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On Certain Topological Structures of Two - Banach Space Valued Paranormed Sequence Space $\ell((S, \|.\,,.\|), \bar{\xi}, \bar{u})$

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Abstract

The aim of this paper is to introduce and study a new class $\ell((S, \|.\,,.\|), \bar{\xi}, \bar{u})$ of sequences with values in *2*-Banach space as a generalization of the familiar sequence space ℓ_p. We explore some of the preliminary results that characterize the linear topological structure of the class $\ell((S, \|.\,,.\|), \bar{\xi}, \bar{u})$ when topologized it with suitable natural paranorm.

Keywords and Phrases: *2- normed space, sequence space, paranormed space, solid space.*

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1. Introduction

So far, a good number of research works have been done on various types of 2-normed space valued sequence spaces. The notion of 2–normed space was initially introduced by S. GÄahler [17] as an interesting linear generalization of a normed linear space, which was subsequently studied by Iseki [9], White and Cho [5], Freese *et al.* [15] , Freese and Cho [14] and many others. Recently a lot of activities have been started by many researchers to study this concept in different directions, for instances, Savas [2] ,Gunawan and Mashadi [3] , Srivastava and Pahari ([7], [8]), Açikgöz [10], and others.

2. Preliminaries

We recall some basic facts and definitions that are used in this paper.

Definition 2.1. Let S be a linear space of dimension > 1 over $\boldsymbol{K}$, the field of real or complex numbers. A 2 - norm on S is a real valued function $\|.\,,.\|$ on $S \times S$ satisfying the following conditions:

(i) $\| s, t \| \geq 0$ and $\| s, t \| = 0$ if and only if s and t are linearly dependent;

(ii) $\| s, t \| = \| t, s \|$, for all $s, t \in S$;

(iii) $\| \alpha s, t \| = |\alpha| \, \| s, t \|$, where $\alpha \in \boldsymbol{K}$ and $s, t \in S$;and

(iv) $\| s_1 + s_2, t \| \leq \| s_1, t \| + \| s_2, t \|$,for all s_1, s_2 and $t \in S$.

The pair $(S, \|.\,,.\|)$ is called a 2–normed space. Thus the notion of 2–normed space is just a two- dimensional analogue of a normed space.

Recall that $(S, \|.\,,.\|)$ is a 2-Banach space if every Cauchy sequence $< s_n >$ in S is convergent to some s_0 in S. Geometrically, a 2-norm function represents the area of the usual parallelogram spanned by the two associated vectors.

Example 2.2. Consider $S = \boldsymbol{R}^2$, being equipped with

$$\| \bar{s}, \bar{t} \| = | s_1 t_2 - s_2 t_1 |, \text{ where } \bar{s} = (s_1, s_2) \text{ and } \bar{t} = (t_1, t_2).$$

Then $(S, \|.\,,.\|)$ forms a 2–normed space and $\| \bar{s}, \bar{t} \|$ represents the area of the parallelogram spanned by the two associated vectors $\bar{s}$ and $\bar{t}$.

Definition 2.3. A sequence $\bar{s} = < s_n >$ in a linear 2–normed space S is *convergent* if there is an $s_0 \in S$ such that $\lim_{n\to\infty} \| s_n - s, t \| = 0$, for each $t \in S$ .It is said to be a *Cauchy* if there are t and w in S such that t and w are linearly independent and

$$\lim_{m,n\to\infty} \|s_m - s_n, t\| = 0 \text{ and } \lim_{m,n\to\infty} \|s_m - s_n, w\| = 0.$$

The notion of convergence was introduced by White and Cho [5].A linear 2–normed space $(S, \|.\,,.\|)$ is called *2–Banach* space if every Cauchy sequence $< s_n >$ in S is convergent to some $s \in S$.

