[Volume 3 issue 5 May 2015]
Page No.973-982 ISSN :2320-7167
INTERNATIONAL JOURNAL OF MATHEMATICS AND COMPUTER RESEARCH

Proof of Goldbach's conjecture

Boris Sklyar
23-2-28, Nelidovskaya st., Moscow, 125363, Russia
E-mail:-brs.sklr@mail.ru

Abstract
Basing on derived earlier "matrix definition" of prime numbers [1] it will be shown that: Any even natural number N=2n, n=5,6,7,… can be presented as a sum of two prime numbers N=2n= Pr1+Pr2.
Employment of "matrix definition" of prime numbers in the course of proof of Goldbach"s conjecture providesadditional characteristic of primes making proof very simple.

Introduction
In the paper [1] "matrix definition" of prime numbers was formulated as follows:Natural numbers that are **not** contained in arrays

$$\mathrm{P1}(i,j) = 6ij + i - j - 1 = \begin{pmatrix} 5 & 10 & 15 & 20 & 25 & 30 & \dots \\ 0 & 23 & 34 & 45 & 56 & 67 & \dots \\ 0 & 0 & 53 & 70 & 87 & 104 & \dots \\ 0 & 0 & 0 & 95 & 118 & 141 & \dots \\ 0 & 0 & 0 & 0 & 149 & 178 & \dots \\ . & . & . & . & . & 215 & \dots \end{pmatrix};\ i = 1,2,3,4\dots;\ j \geq i$$

$$\mathrm{P2}(i,j) = 6ij - i + j - 1 = \begin{pmatrix} 5 & 12 & 19 & 26 & 33 & 40 & \\ 0 & 23 & 36 & 49 & 62 & 75 & \dots \\ 0 & 0 & 53 & 72 & 91 & 110 & \dots \\ 0 & 0 & 0 & 95 & 120 & 145 & \dots \\ 0 & 0 & 0 & 0 & 149 & 180 & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & 215 & \dots \end{pmatrix};\ i = 1,2,3,4\dots;\ j \geq i$$

are indexes P of **all primes** in the sequence $S_1(P)$=6 P +5.
Natural numbers that are **not** contained in arrays

$$P3(i,j) = 6ij - i - j - 1 = \begin{pmatrix} 3 & 8 & 13 & 18 & 23 & 28 & \dots \\ 0 & 19 & 30 & 41 & 52 & 63 & \dots \\ 0 & 0 & 47 & 64 & 81 & 98 & \dots \\ 0 & 0 & 0 & 87 & 110 & 133 & \dots \\ 0 & 0 & 0 & 0 & 139 & 168 & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & 203 & \dots \end{pmatrix} \text{i; } i = 1,2,3,4\dots;\ j \geq i$$

$$P4(i,j) = 6ij + i + j - 1 = \begin{pmatrix} 7 & 14 & 21 & 28 & 35 & 42 & \dots \\ 0 & 27 & 40 & 53 & 66 & 79 & \dots \\ 0 & 0 & 59 & 78 & 97 & 116 & \dots \\ 0 & 0 & 0 & 103 & 128 & 153 & \dots \\ 0 & 0 & 0 & 0 & 159 & 190 & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & 227 & \dots \end{pmatrix};\quad i = 1,2,3,4\dots;\ j \geq i$$

are indexes P of **all primes** in the sequence $S_2(P)$=6 P +7.

Let us modify it; change indexes

i_{new}=i; j_{new}=j-i+1; j= j_{new}+i-1

we have (omitting notation “new”):

P1(i,j)=6i^2-1+(6i-1)(j-1); P2(i,j)=6i^2-1+(6i+1)(j-1);

P3(i,j)=6i^2-1-2i+(6i-1)(j-1); P4(i,j)=6i^2-1+2i+(6i+1)(j-1);

“Matrix definition” of prime numbers can be presented in the following form:

