) + \varepsilon e^{i\omega t} C_{11}(y) \\ U_2(y,t) &= U_{20}(y) + \varepsilon e^{i\omega t} U_{21}(y) \\ \theta_2(y,t) &= \theta_{20}(y) + \varepsilon e^{i\omega t} \theta_{21}(y) \\ C_2(y,t) &= C_{20}(y) + \varepsilon e^{i\omega t} C_{21}(y) \end{aligned}$$

By substituting the above set of equations into equations (13) to (18), equating the periodic and non-periodic terms, and neglecting the terms containing $\boldsymbol{\varepsilon^2}$, we obtain the following set of ordinary differential equations:

REGION-I

Non-Periodic Terms:

$$\frac{\partial^2 U_{10}}{\partial y^2} - \frac{\partial U_{10}}{\partial y} - M^2 U_{10} = -P - Gr\theta_{10} - GcC_{10} \tag{22}$$

$$\frac{\partial^2 \theta_{10}}{\partial y^2} - Pr\frac{\partial \theta_{10}}{\partial y} - F\theta_{10} = 0 \tag{23}$$

$$\frac{\partial^2 C_{10}}{\partial y^2} - Sc\frac{\partial C_{10}}{\partial y} = 0 \tag{24}$$

Periodic terms:

$$\frac{\partial^2 U_{11}}{\partial y^2}-\frac{\partial U_{11}}{\partial y}-(M^2+i\omega)U_{11}$$
$$=\frac{\partial U_{10}}{\partial y}-Gr\theta_{11}-GcC_{11} \qquad (25)$$

$$\frac{\partial^2\theta_{11}}{\partial y^2}-Pr\frac{\partial\theta_{11}}{\partial y}-(F+i\omega Pr)\theta_{11}=Pr\frac{\partial\theta_{10}}{\partial y} \qquad (26)$$

$$\frac{\partial^2 C_{11}}{\partial y^2}-Sc\frac{\partial C_{11}}{\partial y}-i\omega ScC_{11}$$
$$=Sc\frac{\partial C_{10}}{\partial y} \qquad (27)$$

REGION-II

Non-Periodic terms:

$$\frac{\partial^2 U_{20}}{\partial y^2}-\frac{1}{\alpha_1\xi_1}\frac{\partial U_{20}}{\partial y}-\left(\frac{\xi_1 M^2+\alpha_1\xi_1 K^2}{\alpha_1\xi_1}\right)U_{20}$$
$$=-\frac{P}{\alpha_1}-\frac{Grm_1}{\alpha_1\xi_1}\theta_{20}-\frac{Gc\eta_1}{\alpha_1\xi_1}C_{20} \qquad (28)$$

$$\frac{\partial^2\theta_{20}}{\partial y^2}-\frac{Pr}{\beta_1\xi_1}\frac{\partial\theta_{20}}{\partial y}-\frac{F}{\beta_1}\theta_{20}$$
$$=0 \qquad (29)$$

$$\frac{\partial^2 C_{20}}{\partial y^2}-\frac{Sc}{\gamma_1}\frac{\partial C_{20}}{\partial y}=0 \qquad (30)$$

Periodic Terms:

$$\frac{\partial^2 U_{21}}{\partial y^2}-\frac{1}{\alpha_1\xi_1}\frac{\partial U_{21}}{\partial y}-\left(\frac{\xi_1 M^2+\alpha_1\xi_1 K^2+i\omega}{\alpha_1\xi_1}\right)U_{21}$$
$$=\frac{1}{\alpha_1\xi_1}\frac{\partial U_{20}}{\partial y}-\frac{Grm_1}{\alpha_1\xi_1}\theta_{21}-\frac{Gc\eta_1}{\alpha_1\xi_1}C_{21} \qquad (31)$$

$$\frac{\partial^2\theta_{21}}{\partial y^2}-\frac{Pr}{\beta_1\xi_1}\frac{\partial\theta_{21}}{\partial y}-\left(\frac{F\xi_1+i\omega Pr}{\beta_1\xi_1}\right)\theta_{21}$$
$$=\frac{Pr}{\beta_1\xi_1}\frac{\partial\theta_{20}}{\partial y} \qquad (32)$$

$$\frac{\partial^2 C_{21}}{\partial y^2}-\frac{Sc}{\gamma_1}\frac{\partial C_{21}}{\partial y}-\frac{i\omega Sc}{\gamma_1}C_{21}$$
$$=\frac{Sc}{\gamma_1}\frac{\partial C_{20}}{\partial y} \qquad (33)$$

The equations (22) to (33) are ordinary linear coupled differential equations with constant coefficients. The corresponding boundary and interface conditions then become:

Non-Periodic terms:

$$\left.\begin{array}{c} U_{10}(1)=0 \\ U_{20}(-1)=0 \\ U_{10}(0)=U_{20}(0) \\ \frac{\partial U_{10}}{\partial y}=\alpha_1\frac{\partial U_{20}}{\partial y}\ at\ y=0 \end{array}\right\} \qquad (34)$$

$$\left.\begin{array}{c} \theta_{10}(1)=1 \\ \theta_{20}(-1)=0 \\ \theta_{10}(0)=\theta_{20}(0) \\ \frac{\partial \theta_{10}}{\partial y}=\beta_1\frac{\partial \theta_{20}}{\partial y}\ at\ y=0 \end{array}\right\} \qquad (35)$$

$$\left.\begin{array}{c} C_{10}(1)=1 \\ C_{20}(-1)=0 \\ C_{10}(0)=C_{20}(0) \\ \frac{\partial C_{10}}{\partial y}=\gamma_1\frac{\partial C_{20}}{\partial y}\ at\ y=0 \end{array}\right\} \qquad (36)$$

Periodic Terms:

$$\left.\begin{array}{c} U_{11}(1)=0 \\ U_{21}(-1)=0 \\ U_{11}(0)=U_{21}(0) \\ \frac{\partial U_{11}}{\partial y}=\alpha_1\frac{\partial U_{21}}{\partial y}\ at\ y=0 \end{array}\right\} \qquad (37)$$

$$\left.\begin{array}{c} \theta_{11}(1)=1 \\ \theta_{21}(-1)=0 \\ \theta_{11}(0)=\theta_{21}(0) \\ \frac{\partial \theta_{11}}{\partial y}=\beta_1\frac{\partial \theta_{21}}{\partial y}\ at\ y=0 \end{array}\right\} \qquad (38)$$

$$\left.\begin{array}{c} C_{11}(1)=1 \\ C_{21}(-1)=0 \\ C_{11}(0)=C_{21}(0) \\ \frac{\partial C_{11}}{\partial y}=\gamma_1\frac{\partial C_{21}}{\partial y}\ at\ y=0 \end{array}\right\} \qquad (39)$$