Definition 2.4. Let $(S, \|.\,,.\|)$ be the 2- Normed space over the field $\boldsymbol{C}$ of complex numbers and $\bar{\theta} = (\theta, \theta, \theta, \ldots)$ denotes the zero element of S. Let $\omega(S)$ denotes the linear space of all sequences $\bar{s} = < s_k >$ with $s_k \in S$, $k \geq 1$ with usual coordinate wise operations i.e., for each

$\bar{s} = < s_k >$, $\bar{w} = < w_k > \in \omega(S)$ and $\gamma \in \boldsymbol{C}$,

$$\bar{s} + \bar{w} = < s_k + w_k > \text{ and } \gamma \bar{s} = < \gamma s_k > .$$

We shall denote $\omega(\boldsymbol{C})$ by ω. Any linear subspace of ω is then called a *sequence space*.

Further, if $\bar{\gamma} = <\gamma_k> \in \omega$ and $\bar{s} \in \omega(S)$ we shall write $\bar{\gamma}\,\bar{s} = <\gamma_k s_k>$.

The concept of paranorm is closely related to linear metric space (see, Wilansky [1]) and its studies on sequence spaces were initiated by Maddox [4] and many others.

Definition 2.5: A paranormed space (S, Φ) is a linear space S with zero element θ together with a function $\Phi : S \to \boldsymbol{R}^+$ (called a paranorm on S) which satisfies the following axioms:

$PN1$: $\Phi(\theta) = 0$;

$PN2$: $\Phi(s) = \Phi(-s)$, for all $s \in S$;

$PN3$: $\Phi(s_1 + s_2) \le \Phi(s_1) + \Phi(s_2)$, for all $s_1, s_2 \in S$;

and

$PN4$: Scalar multiplication is continuous i.e., if $<\gamma_n>$ is a sequence of scalars with $\gamma_n \to \gamma$ as $n \to \infty$ and $<s_n>$ is a sequence of vectors with $\Phi(s_n - s) \to 0$ as $n \to \infty$, then

$$\Phi(\gamma_n s_n - \gamma s) \to 0 \text{ as } n \to \infty.$$

Note that the continuity of scalar multiplication is equivalent to

(i) if $\Phi(s_n) \to 0$ and $\gamma_n \to \gamma$ as $n \to \infty$, then $\Phi(\gamma_n s_n) \to 0$ as $n \to \infty$; and

(ii) if $\gamma_n \to 0$ as $n \to \infty$ and s be any element in S, then $\Phi(\gamma_n s) \to 0$, see Wilansky [1].

A paranorm is called total if $\Phi(s) = 0 \Rightarrow s = \theta$, see Wilansky [1].

The studies of paranorm on sequence spaces were initiated by Maddox [4] and many others. Basariv and Altundag [11], Pahari [12] , Tiwari and Srivastava [13] , Parasar and Choudhary [16], Khan [18],Bhardwaj and Bala [19], and many others further studied various types of paranormed sequence spaces and function spaces.

Definition 2.6. A sequence space S is said to be *solid* if $\bar{s} = <s_k> \in S$ and $\bar{\gamma} = <\gamma_k>$ a sequence of scalars with $|\gamma_k| \le 1$, for all $k \ge 1$, then

$$\bar{\gamma}\,\bar{s} = <\gamma_k s_k> \in S.$$

3. The Class $\ell((S, \|.,.\|), \bar{\xi}, \bar{u})$ of 2-Normed Space Valued Vector Sequences

Let $\bar{u} = <u_k>$ and $\bar{v} = <v_k>$ be any sequences of strictly positive real numbers and $\bar{\xi} = <\xi_k>$ and $\bar{\mu} = <\mu_k>$ be the sequences of non zero complex numbers.

We now introduce the following classes of 2-normed space S–valued vector sequences

$$\ell((S, \|.,.\|), \bar{\xi}, \bar{u}) = \{\bar{s} = <s_k> \in \omega(S) \text{ satisfying } \sum_{k=1}^{\infty} \|\xi_k s_k, t\|^{u_k} < \infty, \text{for each } t \in S\}.$$

In fact, this class is a generalization of the familiar sequence spaces, studied in Srivastava and Pahari ([6], [7], [8]) , Pahari [12], using 2-norm .