Natural numbers which **do not appear** in any one of two arrays

$$P1(i,j) = 6i^2 - 1 + (6\mathrm{i} - 1)(\mathrm{j} - 1) = \begin{pmatrix} 5 & 10 & 15 & 20 & 25 & 30 & \dots \\ 23 & 34 & 45 & 56 & 67 & 78 & \dots \\ 53 & 70 & 87 & 104 & 121 & 138 & \dots \\ 95 & 118 & 141 & 164 & 187 & 210 & \dots \\ 149 & 178 & 207 & 236 & 265 & 294 & \dots \\ . & . & . & . & . & . & \dots \end{pmatrix};\ i,j = 1,2,3,4\dots$$

$$P2(i,j) = 6i^2 - 1 + (6\mathrm{i} + 1)(\mathrm{j} - 1) = \begin{pmatrix} 5 & 12 & 19 & 26 & 33 & 40 & \\ 23 & 36 & 49 & 62 & 75 & 88 & \dots \\ 53 & 72 & 91 & 110 & 129 & 148 & \dots \\ 95 & 120 & 145 & 170 & 195 & 220 & \dots \\ 149 & 180 & 211 & 242 & 273 & 304 & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & . & \dots \end{pmatrix};\ i,j = 1,2,3,4\dots$$

are **indexes P of all prime numbers** in the sequence S_1(p)=6p+5, natural numbers which **do not appear** in any one of two arrays

$$P3(i,j) = 6i^2 - 1 - 2i + (6i-1)(j-1) = \begin{pmatrix} 3 & 8 & 13 & 18 & 23 & 28 & \dots \\ 19 & 30 & 41 & 52 & 63 & 74 & \dots \\ 47 & 64 & 81 & 98 & 115 & 132 & \dots \\ 87 & 110 & 133 & 156 & 179 & 202 & \dots \\ 139 & 168 & 197 & 226 & 255 & 284 & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & . & \dots \end{pmatrix} ;\ i,j = 1,2,3,4 \dots$$

$$P4(i,j) = 6i^2 - 1 + 2i + (6i+1)(j-1) = \begin{pmatrix} 7 & 14 & 21 & 28 & 35 & 42 & \dots \\ 27 & 40 & 53 & 66 & 79 & 92 & \dots \\ 59 & 78 & 97 & 116 & 135 & 154 & \dots \\ 103 & 128 & 153 & 178 & 203 & 228 & \dots \\ 159 & 190 & 221 & 252 & 283 & 314 & \dots \\ \vdots & \vdots & \vdots & \vdots & \dots & . & \dots \end{pmatrix} ;\quad i,j = 1,2,3,4 \dots$$

are **indexes P of all prime numbers** in the sequence $S_1(p)=6p+7$.

Proof of Goldbach's conjecture[bs1][bs2]

It is well known that all prime numbers (>=5) belong to one of two sequences

$S_1(p)=6p+5;\qquad S_2(p)=6p+7;\quad p=0, 1, 2, 3,\dots$

So the sums of two prime numbers belong to one of three sequences

$Q_1(p_e)= S_1(P_1)+S_1(P_2)= 6P_1+5+6P_2+5= 6(P_1+P_2)+10=6p_e+10=10, 16, 22,\dots$ (a)

$Q_2(p_e) =S_1P_1)+S_2(P_2)= 6P_1+5+6P_2+7= 6(P_1+P_2)+12=6p_e+12=12, 18, 24 \dots$ (b)

$Q_3(p_e)=S_2(P_1)+S_2(P_2)= 6P_1+7+6P_2+7= 6(P_1+P_2)+14 =6p_e+14=14, 20, 26, \dots , p=0,1,2,3,\dots$ (c)

where p_e – index of even number $N_e=2n$, P_1, P_2 – indexes of prime numbers in the sequences $S_1(p)$ and $S_2(p)$. (Capital letters in the notations of indexes refer to prime numbers, line letters - to composite or unknown numbers).

Obviously, the statement (1) is correct if and only if any natural number can be presented as a sum of two indexes of prime numbers:

a) both belonging to the sequence $S_1(p)$, $Q_1(p)=S_1(p_e/2+K)+S_1(p_e/2-K)$, (5)
or
b) one belonging to the sequence S1(p) and other – to S2(p),), $Q_2(p)=S_1(p_e/2+K)+S_2(p_e/2-K)$ (6)

or

c) both belonging to the sequence S2(p),), $Q_3(p)=S_2(p_e/2+K)+S_2(p_e/2-K)$, (7)

where k, K – additional index, starting from $-p_e/2$.

$$S(k)=S(p_e/2+k) , k=-p_e/2, -p_e/2+1,..,p_e/2-1, p_e/2 \quad (8)$$

Case a)

As an example let $N_e=250$, $p_e=(250-10)/6=40=P_1+P_2$.