The analytical solutions of the differential equations (22) to (33) are readily obtainable under the boundary conditions (34) to (39). They are:

$$U_{10}(y)=C_5e^{m_5y}+C_6e^{m_6y}+K_1+K_2e^{m_1y}+K_3e^{m_2y}+K_4e^{m_3y}+K_5e^{m_4y} \qquad (40)$$

$$U_{20}(y) = C_{17}e^{m_{17}y}+C_{18}e^{m_{18}y}+K_{20}+K_{21}e^{m_{13}y}+K_{22}e^{m_{14}y}+K_{23}e^{m_{15}y} + K_{24}e^{m_{16}y} \qquad (41)$$

$$\theta_{10}(y) = C_1 e^{m_1 y} + C_2 e^{m_2 y} \quad (42)$$
$$\theta_{20}(y) = C_{13} e^{m_{13} y} + C_{14} e^{m_{14} y} \quad (43)$$
$$C_{10}(y) = C_3 e^{m_3 y} + C_4 e^{m_4 y} \quad (44)$$
$$C_{20}(y) = C_{15} e^{m_{15} y} + C_{16} e^{m_{16} y} \quad (45)$$

$$U_{11}(y) = C_{11} e^{m_{11} y} + C_{12} e^{m_{12} y} + K_{10} e^{m_1 y} + K_{11} e^{m_2 y} + K_{12} e^{m_3 y} + K_{13} e^{m_4 y} + K_{14} e^{m_5 y} + K_{15} e^{m_6 y} + K_{16} e^{m_7 y} + K_{17} e^{m_8 y} + K_{18} e^{m_9 y} + K_{19} e^{m_{10} y} \quad (46)$$
$$U_{21}(y) = C_{23} e^{m_{23} y} + C_{24} e^{m_{24} y} + K_{29} e^{m_{13} y} + K_{30} e^{m_{14} y} + K_{31} e^{m_{15} y} + K_{32} e^{m_{16} y} + K_{33} e^{m_{17} y} + K_{34} e^{m_{18} y} + K_{35} e^{m_{19} y} + K_{36} e^{m_{20} y} + K_{37} e^{m_{21} y} + K_{38} e^{m_{22} y} \quad (47)$$

$$\theta_{11}(y) = C_7 e^{m_7 y} + C_8 e^{m_8 y} + K_6 e^{m_1 y} + K_7 e^{m_2 y} \quad (48)$$

$$\theta_{21}(y) = C_{19} e^{m_{19} y} + C_{20} e^{m_{20} y} + K_{25} e^{m_{13} y} + K_{26} e^{m_{14} y} \quad (49)$$

$$C_{11}(y) = C_9 e^{m_9 y} + C_{10} e^{m_{10} y} + K_8 e^{m_3 y} + K_9 e^{m_4 y} \quad (50)$$

$$C_{21}(y) = C_{21} e^{m_{21} y} + C_{22} e^{m_{22} y} + K_{27} e^{m_{15} y} + K_{28} e^{m_{16} y} \quad (51)$$

Where

$V_1 = F + i\omega Pr$, $V_2 = i\omega Sc$, $V_3 = M^2 + i\omega$, $V_4 = \frac{Pr}{\beta_1 \xi_1}$, $V_5 = \frac{F}{\beta_1}$, $V_6 = \frac{Sc}{\gamma_1}$, $V_7 = \frac{1}{\alpha_1 \xi_1}$, $V_8 = \frac{\xi_1 M^2 + \alpha_1 \xi_1 K^2}{\alpha_1 \xi_1}$, $V_9 = \frac{F\xi_1 + i\omega Pr}{\beta_1 \xi_1}$, $V_{10} = \frac{i\omega Sc}{\gamma_1}$, $V_{11} = \frac{\xi_1 M^2 + \alpha_1 \xi_1 K^2 + i\omega}{\alpha_1 \xi_1}$,

$m_1 = \frac{Pr + \sqrt{Pr^2 + 4F}}{2}$, $m_2 = \frac{Pr - \sqrt{Pr^2 + 4F}}{2}$, $m_3 = 0$, $m_4 = Sc$, $m_5 = \frac{1 + \sqrt{1 + 4M^2}}{2}$,

$m_6 = \frac{1 - \sqrt{1 + 4M^2}}{2}$, $m_7 = \frac{Pr + \sqrt{Pr^2 + 4V_1}}{2}$, $m_8 = \frac{Pr - \sqrt{Pr^2 + 4V_1}}{2}$, $m_9 = \frac{Sc + \sqrt{Sc^2 + 4V_2}}{2}$,

$m_{10} = \frac{Sc - \sqrt{Sc^2 + 4V_2}}{2}$, $m_{11} = \frac{1 + \sqrt{1 + 4V_3}}{2}$, $m_{12} = \frac{1 - \sqrt{1 + 4V_3}}{2}$, $m_{13} = \frac{V_4 + \sqrt{V_4^2 + 4V_5}}{2}$,

$m_{14} = \frac{V_4 - \sqrt{V_4^2 + 4V_5}}{2}$, $m_{15} = 0$, $m_{16} = V_6$, $m_{17} = \frac{V_7 + \sqrt{V_7^2 + 4V_8}}{2}$, $m_{18} = \frac{V_7 - \sqrt{V_7^2 + 4V_8}}{2}$,

$m_{19} = \frac{V_4 + \sqrt{V_4^2 + 4V_9}}{2}$, $m_{20} = \frac{V_4 - \sqrt{V_4^2 + 4V_9}}{2}$, $m_{21} = \frac{V_6 + \sqrt{V_6^2 + 4V_{10}}}{2}$, $m_{22} = \frac{V_6 - \sqrt{V_6^2 + 4V_{10}}}{2}$,

$, m_{23} = \frac{V_7+\sqrt{V_7^2+4V_{11}}}{2}, \quad m_{24} = \frac{V_7-\sqrt{V_7^2+4V_{11}}}{2}, \ r_1 = e^{-m_2}, \ r_2 = 1- e^{(m_1-m_2)}$