4. Main Results

In this section, we shall investigate some results that characterize the linear topological structure of the class $\ell((S, \|.,.\|), \bar{\xi}, \bar{u})$ of 2-normed space S- valued sequences by endowing it with suitable natural paranorm. Throughout the work, we denote

$\sum_{(k,1)}$ for $\sum_{k=1}^{\infty}$, $\sum_{(k,n)}$ for $\sum_{k=n}^{\infty}$, $z_k = |\xi_k \mu_k^{-1}|^{u_k}$, $\sup u_k = M$ and for scalar α, $A[\alpha] = \max(1, |\alpha|)$.

But when the sequences $<u_k>$ and $<v_k>$ occur, then to distinguish M we use the notations $M(u)$ and $M(v)$ respectively.

Theorem 4.1 . $\ell((S, \|.,.\|), \bar{\xi}, \bar{u})$ *forms a linear space over the field of complex numbers* $\boldsymbol{C}$ *if* $<u_k>$ *is bounded above*.

Proof. Assume that $\sup_k u_k < \infty$ and $\bar{s} = <s_k>$, $\bar{w} = <w_k> \in \ell(S, \|.,.\|, \bar{\xi}, \bar{u})$ .So that for each $t \in S$, we have

$$\sum_{(k,1)} \|\xi_k s_k, t\|^{u_k} < \infty \text{ and } \sum_{(k,1)} \|\xi_k w_k, t\|^{u_k} < \infty.$$

Let $0 < u_k \le \sup_k u_k = M$, $D = \max(1, 2^{M-1})$ and setting $2D\max(1, |\alpha|^M) \le 1$ and $2D\max(1, |\beta|^M) \le 1$ and using

$$|a+b|^{u_k} \le D\{|a|^{u_k} + |b|^{u_k}\} \text{ for all } a, b \in \mathbf{C}.$$

Then we have

$$\sum_{(k,1)} \|\xi_k(\alpha s_k + \beta w_k), t\|^{u_k} \le \sum_{(k,1)} [D|\alpha|^{u_k} \|\xi_k s_k, t\|^{u_k} + D|\beta|^{u_k} \|\xi_k w_k, t\|^{u_k}]$$

$$\le \sum_{(k,1)} [D\, A[|\alpha|^M] \|\xi_k s_k, t\|^{u_k} + D\, A[|\beta|^M] \|\xi_k w_k, t\|^{u_k}]$$

$$\le \frac{1}{2} \sum_{(k,1)} \|\xi_k s_k, t\|^{u_k} + \frac{1}{2}$$

$$\sum_{(k,1)} \|\xi_k w_k, t\|^{u_k} < \infty,$$

for each $t \in S$ and therefore $\alpha\bar{s} + \beta\bar{w} \in \ell((S, \|. , .\|), \bar{\xi}, \bar{u})$.

This implies that $\ell((S, \|. , .\|), \bar{\xi}, \bar{u})$ forms a linear space over $\boldsymbol{C}$.

Theorem 4.2 . *If $\ell((S, \|. , .\|), \bar{\xi}, \bar{u})$ forms a linear space over $\boldsymbol{C}$ then $< u_k >$ is bounded above.*

Proof.

Suppose that $\ell((S, \|. , .\|), \bar{\xi}, \bar{u})$ forms a linear space over $\boldsymbol{C}$ but $sup_k\, u_k = \infty$. Then there exists a sequence $< k(n) >$ of positive integers satisfying $1 \le k(n) < k(n+1)$, $n \ge 1$ for which

$$u_{k(n)} > n \text{ , for each } n \ge 1 \qquad (4.1)$$

Now, corresponding to $s_0 \in S$ and $s_0 \ne \theta$, we define the sequence $\bar{s} = < s_k >$ by

$$s_k = \begin{cases} \xi_{k(n)}^{-1}\, n^{-2/u_{k(n)}}\, s_0, \text{ if } k = k(n) \text{ , } n \ge 1 \text{ and} \\ \theta, \text{ otherwise.} \end{cases} \qquad (4.2)$$