Wright down in line the row of indexes P of prime numbers of the sequence $S_1(P)$ (denote by corresponding numbers) and indexes p of composite numbers (denote by X)

0 1 2 3 4 X 6 7 8 9 X 11 X 13 14 X 16 17 18 X X 21 22 24 X X 27 28 29 X…

Under this line wright down the same row in invert order:(A)

0 1 2 3 4 X 6 7 8 9 X 11 X 13 14 X 16 17 18 X **X** 21 22 X 24 X X … 38 39

+ 39 38 37 X X X X 32 31 X 29 28 27 X X 24 X 22 21**X**X 18 17 16 X 14 …2 1

= 40 40 40 40 40 40 40 40 40 40 40 40 40

In general for given natural number $N_e=6p_e+10$ we have:

0 1 2 3 4 X 6 7 …$p_e/2$-1$p_e/2$$p_e/2$+1$p_e/2$+2…$p_e/2$+k…$p_e$-5 p_e-4 p_e-3 p_e-2 p-1 p

+ p_e-1 p_e-2 p_e-3 … … … $p_e/2$+1 $p_e/2$ $p_e/2$-1 $p_e/2$- 2… $p_e/2$-k… X4 3 2 1 0

= p_e … p_e ….

In order to satisfy condition (5) at least one pair $P_1 = p_e/2+K$ and $P_2=p_e/2-K$; $k=1,2,..,p_e/2$ must be the pair of indexes of prime numbers.

Prove by contradiction that there is always at least one such pair. Suppose that for all primes $S_1(p_e/2+K)$ all members $S_1(p_e/2-K)$ are composite. That means that all indexes $(p_e/2-K)$ must appear in array P1(i,j) or in array P2(i,j) and **all** equations

$$6i^2-1+(6i-1)(j-1)=p_e/2-K \text{ or} \quad (9)$$

$$6i^2-1 +(6i+1)(j-1)=p_e/2-K \quad (10)$$

have integer solutions.

From (9) we have

$K=p_e/2-(6i^2-1)-(6i-1)(j-1)$ (11)

From (10) we have

$K= p_e/2 -(6i^2-1)-(6i+1)(j-1)$ (12)

Denote***N(u)* ={**1,2,3,4,….,…,p_e-1,p_e**}**- array of natural numbers, ***N(u)=u.***

P1(u) =**{**1,2,3,4, 0, 6,7,8,9, 0, 11, 0,13,14, 0,16,17,18, 0, 0, 21,22, 0, 24, 0, 0, 27,28,29, 0,…,p_e-1,p_e**}** – array of indexes of primes in S1(P), ***P1(u)=u*** if ***u* –** index of a prime, ***u***=0 if corresponding number is composite.

P12(u)=**{**0,0,0,0,5,0,0,0,0,10,0,12,0,0,15,0,0,0,19,20,0, 0,23,0,25,26,0,0,0,30,…, p_e-1,p_e**}**, ***P12(u)=u*** if number ***u*** presents in arrays P1(i,j)or in array P2(i,j), ***P12(u)***=0, if corresponding number does not appear in array P1(i,j) or in arrayP2(i,j).

$p_e/2$ = **{** $p_e/2$, $p_e/2$,… $p_e/2$**}** **=**const

We have:

P1(u)=N(u)-P12(u) (in accordance with matrix definition of primes).

P1(u)= $p_e/2$+***K1(u);*** ***K1(u)=P1(u)-*** $p_e/2$=***N(u)-P12(u)-*** $p_e/2$; ( )

From the other hand (since we suppose that all numbers ($p_e/2$ –***K(u)***) are composite)

$p_e/2$ –***K(u)=P12(u); K(u)=*** $p_e/2$-***P12(u)***

N(u)-P12(u)- $p_e/2$= $p_e/2$-***P12(u);***

N= $p_e/2$ –contradiction,

so all numbers ($p_e/2$-K) cannot be composite and there is at least one pair of indexes of primes P_1 = p/2+K and P_2=p/2-K.