$r_3 = e^{(m_{14}-m_{13})} - 1, \ r_4 = m_2 e^{-m_2}, \ r_5 = m_1 - m_2 e^{(m_1-m_2)},$

$r_6 = \beta_1 (m_{14} e^{(m_{14}-m_{13})} - m_{13}), \quad r_7 = e^{-m_4}, \ r_8 = 1- e^{(m_3-m_4)}, \quad r_9 = e^{(m_{16}-m_{15})} - 1,$

$r_{10} = m_4 e^{-m_4}, \ r_{11} = m_3 - m_4 e^{(m_3-m_4)}, \ r_{12} = \gamma_1 (m_{16} e^{(m_{16}-m_{15})} - m_{15}),$

$r_{13} = 1- e^{(m_5-m_6)}, \quad r_{14} = e^{(m_{18}-m_{17})} - 1, \ r_{15} = m_5 - m_6 e^{(m_5-m_6)},$

$r_{16} = \alpha_1 (m_{18} e^{(m_{18}-m_{17})} - m_{17}), \quad r_{17} = 1- e^{(m_7-m_8)}, \quad r_{18} = e^{(m_{20}-m_{19})} - 1,$

$r_{19} = m_7 - m_8 e^{(m_7-m_8)}, \ r_{20} = \beta_1 (m_{20} e^{(m_{20}-m_{19})} - m_{19}), \ r_{21} = 1- e^{(m_9-m_{10})},$

$r_{22} = e^{(m_{22}-m_{21})} - 1, \ r_{23} = m_9 - m_{10} e^{(m_9-m_{10})}, \ r_{24} = \gamma_1 (m_{22} e^{(m_{22}-m_{21})} - m_{21}),$

$r_{25} = 1- e^{(m_{11}-m_{12})}, \ r_{26} = e^{(m_{24}-m_{23})} - 1, \ r_{27} = m_{11} - m_{12} e^{(m_{11}-m_{12})},$

$r_{28} = \alpha_1 (m_{24} e^{(m_{24}-m_{23})} - m_{23}), \quad C_1 = \frac{r_1 r_6 - r_3 r_4}{r_3 r_5 - r_2 r_6}, \ C_2 = e^{-m_2} - C_1 e^{(m_1-m_2)},$

$C_3 = \frac{r_7 r_{12} - r_{10} r_{11}}{r_9 r_{11} - r_8 r_{12}}, \ C_4 = e^{-m_4} - C_3 e^{(m_3-m_4)}, \ C_{13} = \frac{r_1 - C_1 r_2}{r_3}, \ C_{14} = - C_{13} e^{(m_{14}-m_{13})},$

$C_{15} = \frac{-r_7 - C_3 r_8}{r_9}, \ C_{16} = - C_{15} e^{(m_{16}-m_{15})}, \ K_1 = \frac{P}{M^2}, \quad K_2 = -\frac{GrC_1}{m_1^2 - m_1 - M^2},$

$K_3 = -\frac{GrC_2}{m_2^2 - m_2 - M^2},$

$K_4 = -\frac{GcC_3}{m_3^2 - m_3 - M^2}, \quad K_5 = -\frac{GcC_4}{m_4^2 - m_4 - M^2}, \quad K_6 = \frac{PrC_1 m_1}{m_1^2 - Prm_1 - V_1}, \quad K_7 = \frac{PrC_2 m_2}{m_2^2 - Prm_2 - V_1},$

$K_8 = \frac{ScC_3 m_3}{m_3^2 - Scm_3 - V_2}, \quad K_9 = \frac{ScC_4 m_4}{m_4^2 - Scm_4 - V_2}, \ K_{10} = \frac{K_2 m_1 - GrK_6}{m_1^2 - m_1 - V_3}, \quad K_{11} = \frac{K_3 m_2 - GrK_7}{m_2^2 - m_2 - V_3},$

$K_{12} = \frac{K_4 m_3 - GcK_8}{m_3^2 - m_3 - V_3},$

$K_{22} = -\frac{V_7 Grm_1 C_{14}}{m_{14}^2 - V_7 m_{14} - V_8}, \ K_{23} = -\frac{V_7 Gc\eta_1 C_{15}}{m_{15}^2 - V_7 m_{15} - V_8}, \quad K_{24} = -\frac{V_7 Gc\eta_1 C_{16}}{m_{16}^2 - V_7 m_{16} - V_8},$

$K_{25} = \frac{V_4 C_{13} m_{13}}{m_{13}^2 - V_4 m_{13} - V_9}, \quad K_{26} = \frac{V_4 C_{14} m_{14}}{m_{14}^2 - V_4 m_{14} - V_9}, \quad K_{27} = \frac{V_6 C_{15} m_{15}}{m_{15}^2 - V_6 m_{15} - V_{10}},$

$K_{28} = \frac{V_6 C_{16} m_{16}}{m_{16}^2 - V_6 m_{16} - V_{10}}, \quad K_{29} = \frac{V_7 (K_{21} m_{13} - GrK_{25} m_1)}{m_{13}^2 - V_7 m_{13} - V_{11}},$

$K_{30} = \frac{V_7 (K_{22} m_{14} - GrK_{26} m_1)}{m_{14}^2 - V_7 m_{14} - V_{11}}, \ K_{31} = \frac{V_7 (K_{23} m_{15} - GcK_{27} \eta_1)}{m_{15}^2 - V_7 m_{15} - V_{11}}, \quad K_{32} = \frac{V_7 (K_{24} m_{16} - GcK_{28} \eta_1)}{m_{16}^2 - V_7 m_{16} - V_{11}},$

$A_1 = K_1 + K_2 e^{m_1} + K_3 e^{m_2} + K_4 e^{m_3} + K_5 e^{m_4},$

$A_2 = K_{20} + K_{21} e^{-m_{13}} + K_{22} e^{-m_{14}} + K_{23} e^{-m_{15}} + K_{24} e^{-m_{16}},$

$A_3 = K_{20} + K_{21} + K_{22} + K_{23} + K_{24} - (K_1 + K_2 + K_3 + K_4 + K_5),$

$A_4 = \alpha_1 (K_{21} m_{13} + K_{22} m_{14} + K_{23} m_{15} + K_{24} m_{16}) - (K_2 m_1 + K_3 m_2 + K_4 m_3 + K_5 m_4),$