Then for $k = k(n)$, $n \ge 1$, we have

$$\sum_{(k,1)} \|\xi_k s_k, t\|^{u_k} = \sum_{(n,1)} \|n^{-2/u_{k(n)}} s_0, t\|^{u_{k(n)}} = \sum_{(n,1)} \frac{\|s, s_0\|^{u_{k(n)}}}{n^2}$$

$$\le A[\|s_0, t\|^{M(u)}] \sum_{(n,1)} \frac{1}{n^2} < \infty,$$

and $\|\xi_k s_k, t\|^{u_k} = 0$, for $k \ne k(n)$, $n \ge 1$,

showing that $\bar{s} \in \ell((S, \|. , .\|), \bar{\xi}, \bar{u})$. But on the other hand, let us choose $t_0 \in S$ such that $\|s_0, t_0\| = 1$. Then for such t_0 and scalar $\alpha = 4$, for $k = k(n)$, $n \ge 1$, in view of (4.1) and (4.2), we have

$$\sum_{(k,1)} \|\alpha\, \xi_k s_k, t_0\|^{u_k} = \sum_{(n,1)} \|\xi_{k(n)}\, \alpha\, s_{k(n)}, t_0\|^{u_{k(n)}}$$

$$= \sum_{(n,1)} \|4\; n^{-2/u_{k(n)}} s_0, t_0\|^{u_{k(n)}} = \sum_{(n,1)} \frac{4^{u_{k(n)}}}{n^2} \|s_0, t_0\|$$

$$\ge \sum_{(n,1)} \frac{4^n}{n^2} > 1 .$$

This shows that $\alpha\bar{s} \notin \ell((S, \|. , .\|), \bar{\xi}, \bar{u})$, a contradiction. This completes the proof.

The following result is an immediate consequence of Theorems 4.1 and 4.2.

Theorem 4.3 . *$\ell((S, \|. , .\|), \bar{\xi}, \bar{u})$ is a linear space over $\boldsymbol{C}$ if and only if $sup_k\, u_k < \infty$.*

Theorem 4.4 . *The space $\ell((S, \|. , .\|), \bar{\xi}, \bar{u})$ forms a solid.*

Proof. Let $\bar{s} = < s_k > \in \ell((S, \|. , .\|), \bar{\xi}, \bar{u})$. So that for each $t \in S$,

$$\sum_{(k,1)} \|\xi_k s_k, t\|^{u_k} < \infty.$$

Let $< \gamma_k >$ be a sequence of scalars satisfying $|\gamma_k| \le 1$ for all $k \ge 1$.Then we have

$$\sum_{(k,1)} \|\xi_k \gamma_k s_k, t\|^{u_k} = \sum_{(k,1)} |\gamma_k|^{u_k} \|\xi_k s_k, t\|^{u_k}$$

$$\le \sum_{(k,1)} \|\xi_k s_k, t\|^{u_k} < \infty,$$

for each $t \in S$. This shows that $< \gamma_k s_k > \in \ell((S, \|. , .\|), \bar{\xi}, \bar{u})$ and hence $\ell((S, \|. , .\|), \bar{\xi}, \bar{u})$ is normal.

Let $\bar{u} = < u_k >$ such that $\sup_k u_k < \infty$ and $\bar{s} = < s_k > \in \ell((S, \|. , .\|), \bar{\xi}, \bar{u})$. We define a real valued function

$$\Phi_{\xi,u}(\bar{s}) = \{ (\sum_{(k,1)} \|\xi_k s_k, t\|^{u_k})^{1/M} \text{ , for each } t \in S \}. \qquad (4.3)$$

Throughout the work, Φ will denote $\Phi_{\xi,u}$ and $\bar{u} = < u_k >$, $\bar{v} = < v_k >$ such that $\sup_k u_k < \infty$ and $\sup_k v_k < \infty$. We prove below that $\ell((S, \|. , .\|), \bar{\xi}, \bar{u})$ with respect to Φ forms a paranormed space.

