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CHOOSING A CONSTANT 'α' IN EXPONENTIAL SMOOTHING USING ARMA MODEL

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ABSTRACT

Time series analysis and forecasting process plays an important role in business, atmospheric studies, insurance companies, banking sectors etc. There are many forecasting techniques like moving averages, double moving average, multiple moving averages, simple exponential smoothing, adaptive smoothing, double exponential smoothing, triple exponential smoothing, autoregressive integrated moving averages, etc. In simple exponential smoothing, constant 'α' is not fixed, it may vary from 0 to 1. In this paper, we discuss about 'α' where it is estimated through some process. We estimating the constant of exponential smoothing using autoregressive moving average models by various autoregressive and moving average parameters. We fitting a new exponential smoothing model by fixing value to 'α'. To check for a goodness of fit, we use Kolmogrov - Smirnov test to simple exponential smoothing and new exponential smoothing models. Mean square error criteria are used for the purpose of choosing best model between simple exponential smoothing model and new exponential smoothing model.

KEY WORDS: Simple Exponential Smoothing, Autoregressive Moving Average, Errors, Absolute errors, Mean Square Error.

INTRODUCTION

Forecasting plays an important role in national economy, customers, products in industry, marketing organization, executive offices, atmospheric studies, financial organizations, etc. There are many forecasting models like moving averages, exponential smoothing, auto regression, autoregressive moving averages (ARMA), autoregressive integrated moving averages (ARIMA), vector autoregressive models, autoregressive conditional heteroscedasticity (ARCH), generalized autoregressive conditional heteroscedasticity (GARCH), etc. In moving averages, we take same weight for each observation.

Exponential smoothing is a procedure for continually revising a forecast in the light of more recent emporia. Exponential smoothing assigns exponentially decreasing weights as the observation get older. In other words, relatively more weights are given for recent observations in forecasting than the older observations. Exponential smoothing is a simple technique used to smooth and forecast a time-series without the necessity of fitting a parametric model.

SIMPLE EXPONENTIAL SMOOTHING

Simple exponential smoothing is also called a single exponential smoothing. The parameter in simple exponential smoothing is 'α'. Simple exponential smoothing is used for short range forecasting, usually just for one month to the future. The model of the simple exponential smoothing is

$$S_t = Y_t + (1- \alpha)S_{t-1}$$

where S_t = forecast for time 't'

S_{t-1} = forecast for time 't-1'

Y_t = time series observation at time 't'

α = constant

α lies between 0 and1

$\alpha + \beta = 1$

DOUBLE EXPONENTIAL SMOOTHING

It is an extension of single exponential smoothing method. If we compute single exponential smoothing to earlier single exponential smoothing, then we get double exponential smoothing. Under double exponential smoothing, the two smoothing averages are defined by

$S_T = \alpha\, Y_T + (1 - \alpha)\, S_{T-1}$

$S_T^{(2)} = \alpha\, S_T + (1 - \alpha)\, S_{T-1}^{(2)}$

where T is current time period

TRIPLE EXPONENTIAL SMOOTHING

It is an extension to double exponential smoothing. If we compute single exponential Smoothing to double exponential smoothing, then we get triple exponential smoothing, the three smoothing averages are defined by

$S_T = \alpha\, Y_T + (1 - \alpha)\, S_{T-1}$

$S_T^{(2)} = \alpha\, S_T + (1 - \alpha)\, S_{T-1}^{(2)}$

$S_T^{(3)} = \alpha\, S_T^{(2)} + (1 - \alpha)\, S_{T-1}^{(3)}$

ARMA (AUTO REGRESSIVE MOVING AVERAGE)

The combination of auto regression of ‘p’ recent time-series observations Y_{t-1}, $Y_{t-2} \ldots Y_{t-p}$ with moving averages of a recent error terms e_t, $e_{t-1} \ldots e_{t-q}$ is known as ARMA (p, q)

The auto regressive moving average model of order (p, q) is

$$Y_t = c + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q} + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p}$$

In this paper, we estimated the parameter of simple exponential smoothing by using ARMA models. After estimating parameter, we fitted a new exponential smoothing and comparing with simple exponential smoothing based on forecasting accuracy mean square error (MSE)