$A_5 = 1 - (K_6 e^{m_1} + K_7 e^{m_2}), \ A_6 = K_{25} e^{-m_{13}} + K_{26} e^{-m_{14}}, \ A_7 = K_{25} + K_{26} - (K_6 + K_7),$

$A_8 = \beta_1 (K_{25} m_{13} + K_{26} m_{14}) - (K_6 m_1 + K_7 m_2), \quad A_9 = 1 - (K_8 e^{m_3} + K_9 e^{m_4}),$

$A_{10} = K_{27} e^{-m_{15}} + K_{28} e^{-m_{16}}, \quad A_{11} = K_{27} + K_{28} - (K_8 + K_9),$

$A_{12} = \gamma_1 (K_{27} m_{15} + K_{28} m_{16}) - (K_8 m_3 + K_9 m_4), \quad Q_1 = A_3 + A_1 e^{-m_6} - A_2 e^{m_{18}},$

$Q_2 = A_4 r_{14} + A_1 r_{14} m_6 e^{-m_6} - \alpha_1 A_2 r_{14} m_{18} e^{m_{18}}, \quad Q_3 = A_7 - A_5 e^{-m_8} - A_6 e^{m_{20}},$

$Q_4 = A_8 r_{18} - A_5 r_{18} m_8 e^{-m_8} - \beta_1 A_6 r_{18} m_{20} e^{m_{20}}, \quad Q_5 = A_{11} - A_9 e^{-m_{10}} - A_{10} e^{m_{22}},$

$Q_6 = A_{12} r_{22} - A_9 r_{22} m_{10} e^{-m_{10}} - \gamma_1 A_{10} r_{22} m_{22} e^{m_{22}}, \quad C_5 = \frac{Q_2 - Q_1 r_{16}}{r_{14} r_{15} - r_{13} r_{16}},$

$$C_6 = -A_1\, e^{-m_6} - C_5 e^{(m_5-m_6)}\,, \quad C_7 = \frac{Q_4 - Q_3 r_{20}}{r_{18} r_{19} - r_{17} r_{20}}\,,$$

$$C_8 = A_5\, e^{-m_8} - C_7 e^{(m_7-m_8)}\,, \quad C_9 = \frac{Q_6 - Q_5 r_{24}}{r_{22} r_{23} - r_{21} r_{24}}\,,$$

$$C_{10} = A_9\, e^{-m_{10}} - C_9 e^{(m_9-m_{10})}\,, \quad C_{17} = \frac{Q_1 - C_5 r_{13}}{r_{14}}\,,$$

$$C_{18} = -A_2 e^{m_{18}} - C_{17} e^{(m_{18}-m_{17})}\,, \quad C_{19} = \frac{Q_3 - C_7 r_{17}}{r_{18}}\,,$$

$$C_{20} = -A_6 e^{m_{20}} - C_{19} e^{(m_{20}-m_{19})}\,, \quad C_{21} = \frac{Q_5 - C_9 r_{21}}{r_{22}}\,,$$

$$C_{22} = -A_{10} e^{m_{22}} - C_{21} e^{(m_{22}-m_{21})}, \quad K_{14} = \frac{C_5 m_5}{m_5^2 - m_5 - V_3}\,, \quad K_{15} = \frac{C_6 m_6}{m_6^2 - m_6 - V_3}\,,$$

$$K_{16} = -\frac{\square\square C_7}{m_7^2 - m_7 - V_3}\,, \quad K_{17} = -\frac{\square\square C_8}{m_8^2 - m_8 - V_3}\,, \quad K_{18} = -\frac{\square\square C_9}{m_9^2 - m_9 - V_3}\,, \quad K_{19} = -\frac{\square\square C_{10}}{m_{10}^2 - m_{10} - V_3}\,,$$

$$K_{33} = \frac{V_7 C_{17} m_{17}}{m_{17}^2 - V_7 m_{17} - V_{11}}\,, \quad K_{34} = \frac{V_7 C_{18} m_{18}}{m_{18}^2 - V_7 m_{18} - V_{11}}\,, \quad K_{35} = -\frac{V_7 Grm_1 C_{19}}{m_{19}^2 - V_7 m_{19} - V_{11}}\,,$$

$$K_{36} = -\frac{V_7 Grm_1 C_{20}}{m_{20}^2 - V_7 m_{20} - V_{11}}\,, \quad K_{37} = -\frac{V_7 Gc\eta_1 C_{21}}{m_{21}^2 - V_7 m_{21} - V_{11}}\,, \quad K_{38} = -\frac{V_7 Gc\eta_1 C_{22}}{m_{22}^2 - V_7 m_{22} - V_{11}}\,,$$

$$A_{13} = K_{10} e^{m_1} + K_{11} e^{m_2} + K_{12} e^{m_3} + K_{13} e^{m_4} + K_{14} e^{m_5} + K_{15} e^{m_6} + K_{16} e^{m_7} + K_{17} e^{m_8} + K_{18} e^{m_9} + K_{19} e^{m_{10}}\,,$$

$$A_{14} = K_{29} e^{-m_{13}} + K_{30} e^{-m_{14}} + K_{31} e^{-m_{15}} + K_{32} e^{-m_{16}} + K_{33} e^{-m_{17}} + K_{34} e^{-m_{18}} + K_{35} e^{-m_{19}} + K_{36} e^{-m_{20}} + K_{37} e^{-m_{21}} + K_{38} e^{-m_{22}}\,,$$

$$A_{15} = K_{29} + K_{30} + K_{31} + K_{32} + K_{33} + K_{34} + K_{35} + K_{36} + K_{37} + K_{38} - (K_{10} + K_{11} + K_{12} + K_{13} + K_{14} + K_{15} + K_{16} + K_{17} + K_{18} + K_{19})\,,$$

$$A_{16} = \alpha_1 (K_{29} m_{13} + K_{30} m_{14} + K_{31} m_{15} + K_{32} m_{16} + K_{33} m_{17} + K_{34} m_{18} + K_{35} m_{19} + K_{36} m_{20} + K_{37} m_{21} + K_{38} m_{22}) - (K_{10} m_1 + K_{11} m_2 + K_{12} m_3 + K_{13} m_4 + K_{14} m_5 + K_{15} m_6 + \square_{16} m_7 + \square_{17} m_8 + K_{18} m_9 + K_{19} m_{10})\,, \quad Q_7 = A_{15} + A_{13} e^{-m_{12}} - A_{14} e^{m_{24}}\,,$$

$$Q_8 = A_{16} r_{26} + A_{13} r_{26} m_{12} e^{-m_{12}} - \alpha_1 A_{14} r_{26} m_{24} e^{m_{24}}\,, \quad C_{11} = \frac{Q_8 - Q_7 r_{28}}{r_{26} r_{27} - r_{25} r_{28}}\,,$$

$$C_{12} = -A_{13} e^{-m_{12}} - C_{11} e^{(m_{11}-m_{12})}\,, \quad C_{23} = \frac{Q_7 - C_{11} r_{25}}{r_{26}}\,, \quad C_{24} = -A_{14} e^{m_{24}} - C_{23} e^{(m_{24}-m_{23})}.$$