Theorem 4.5. *$\ell((S, \|. , .\|), \bar{\xi}, \bar{u})$ forms a total paranormed - space with respect to Φ.*

Proof.Let $\alpha \in C$ and $\bar{s} = < s_k >$, $\bar{w} = < w_k > \in \ell((S, \|. , .\|), \bar{\xi}, \bar{u})$.Then we can easily verify that Φ satisfy the following properties of paranorm.

PN_1. $\Phi(\bar{s}) \ge 0$, and $\Phi(\bar{s}) = 0$ if and only if $\bar{s} = \bar{\theta}$;

PN_2. $\Phi(\bar{s} + \bar{w}) \le \Phi(\bar{s}) + \Phi(\bar{w})$;

PN_3. $\Phi(\alpha\bar{s}) \le A(\alpha)\, \Phi(\bar{s})$;

PN_4. Finally for continuity of scalar multiplication, it is sufficient to show that

(a) $\Phi(\overline{s}^{(n)}) \to 0$ and $\gamma_n \to \gamma$ imply $\Phi(\gamma_n \overline{s}^{(n)}) \to 0$; and

(b) $\gamma_n \to 0$ implies $\Phi(\gamma_n \overline{s}) \to 0$ for each $\overline{s} \in \ell((S, \|.,.\|), \overline{\xi}, \overline{u})$.

Now to prove (a) suppose $|\gamma_n| \le L$ for all $n \ge 1$, then in view of (4.3), we have

$$\Phi(\gamma_n \overline{s}^{(n)}) = \{ (\sum_{(k,1)} \|\gamma_n \xi_k s_k, t\|^{u_k})^{1/M}, \text{ for each } t \in S \}$$

$$\le \sup_k |\gamma_n|^{u_k/M} \{ (\sum_{(k,1)} \|\xi_k s_k, t\|^{u_k})^{1/M}, \text{ for each } t \in S \}$$

$$\le A(L)\, \Phi(\overline{s}^{(n)}),$$

whence (a) follows.

Next if $\overline{s} \in \ell((S, \|.,.\|), \overline{\xi}, \overline{u})$, then for $\varepsilon > 0$ there exists an integer K such that

$$\sum_{(k,K)} \|\xi_k s_k, t\|^{u_k} < \left(\frac{\varepsilon}{2}\right)^M, \text{ for each } t \in S.$$

Further if $\gamma_n \to 0$, we can find N such that for $n \ge N$, then for each $t \in S$, we have

$$\sum_{(k,K\text{-}1)} |\gamma_n|^{u_k} \|\xi_k s_k, t\|^{u_k} < \left(\frac{\varepsilon}{2}\right)^M \text{ and } |\gamma_n| \le 1.$$

Thus for each $t \in S$,

$$\Phi(\gamma_n \overline{s}) \le \left(\sum_{k=1}^{K\text{-}1} \|\gamma_n \xi_k s_k, t\|^{u_k}\right)^{1/M} + \left(\sum_{(k,K)} \|\xi_k s_k, t\|^{u_k}\right)^{1/M} < \varepsilon,$$

for all $n \ge N$, and hence (b) follows.

Theorem 4.6. *If S is a Banach space, then $(\ell((S, \|.,.\|), \overline{\xi}, \overline{u}), \Phi)$ is complete.*

Proof. We prove the completeness of $\ell((S, \|.,.\|), \overline{\xi}, \overline{u})$ with respect to the metric $d(\overline{s}, \overline{t}) = \Phi(\overline{s} - \overline{t})$.