METHODOLOGY

By using time series observations, we forecast future values. There are many models for forecasting, we now fitting an equation by using ARMA. In general, the time series data is in the form

t	t_1 t_2 t_3 … t_n
Y_t	Y_1 Y_2 Y_3 … Y_n

AUTO REGRESSION (AR)

p^{th} order auto regression equation is formed by taking regression equation of time series values of order t-1, t-2, … t-p with their 1, 2, … p^{th} order auto regressive parameter, added with a constant term ‘c’ and also error term ‘e_t’.

p^{th} order AR model is

$$Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} +$$

where c = constant term,

$\phi_p = p^{th}$ auto regressive parameter

e_t = the error term at time ‘t’.

MOVING AVERAGE

Moving average is the relation of error terms with time series value. The q^{th} order moving average model is multiplication of error terms of time points t-1, t-2, … t-q with θ_1, θ_2, …, θ_q moving average parameters subtracted from error term ‘e_t’ and constant ‘c’.

The q^{th} order moving average model is

$$Y_t = c + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \cdots - \theta_q e_{t-q}$$

where

$c =$ constant term

$\theta_q = q^{th}$ moving average parameter

$e_{t-k} =$ the error term at time $t - k$

AUTO REGRESSIVE MOVING AVERAGE MODEL (p, q)

Auto regressive moving average model (p, q) is a mixture of auto regression of order ‘p’ and moving averages of order ‘q’. The equation of ARMA (p, q) is

$$Y_t = c + \phi_1 Y_{t-1} + \cdots + \phi_p Y_{t-p} + e_t - \theta_1 e_{t-1} - \cdots - \theta_q e_{t-q}$$

where

Y_t = Forecast time series value at time 't'

$\phi_1, \phi_2, \cdots \phi_p$ = Auto regressive parameters

$\theta_1, \theta_2, \cdots \theta_q$ = Moving average parameters

$Y_{t-1}, Y_{t-2}, \cdots Y_{t-p}$ are time series values of $(t-1)^{th}$, $(t-2)^{th} \cdots (t-p)^{th}$ time point.

$e_{t-1}, e_{t-2}, \cdots e_{t-q}$ are error terms at time points $(t\text{-}1)^{th}$, $(t\text{-}2)^{th} \cdots (t\text{-}q)^{th}$.

$B, B^2, \cdots B^p$ are back shift operators of time series values

$B, B^2, \cdots B^q$ are back shift operators of error terms.

We fitted 9 ARMA models and they are

ARMA (1, 1): $Y_t = c + \phi_1 Y_{t-1} + e_t - \theta_1 e_{t-1}$

ARMA (1, 2): $Y_t = c + \phi_1 Y_{t-1} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}$

ARMA (1, 3): $Y_t = c + \phi_1 Y_{t-1} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3}$

ARMA (2, 1): $Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t - \theta_1 e_{t-1}$

ARMA (2, 2): $Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}$

ARMA (2, 3):

$$Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3}$$

ARMA (3, 1): $Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-3} + e_t - \theta_1 e_{t-1}$

ARMA (3, 2):

$$Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-3} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}$$

ARMA (3, 3):

$$Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \phi_3 Y_{t-3} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} - \theta_3 e_{t-3}$$

For determination of the best model among 9 ARMA models i.e., ARMA (1, 1), ARMA (1, 2), ARMA (1, 3), ARMA (2, 1), ARMA (2, 2), ARMA (2, 3), ARMA (3, 1), ARMA (3, 2), ARMA (3, 3) we use mean square error (MSE) criteria. We computed MSE for all nine ARMA models. A model possessing minimum MSE value that model is the best model among these fitted nine models.

Mean square error criteria formula is

$$MSE = \frac{\sum_{i=1}^{n} \left(Y_t - \hat{Y}_t\right)^2}{n}$$

where

Y_t: actual value of time point 't'

$\hat{Y}_t$: estimated value of time point 't'

n: number of time points

By calculating the error and absolute errors of the model, we fitting a new exponential smoothing model.