The coefficient of skin friction, Nusselt number and Sherwood number are given as:

$$C_f(U) = \left[\frac{\square\square_{10}}{\square\square}\right]_{\square=1} + \square\square^{\square\square\square}\left[\frac{\square\square_{11}}{\square\square}\right]_{\square=1}$$

$$C_f(L) = \left[\frac{\square\square_{20}}{\square\square}\right]_{\square=-1} + \square\square^{\square\square\square}\left[\frac{\square\square_{21}}{\square\square}\right]_{\square=-1}$$

$$Nu(U) = \left[\frac{\square\square_{10}}{\square\square}\right]_{\square=1} + \square\square^{\square\square\square}\left[\frac{\square\square_{11}}{\square\square}\right]_{\square=1}$$

$$Nu(L) = \left[\frac{\square\square_{20}}{\square\square}\right]_{\square=-1} + \square\square^{\square\square\square}\left[\frac{\square\square_{21}}{\square\square}\right]_{\square=-1}$$

$$Sh(U) = \left[\frac{\square\square_{10}}{\square\square}\right]_{\square=1} + \square\square^{\square\square\square}\left[\frac{\square\square_{11}}{\square\square}\right]_{\square=1}$$

$$\mathrm{Sh(L)} = \left[\frac{\square\square_{20}}{\square\square}\right]_{\square=-1} + \square\square^{\square\square\square}\left[\frac{\square\square_{21}}{\square\square}\right]_{\square=-1}$$

RESULTS AND DISCUSSION

The degree of diversity of the velocity, temperature and concentration of the unsteady movement of two electrically conducting free convective immiscible fluids flowing in a horizontal channel with heat and mass transfer by assigning different numerical values to the varying parameter when $\mathrm{Gr} = 5, \mathrm{Gc} = 5, \mathrm{Pr} = 1, \mathrm{Sc} = 0.78, \mathrm{F} = 3, \mathrm{K} = 5, \mathrm{M} = 1, \alpha_1 = 1, \beta_1 = 1, \gamma_1 = 1, \omega = 10, \xi_1 = 1, \varphi_1 = 1, \eta_1 = 1, \mathrm{P} = 1, \omega t = 30$ were conducted using MATLAB and the following are the results obtained.

Figure 1 and 2 depicts the effect of Grashof numbers (□□ □□□ □□) for heat and mass transfer respectively on the velocity field. In both figures, it can be seen that the velocity increases as the domination of buoyancy force over viscous force increases. Also, it can be clearly seen that, this increase occurs in the porous region and upper part of the clear region. However, it would be worthy to note that the Grashof number for mass transfer (□□) increases the velocity of the fluid more than the Grashof number for heat transfer (□□).

Figure 3 and 4 which describes the effect of Prandtl number (□□) and Permeability parameter (□) respectively on the velocity field shows that as the momentum diffusivity (kinematic viscosity) gradually dominates the thermal diffusivity, the velocity of the flow decreases with slight alteration in the porous region while the variation in the velocity is not significant even if the Prandtl number (□□) increases for region II. In the case of Figure 4, the velocity is low for a less than unity Permeability parameter while further increase above unity causes a rapid increase in the velocity.

Figure 5 and 6 shows the effect of Viscosity ratio and Radiation parameter on the velocity of the flow. It is observed that as the viscosity ratio increases, the velocity decreases with lager velocity boundary layer in region II as compared to region I. this observation concurs with the fact that increase in the thickness of a fluid reduces the velocity of that fluid. For Figure 6, the Radiation parameter has no significant effect on the velocity field as it can be seen, although an increase in the Radiation parameter decreases the velocity in the porous region.

Figure 7, 8 and 9 highlights the effect of the Hartmann number, Schmidt number and diffusivity ratio respectively on the velocity field. Figure 7 shows that as the electromagnetic force increasingly dominates over the viscous force, the velocity decreases in the upper region while near the interface, the velocity profiles cut across each other in order to cause an increase in the velocity in the lower region, hence the reverse of what happens in the upper region is seen in the lower region. The decrease in the velocity in the upper region can be attributed to the presence of magnetic field applied transverse to the flow which would suppress turbulence thereby causing decrease in the velocity. Figure 8 depicts that gradual domination of the viscous diffusion rate over the molecular (mass) diffusion rate contributes to a decrease in the velocity field in region I and upper part of region II while increase in the Schmidt number has no significance on the velocity in the lower part of region II. In figure 9, it can be seen that as the diffusivity ratio increases, the velocity of the flow barely decreases in the porous region and upper part of the clear region while no contribution is made to the velocity at the lower part of the clear region.

Fig 1: Effect of Gr on velocity profile.

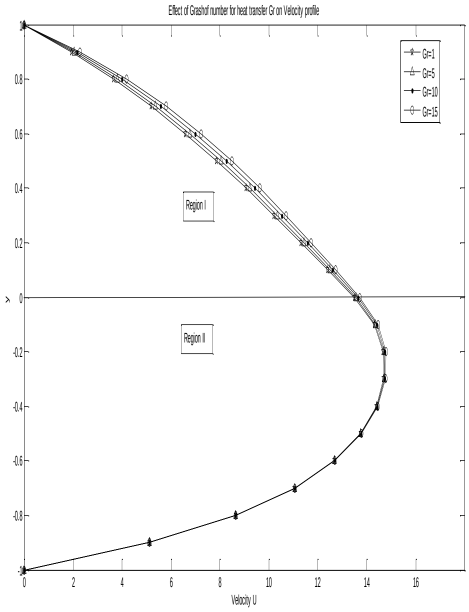
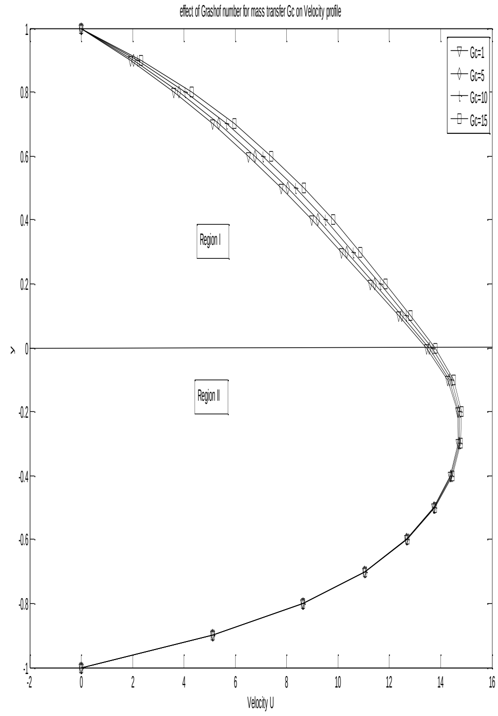