Let $< \overline{s}^{(n)} >$ be a Cauchy sequence in $\ell((S, \|.,.\|), \overline{\xi}, \overline{u})$. Then for $0 < \varepsilon < 1$, there exists N such that for all $n, m \ge N$ and for each $t \in S$, we have

$$\Phi(\overline{s}^{(n)} - \overline{s}^{(m)}) = (\sum_{(k,1)} \|\xi_k s_k^{(n)} - \xi_k s_k^{(m)}, t\|^{u_k})^{1/M} < \varepsilon. \quad (4.4)$$

and so for all $n, m \ge N$ and $k \ge 1$ and for each $t \in S$, we have

$$\|s_k^{(n)} - s_k^{(m)}, t\| < |\xi_k|^{-1} \varepsilon^{M/u_k} < |\xi_k|^{-1} \varepsilon.$$

This shows that for each k, $< s_k^{(n)} >$ is a Cauchy sequence in S and because of completeness of S, $s_k^{(n)} \to s_k \in S$ (say) for each k. Being a Cauchy sequence $< s_k^{(n)} >$ is bounded, i.e. $\Phi(s_k^{(n)}) \le L$ for some $L > 0$ and for all $n \ge 1$. Thus for every n and r,

$$(\sum_{k=1}^{r} \|\xi_k s_k^{(n)} - \xi_k, t\|^{u_k})^{1/M} \le L.$$

First taking $n \to \infty$ and then $r \to \infty$, then for each $t \in S$,

$$(\sum_{(k,1)} \|\xi_k s_k, t\|^{u_k})^{1/M} \le L$$

which implies that $\overline{s} = < s_k > \in \ell((S, \|.,.\|), \overline{\xi}, \overline{u})$.

Now for any r, by (4.4) we have

$$(\sum_{k=1}^{r} \|\xi_k s_k^{(n)} - \xi_k s_k^{(m)}, t\|^{u_k})^{1/M} < \varepsilon, \text{ for } n, m \ge N,$$

and so letting $m \to \infty$ first and then $r \to \infty$, we get

$$\Phi(\overline{s}^{(n)} - \overline{s}) = (\sum_{(k,1)} \|\xi_k s_k^{(n)} - \xi_k s_k, t\|^{u_k})^{1/M}$$

$$\le \varepsilon, \text{ for all } n \ge N \text{ and for each } t \in S$$

i.e. $\overline{s}^{(n)} \to \overline{s}$ in $\ell((S, \|.,.\|), \overline{\xi}, \overline{u})$, as $n \to \infty$. This proves the completeness of $\ell((S, \|.,.\|), \overline{\xi}, \overline{u})$.

Theorem 4.7. *For any $\overline{u} = < u_k >$, $\ell((S, \|.,.\|), \overline{\xi}, \overline{u}) \subset \ell((S, \|.,.\|), \overline{\mu}, \overline{u})$ if*

$$\liminf_k z_k > 0.$$

Proof. Assume that $\liminf_k z_k > 0$ and $\overline{s} = < s_k > \in \ell((S, \|.,.\|), \overline{\xi}, \overline{u})$. Then there exist $m > 0$ and a positive integer K such that $m|\mu_k|^{u_k} < |\xi_k|^{u_k}$ for all $k \ge K$ and for each $t \in S$,

$$\sum_{(k,K)} \|\xi_k s_k, t\|^{u_k} < \infty.$$

Thus for each $t \in S$, we have

$$\sum_{(k,K)} \|\mu_k s_k, t\|^{u_k} \le \sum_{(k,K)} \frac{|\xi_k|^{u_k}}{m} \|s_k, t\|^{u_k}$$

$$= \frac{1}{m} \sum_{(k,K)} \|\xi_k s_k, t\|^{u_k} < \infty.$$

This clearly implies that $\overline{s} \in \ell((S, \|.,.\|), \overline{\mu}, \overline{u})$ and hence

$$\ell((S, \|.,.\|), \overline{\xi}, \overline{u}) \subset \ell((S, \|.,.\|), \overline{\mu}, \overline{u}).$$

This completes the proof.