For example, from (11) and (12) for p_e=40 possible values of K are 1, 5, 8, 10,15, but from

(A): ***K(u)=P1(u)***-$p_e/2$=1, 2, 4, 7, 8, 9…(13)

For p_e=800 possible values for K are:

0, 3, 5, 6, 7, 10, 13, 14, 15, 17, 19, 21, 24, 25, 26, 29, 30…

But***K(u)=P1(u)***-$p_e/2$=1,2,3,6,7,9,12,21,23,…

So, it is obvious that values of K, calculated in accordance with (11) ,(12) and (13) cannot coincide fully.

Case c)
The same as case a) but for sequence S2(P).

Case b)

As an example let $N_e=252$, $p_e=(252-12)/6=40=P_1+P_2$.

Wright down in line the row of indexes P of prime numbers of the sequence $S_1(P)$ (denote by corresponding numbers) and indexes p of composite numbers (denote by X)

0 1 2 3 4 X 6 7 8 9 X 11 X 13 14 X 16 17 18 X X 21 22 24 X X 27 28 29 X…

Under this line wright down the row of indexes P of prime numbers of the sequence S2(P) in invert order:

0 1 2 3 4 X 6 7 8 9 X 11 X 13 14 X 16 17 18 X **X** 21 22 X 24 X … 37 38 39

\+ 39 X 37 36 X 34 X 32 31 X 29 X X 26 25 24 X 22 X 20 X X 17 16 15.... X21 --- = 40 40 40 40 40 40 40 4040 40 40 40 40

In general for given natural number $N_e=6p_e+12$ we have:

0 1 2 3 4 X 6 7 …$p_e/2-1$ $p_e/2$ $p_e/2+1$ $p_e/2+2$… $p_e/2+k$… p_e-5 p_e-4 p_e-3 p_e-2 p-1 p

\+ p_e-1 p_e-2 p_e-3 … … … $p_e/2+1$ $p_e/2$ $p_e/2-1$ $p_e/2-2$… $p_e/2-k$… 5 4 X 2 1 0

= p_e … p_e

We have:

P1(u)=N(u)-P12(u) (in accordance with matrix definition of primes).

P1(u)= $p_e/2$***+K(u); K(u)=P1(u)-*** $p_e/2$=***N(u)-P12(u)-*** $p_e/2$; (14)

From the other hand (since we suppose that all numbers ($p_e/2$ –***K(u)***) are composite)

$p_e/2$ –***K(u)=P12(u); K(u)=*** $p_e/2$-***P12(u);*** (15)

N(u)-P12(u)- $p_e/2$= $p_e/2$-***P12(u);***

N(u)= $p_e/2$ – contradiction,

So, it is obvious that values of K, calculated in accordance with (14) and (15) cannot coincide fully and there is always at least one pair of indexes such that $S1(P1)+S2(P2)=Q2(p_e)$; $p_e=P1+P2$.

Conclusion

Goldbach's conjecture has been proved.
C++ program for finding primes satisfying Goldbach's conjecture is presented in Attachment.

References

[1]]. http://ijmcr.in/index.php/current-issue/86-title-matrix-sieve-new-algorithm-for-finding-prime-numbers