Fig 2: Effect of Gc on Velocity profile.



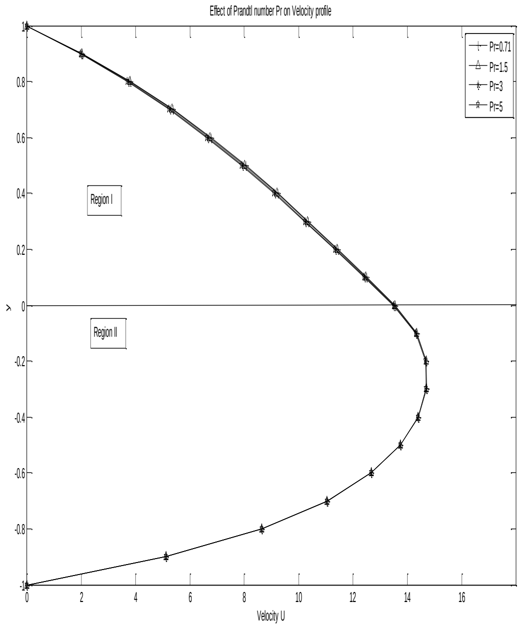


Fig 3: Effect of Pr on Velocity profile.

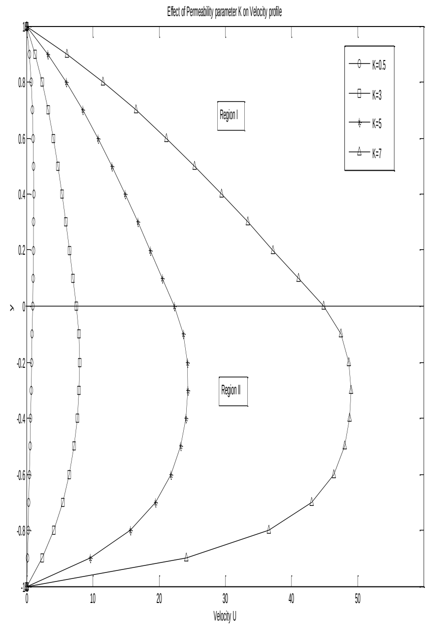


Fig 4: Effect of K on Velocity profile.

Fig 5: Effect of α_1 on Velocity profile

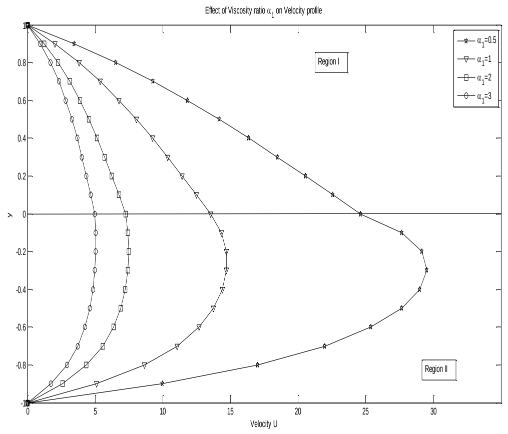
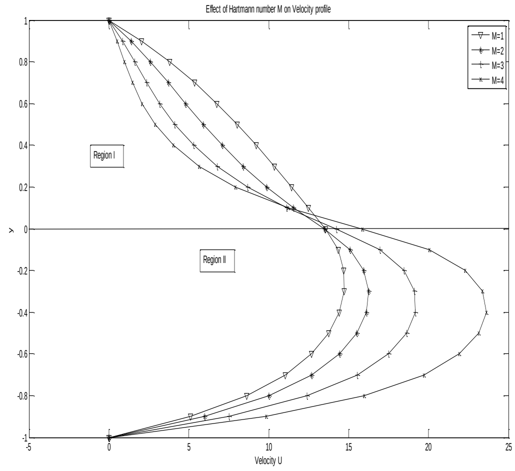


Fig 6: Effect of F on Velocity profile.



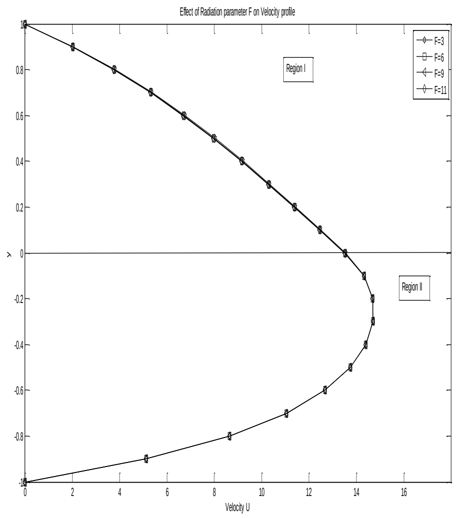


Fig 7: Effect of M on velocity profile.