Generally a simple exponential smoothing equation is given as a forecast at time point'$t+1$' and is a combination of time series value at time point t is Y_t and forecast at time point t i.e., F_t with constant terms α and β.

$$F_{t+1} = \alpha Y_t + (1-\alpha) F_t$$

where α is a smoothing constant lies between 0 and 1

$\beta = 1 - \alpha$

Y_t = time series value at time't'

F_t = forecast values at time't'

F_{t+1} = forecast values at time t+1

In simple exponential smoothing α is not a fixed value, it may various between 0 and 1. In our new exponential smoothing model, we fitting the parameter 'α' by using autoregressive moving average models. For the purpose of fixing 'α' we fitted nine ARMA models i.e. ARMA (1, 1), ARMA (1, 2), ARMA (1, 3), ARMA (2, 1), ARMA (2, 2), ARMA (2, 3), ARMA (3, 1), ARMA (3, 2), ARMA (3, 3). We computed mean square error values for the nine ARMA models, from that we estimating 'α^*' value.

We fitting new exponential smoothing equation of the form

$$F_{t+1}^{*} = \alpha^{*} Y_t + \left(1-\alpha^{*}\right) F_t$$

where

α^{*} is estimated by using the formulae

$$\alpha^{*} = \frac{\sum \varepsilon_t}{\sum |\varepsilon_t|}$$

$\sum |\varepsilon_t|$ is sum of absolute errors of ARMA(3, 2) model

$\sum \varepsilon_t$ is sum of errors of ARMA(3, 2) model

The sum of squares of errors divided by number of observations gives MSE. MSE is calculated for two models i.e. exponential smoothing model and new exponential smoothing model.

MSE of simple exponential smoothing model is

$$MSESES = \frac{\sum_{t=1}^{n} (Y_t - F_t)^2}{n}$$

MSE of new exponential smoothing model is

$$MSENES = \frac{\sum_{t=1}^{n} \left(Y_t - F_t^{*}\right)^2}{n}$$

EMPIRICAL INVESTIGATIONS

In single exponential smoothing model, α is not a fixed value. It may take value from 0 to 1. By using ARMA models, we estimated α^{*} value, and we fitting new exponential smoothing model by using estimated 'α^{*}'.

We fitted nine ARMA models for the given data. Various models are

ARMA (1, 1) Model: $Y_t + 0.997Y_{t-1} - \varepsilon_t - 0.978\ \varepsilon_{t-1} + 122.722 = 0$

ARMA (1, 2) Model: $Y_t - 0.460Y_{t-1} - \varepsilon_t + 0.677\ \varepsilon_{t-1} + 0.321\ \varepsilon_{t-2} + 120.221 = 0$

ARMA (1, 3) Model: $Y_t + 0.077Y_{t-1} - \varepsilon_t + 0.043\ \varepsilon_{t-1} + 0.361\ \varepsilon_{t-2} + 0.593\ \varepsilon_{t-3} + 122.637 = 0$

ARMA (2, 1) Model: $Y_t - 0.825Y_{t-1} + 0.354Y_{t-2} - \varepsilon_t + 0.998\ \varepsilon_{t-1} + 121.054 = 0$

ARMA (2, 2) Model: $Y_t - 1.086Y_{t-1} + 0.675Y_{t-2} - \varepsilon_t + 1.182\ \varepsilon_{t-1} + 0.451\ \varepsilon_{t-2} + 121.924 = 0$

ARMA (2, 3) Model: $Y_t + 0.032Y_{t-1} + 0.147Y_{t-2} - \varepsilon_t + 0.105\ \varepsilon_{t-1} + 0.278\ \varepsilon_{t-2} + 0.608\ \varepsilon_{t-3} + 122.765 = 0$

ARMA (3, 1) Model: $Y_t - 0.506Y_{t-1} + 0.238Y_{t-2} + 0.252Y_{t-3} - \varepsilon_t + 0.597\ \varepsilon_{t-1} + 121.140 = 0$

ARMA (3, 2) Model: $Y_t + 0.131Y_{t-1} - 0.407Y_{t-2} + 0.461Y_{t-3} - \varepsilon_t - 0.180\ \varepsilon_{t-1} + 0.815\ \varepsilon_t - 2 + 122.774 = 0$

ARMA (3, 3) Model: $Y_t + 0.126Y_{t-1} - 0.341Y_{t-2} + 0.532Y_{t-3} - \varepsilon_t - 0.192\ \varepsilon_{t-1} + 0.682\ \varepsilon_{t-2} - 0.118\ \varepsilon_{t-3} + 122.748 = 0$