Attachment 1

```
#include <cstdlib>

#include <iostream>

#include <math.h>

#include <ctime>

using namespace std;

 main( )

{

/* PROOF OF GOLDBACH CONJECTURE*/

 /*CALCULATING PRIMES, SUM OF TWO PRIMES EQUALS GIVEN EVEN NUMBER N<10^18*/

 /*N=Pr1+Pr2*/

 unsigned long long int N=11122233345566; int nd=3000;

 if (N<1000000) nd=300;

 if (N<1000) nd=150;

unsigned long long int N1=N/2-nd; unsigned long long int N2 =N1+2*nd;

unsigned long long int p1=floor(N1/6); unsigned long long int p2=ceil( N2/6);

 int r=84000; int R2[r]; int rm=p2-p1;unsigned long long int S2[r];int r3, r4, v, k;

 int q=84000; int R1[q] ; int qm=rm; unsigned long long int S1[q], Q1, Q2, Ne, Nd, Nd1, Nd2; int q2, q1;

 for (q=1;q<qm;q++)

 R1[q] =1;

 for (r=1;r<rm;r++)

 R2[r] =1;
```

```
unsigned long long        int i, j, P1, P2, P3, P4, B, K;
        unsigned long long        int i2= sqrt( p2/6)+2;
long long int j1, j2;
        int l1=0;int l2=0;
float m1=(long double ) (N-10)/6-(N-10)/6;
float m2=(long double ) (N-12)/6-(N-12)/6;
float m3=(long double ) (N-14)/6-(N-14)/6;
        for ( i=1;i<i2;i++)
        { j2=(p2+i+1)/( 6*i+1)+1;j1=(p1+i+1)/( 6*i+1);
        B=5+5*( i-1); K=7+6*( i-1);
        if ( i>j1) j1=i;
        for(j=j1; j<j2;j++)
{        P1=B+K*( j-1);
        if(( P1>p1)&&( P1<p2))
        { q1=P1-p1; R1[ q1] =0;
} }
j2=(p2-i+1)/( 6*i-1)+1;j1=(p1-i+1)/( 6*i-1);
if (j1<1) j1=1;
        B=5+7*( i-1); K=5+6*( i-1);
 if ( i>j1-1) j1=i+1;
        for(j=j1; j<j2;j++)
{        P2=B+K*( j-1);
if(( P2>p1)&&( P2<p2))
{ q2=P2-p1; R1[ q2] =0;
 } }
 j2=(p2+i+1)/( 6*i-1)+1;j1=(p1+i+1)/( 6*i-1);
B=3+5*( i-1); K=5+6*( i-1);
if ( i>j1) j1=i;
```

```
for(j=j1; j<j2;j++)
{       P3=B+K*( j-1);
if(( P3>p1)&&( P3<p2))
{ r3=P3-p1; R2[ r3]=0;
 } }
j2=(p2-i+1)/( 6*i+1)+1;j1=(p1-i+1)/( 6*i+1);
B=7+7*(i-1); K=7+6*(i-1);
        if ( i>j1) j1=i;
                for(j=j1; j<j2;j++)
{ P4=B+K*( j-1);
if(( P4>p1)&&( P4<p2))
{ r4=P4-p1; R2[ r4] =0;
} } }
 for ( q=1;q<qm;q++) { S1[q] =R1[q]*((p1+q)*6+5); if (S1[q]%5==0) continue; l1=l1+1;}
 for ( r=1;r<rm;r++) { S2[r] =R2[r]*((p1+r)*6+7);if (S2[r]%5==0) continue;l2=l2+1;}
 if (m1==0){ cout<<"N belongs to the sequence N=6p+10; m1="<<m1<<" \n\n";
        for (v=1;v<1000;v++) {
         Q1=S1[v];
         for (k=1;k<1000;k++) {
         Q2=S1[k]; Ne=Q1+Q2;
        if (Ne==N){Nd=N; Nd1=Q1;Nd2=Q2;
                break;}}}}
         if (m2==0){ cout<<"N belongs to the sequence N=6p+12; m2="<<m2<<" \n \n";;
        for (v=1;v<1000;v++) {
         Q1=S1[v];
         for (k=1;k<1000;k++) {
         Q2=S2[k]; Ne=Q1+Q2;
        if (Ne==N){Nd=N; Nd1=Q1;Nd2=Q2;
```

```
                break;}}}}
if (m3==0){ cout<<"N belongs to the sequence N=6p+14; m3="<<m3<<" \n \n";
       for (v=1;v<1000;v++) {
        Q1=S2[v];
        for (k=1;k<1000;k++) {
        Q2=S2[k]; Ne=Q1+Q2;
if (Ne==N){Nd=N; Nd1=Q1;Nd2=Q2;
                break;}}}}
cout<<" N=Pr1+Pr2="<<Nd<<"; Pr1="<<Nd1<<"; Pr2="<<Nd2<<"; \n\n";
       cout<<"run time(ms)="; cout<<clock();
 system("PAUSE");
 return EXIT_SUCCESS;
 }
```