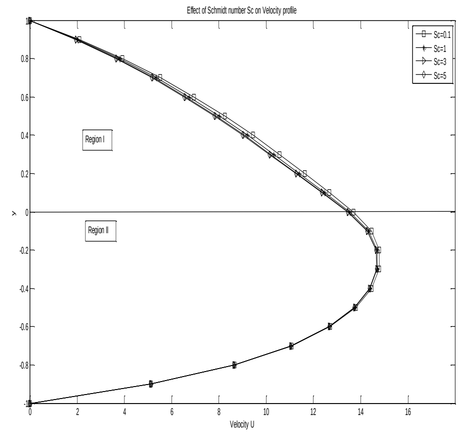
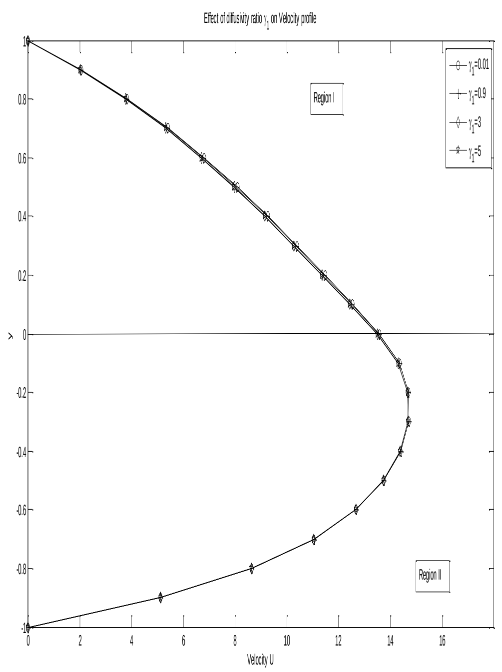


Fig 8: Effect of Sc on Velocity

Figure 10, 11 and 12 demonstrates the effect of Prandtl number, Radiation parameter and Thermal conductivity ratio respectively on the temperature field. It is generally observed that the temperature decreases with increase in the aforementioned parameters. Increase in the Prandtl number causes a decrease in the temperature which is more evident in the upper region where this increase causes the profile to assume a parabolic shape. The effect is also seen in region II but not as evident as in region I. increase in the radiation parameter causes a decrease in the temperature but this decrease is less when compared to that for Prandtl number as the distance between profiles is less in figure 11 considering the values used to obtain that for the radiation parameter. Increase in the thermal conductivity ratio causes a slight decrease in the temperature near the lower part of region I and upper part of region II, while it has no much significance on the upper part of region I and lower part of region II and near the walls.

Figure 13 and 14 depicts the effect of Schmidt number and diffusivity ratio respectively on the concentration field. It is observed that for the same value of viscous diffusion rate and molecular (mass) diffusion rate, the profile assumes a linear shape in both regions, but as the viscous diffusion rate dominates, the profile assumes a parabolic shape and in general causes the concentration to decrease. Also, increase in the diffusivity ratio causes a decrease in the concentration with the profiles assuming a linear shape all through in both regions while distances between the profiles continue to decrease towards the walls in both situations.

Fig 9: Effect of γ_1 on Velocity profile.



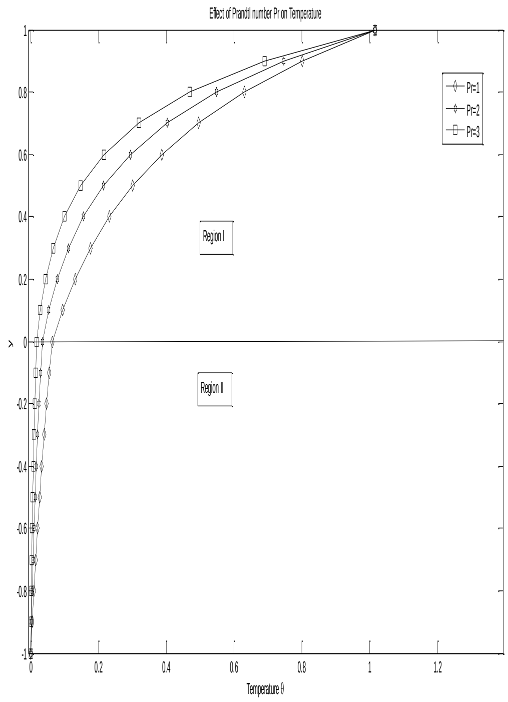


Fig 10: Effect of Pr on Temperature profile.

The table for the skin friction shows that the coefficient of skin friction at the upper plate in case of unsteady flow is greater than that of the mean flow while opposite behavior is noticed at the lower plate. Also, it is observed that increase in the Hartmann number increases the coefficient of skin friction at both plates while increase in the Prandtl number, radiation parameter, Schmidt number, viscosity ratio, thermal conductivity ratio and diffusivity ratio all cause an increase in the coefficient of skin friction at the upper plate while opposite behavior is noticed at the lower plate except for the Prandtl number which has little effect at the lower plate. However, increase in the thermal and mass Grashof numbers and the permeability parameter all cause a decrease in the skin friction coefficient at the upper plate with opposite behaviors at the lower plate where the permeability parameter has the highest effect on the coefficient of skin friction.

In table 2, it can be observed that the Nusselt number at the upper and lower plate in case of unsteady flow is greater than that of the mean flow. The Nusselt number at the upper plate increases due to increase in the Prandtl number, radiation parameter and thermal conductivity ratio with opposite behavior noticed at the lower plate except for the thermal conductivity ratio which increases the Nusselt number at the lower plate.

Table 3 shows that the Sherwood number at the upper plate and lower plat in case of unsteady flow is greater than that of the mean flow. The Sherwood number at the upper plate increases due to increase in the Schmidt number and diffusivity ratio with opposite behavior noticed at the lower plate.

Fig 11: Effect of F on Temperature profile.

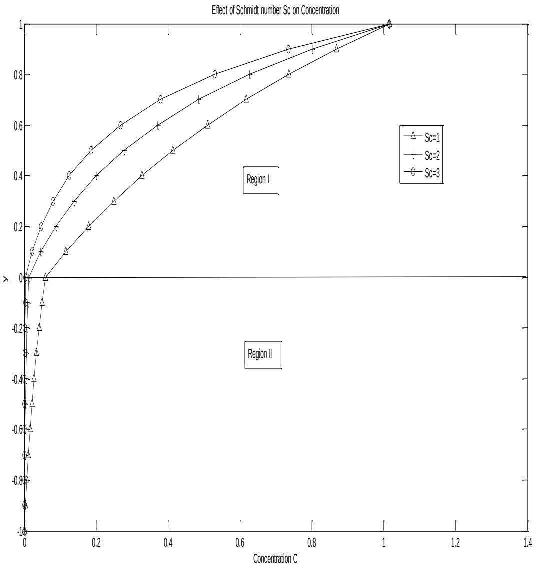


Fig 12: Effect of β_1 on Temperature profile.