Theorem 4.8 . *For any* $\bar{\xi} = <\xi_k>$, *if* $u_k \le v_k$ *for all but finitely many values of* k, *then*

$$\ell((S, \|. , .\|), \bar{\xi}, \bar{u}) \subset \ell((S, \|. , .\|), \bar{\xi}, \bar{v}).$$

Proof. Suppose $0 < u_k \le v_k < \infty$ for all but finitely many values of k. Let $\bar{s} = < s_k >$ $\in \ell((S, \|. , .\|), \bar{\xi}, \bar{u})$ .Then we have

$$\sum_{(k,1)} \|\xi_k s_k, t\|^{u_k} < \infty, \text{for each } t \in S.$$

This shows that there exists $K \ge 1$ such that $\|\xi_k\ s_k, t\| < 1$ for all $k \ge K$ and for each $t \in S$.

Thus $\|\xi_k s_k , t\|^{v_k} \le \|\xi_k s_k , t\|^{u_k}$ for all $k \ge K$ and for each $t \in S$ and consequently

$$\sum_{(k,K)} \|\xi_k s_k, t\|^{v_k} \le \sum_{(k,K)} \|\xi_k s_k, t\|^{u_k} < \infty \text{ ,for each } t \in S$$

and hence $\bar{s} \in \ell((S, \|. , .\|), \bar{\xi}, \bar{v})$.This completes the proof of the theorem.

The following result is an immediate consequence of Theorems 4.7 and 4.8.

Theorem 4.9. If *lim inf*$_k$ $z_k > 0$; and $u_k \le v_k$ *,for all but finitely many values of* k, *then*

$$\ell((S, \|. , .\|), \bar{\xi}, \bar{u}) \subset \ell((S, \|. , .\|), \bar{\mu}, \bar{v}) .$$

In the following example, we conclude that $\ell((S, \|. , .\|), \bar{\xi}, \bar{u})$ may strictly be contained in $\ell((S, \|. , .\|), \bar{\mu}, \bar{v})$ inspite of the satisfaction of both conditions of Theorem 4.9.

Example 4.10.

Let $(S, \|. , .\|)$ be a 2- normed space and consider a sequence $\bar{s} = < s_k >$ defined by

$s_k = k^{-2k}\, s$, if $k = 1, 2, 3,\ldots$,where $s \in S$ and $s \neq \theta$.

Further, let $u_k = k^{-1}$, if k is odd integer, $u_k = k^{-2}$, if k is even integer, $v_k = k^{-1}$ for all values of k,

$\xi_k = 3^k$, $\mu_k = 2^k$ for all values of k.

Then, $z_k = \left|\frac{\xi_k}{\mu_k}\right|^{u_k} = \frac{3}{2}$ or $\left(\frac{3}{2}\right)^{1/k}$ according as k is odd or even integers and hence lim inf$_k$ $z_k > 0$.

Further, $\frac{v_k}{u_k} = 1$, if k is odd integers, $\frac{v_k}{u_k} = k$, if k is even integers. Therefore $0 < u_k \le v_k < \infty$ for all k.

Hence both conditions of Theorem 4.9 are satisfied.

Now for each $t \in S$, we have

$$\sum_{(k,1)} \|\mu_k s_k, t\|^{v_k} = \sum_{(k,1)} \|2^k k^{-2k} s, t\|^{1/k} = \sum_{(k,1)} 2\, k^{-2} \|s ,t\|^{1/k}$$

$$\le 2A\, [\, \|s , t\|\,] \sum_{(k,1)} k^{-2} < \infty.$$

This shows that $\bar{s} \in \ell((S, \|. , .\|), \bar{\mu}, \bar{v})$. But on the other hand, let us choose $t \in S$ such that $\|s, t\| = 1$. Then for each even integer k, we have

$$\|\xi_k s_k, t\|^{u_k} = \|3^k k^{-2k} y , t\|^{1/k^2} = \left(\frac{3}{k^2}\right)^{1/k} \|s, t\|^{1/k^2} > \frac{1}{2}$$

.

This implies that $\bar{s} \notin \ell((S, \|. , .\|), \bar{\xi}, \bar{u})$ and hence the containment of $\ell((S, \|. , .\|), \bar{\xi}, \bar{u})$ in $\ell((S, \|. , .\|), \bar{\mu}, \bar{v})$ is strict.

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