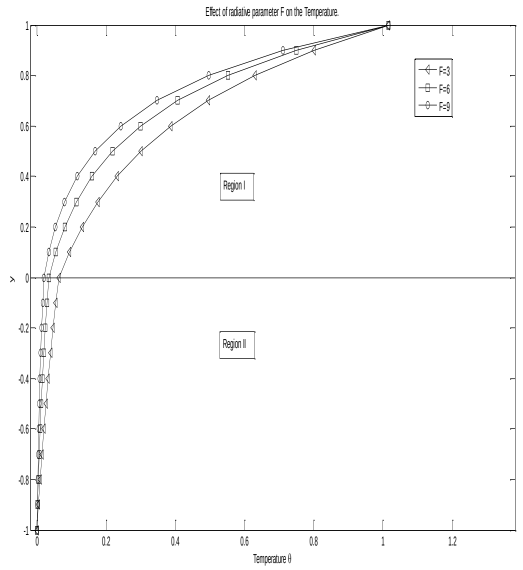


Fig 13: Effect of Sc on Concentration profile.

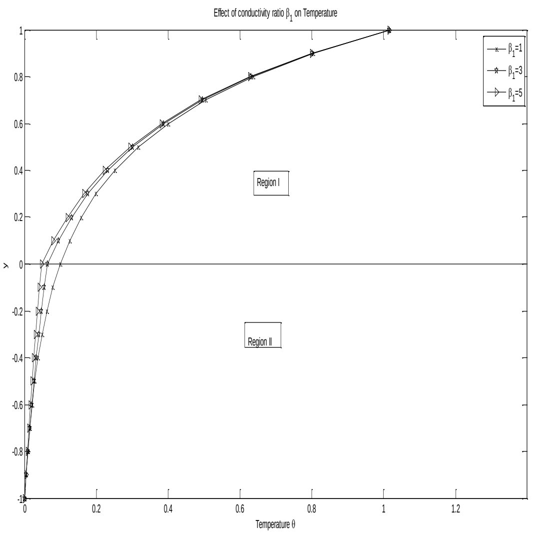


Fig 14: Effect of γ_1 on Concentration profile.

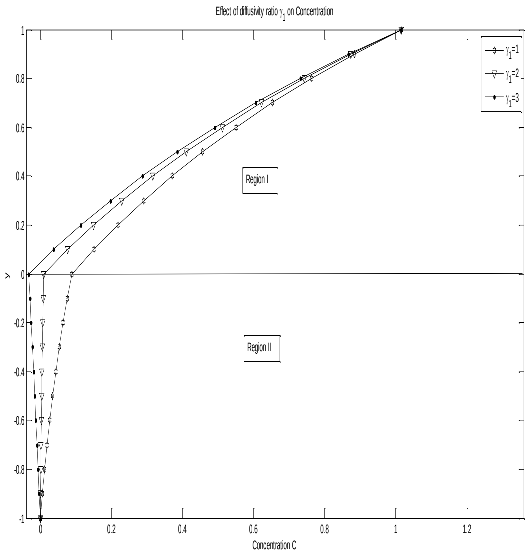


Table 1: Values of the coefficient of skin-friction at the upper plate and lower plate for various values of physical parameters, where $\omega = 10, \xi_1 = 1, \varphi_1 = 1, \eta_1 = 1, P = 1, \omega t = 30$

E	Gr	Gc	Pr	F	K	M	Sc	α_1	β_1	γ_1	$C_f(U)$	$C_f(L)$
0	5	5	1	3	5	1	0.78	1	1	1	-33.6810	120.1005
0.025	5	5	1	3	5	1	0.78	1	1	1	-33.5524	120.0800
0.025	10	5	1	3	5	1	0.78	1	1	1	-35.0286	120.0900
0.025	5	10	1	3	5	1	0.78	1	1	1	-35.3561	120.1000
0.025	5	5	2	3	5	1	0.78	1	1	1	-33.3219	120.0800
0.025	5	5	1	5	5	1	0.78	1	1	1	-33.3832	120.0800
0.025	5	5	1	3	10	1	0.78	1	1	1	-129.0300	965.6400
0.025	5	5	1	3	5	3	0.78	1	1	1	-14.0563	181.7600
0.025	5	5	1	3	5	1	2.62	1	1	1	-32.9818	120.0700
0.025	5	5	1	3	5	1	0.78	2	1	1	-19.3303	61.2309
0.025	5	5	1	3	5	1	0.78	1	2	1	-33.5269	120.0800
0.025	5	5	1	3	5	1	0.78	1	1	2	-33.4237	120.0700

Table 2: Values of Nusselt number at the upper plate and lower plate for various values of physical parameters, where $\omega = 10,\ \xi_1 = 1,\ \omega t = 30$

E	Pr	F	β_1	Nu(U)	Nu(L)
0	1	3	1	2.3054	0.0361
0.025	1	3	1	2.3616	0.0364
0.025	2	3	1	3.0816	0.0100
0.025	1	5	1	2.8456	0.0174
0.025	1	3	2	2.3826	0.0465

Table 3: Values of Sherwood number at the upper plate and lower plate for various values of physical parameters, where $\omega = 10,\ \omega t = 30$

ε	Sc	γ_1	Sh(U)	Sh(L)
0	0.78	1	1.3069	0.0611
0.025	0.78	1	1.3605	0.0615
0.025	2.62	1	2.9072	0.0008
0.025	0.78	2	1.4775	0.0089

SUMMARY, CONCLUSION AND RECOMMENDATION

The unsteady MHD free convective two immiscible fluid flows in a horizontal channel with heat and mass transfer been studied. The governing equations, that is, the momentum, energy and species concentration equations are written in a dimensionless form using the dimensionless parameters. A perturbation method has been employed to evaluate and solved for the velocity, temperature and concentration distribution, the skin frictions, Nusselt numbers and Sherwood numbers.

It can be concluded that increase in the Hartmann number decreased the velocity, while increase in the heat and mass Grashof numbers both support the increase of the flow velocity whereas increase in the viscosity ratio as expected supports the decrease in velocity. The temperature decreases with increase in the Prandtl number, Thermal conductivity ratio and radiation parameter while the concentration decreases with increase in Schmidt number and diffusivity ratio.
This study is expected to be useful in understanding the concept of double phase flow and the effect of heat and mass transfer on MHD free convective two immiscible fluid flows in a horizontal channel.
Finally, the study has potential applications in MHD power generators, MHD pumps, liquid metal cooling of reactors and magnetic drug targeting etc.

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