Among the above nine models, a model having lowest mean square error is the best model.

$$MSE = \frac{\sum (x_i - \hat{x}_i)^2}{n}$$

where $\sum \left(x_i - \hat{x}_i\right)^2$ is the sum of squares of deviations

n is the number of observations.

x_i is time series observation and it is annual values

x_i is observed or estimated values by using ARMA model

Annual	ARMA (1,1)	ARMA (1,2)	ARMA (1,3)	ARMA (2,1)	ARMA (2,2)	ARMA (2,3)	ARMA (3,1)	ARMA (3,2)	ARMA (3,3)
64.4	0.0217	0.0099	0.0054	0.0071	0.0161	0.0048	0.0155	0.0072	0.0112
66.2	2.3963	2.5773	2.6798	2.6549	2.5233	2.6945	2.5335	2.6436	2.5796
64.4	0.0531	0.0768	0.1665	0.2338	0.2445	0.1981	0.2506	0.1659	0.1902
63.9	1.1057	0.2362	0.3525	0.1961	0.3502	0.2309	0.4191	0.4554	0.4740
66.8	3.7415	3.9780	6.4034	4.8272	4.7873	6.3142	5.3870	6.4277	6.1385
65.5	0.2706	0.3868	0.0241	0.2453	0.3949	0.0179	0.3657	0.0968	0.0893
65.4	0.0722	1.0248	0.4889	1.0329	0.6854	0.6889	0.4050	0.7702	0.5640
63.3	3.5446	1.5488	0.3009	1.0885	1.4040	0.1960	1.1270	0.9964	1.3017

65.7	0.0783	0.7508	0.3901	1.3832	0.8131	0.4811	0.8940	1.1124	0.8571
65.1	0.0303	0.0883	0.0001	0.0401	0.1633	0.0090	0.0838	0.0136	0.0871
66.2	0.3266	0.9094	0.4153	0.8979	0.1830	0.5483	0.1058	0.2397	0.0965
65.5	0.0044	0.0124	0.1347	0.0175	0.1261	0.1355	0.0376	0.0561	0.0015
65.7	0.0190	0.1979	0.0745	0.2501	0.0001	0.1071	0.0047	0.0000	0.0139
64.5	1.2819	0.8896	0.5491	0.6706	1.4216	0.4761	1.1185	0.4348	0.6574
66.8	0.5435	1.2102	1.4542	1.5906	0.8740	1.5124	0.9147	0.7437	0.6419
66.1	0.1192	0.0201	0.0381	0.0195	0.0062	0.0250	0.0011	0.0561	0.0155
65.9	0.1335	0.0562	0.0136	0.0519	0.0323	0.0013	0.1014	0.1648	0.2507
65.4	0.3250	0.2048	0.0022	0.1025	0.4265	0.0080	0.3196	0.0088	0.0447
66.1	0.1296	0.0005	0.0032	0.0234	0.0133	0.0004	0.0058	0.1248	0.1408
66	0.0236	0.1316	0.1082	0.0912	0.2688	0.1386	0.2506	0.0072	0.0269
65.8	0.7080	0.4436	0.3355	0.4290	0.8328	0.3140	0.9128	1.3577	1.3884
64.5	3.4808	5.0024	4.5950	4.8960	5.7730	4.6423	5.5937	3.5261	3.6355
68.3	2.0739	1.0268	1.4709	1.0968	1.0681	1.2199	1.1809	0.8690	1.1002
68.6	4.5882	1.2224	1.1402	0.6803	1.0217	0.8870	1.1462	1.1903	1.4643
67.4	0.1382	0.6125	0.0003	0.2107	0.1741	0.0026	0.0367	0.0218	0.0053
65.6	1.2113	0.4801	0.0227	0.4160	0.3904	0.0023	0.3446	0.0777	0.0644
65.3	3.8322	1.5987	1.2701	0.9930	0.6410	1.0276	0.5510	1.5770	1.3661
66	0.8365	1.7020	1.5036	1.1550	0.6129	1.7361	0.6501	0.2356	0.2281
66.5	0.8403	1.9113	1.3811	1.9179	1.4417	1.6266	1.7348	2.5638	2.4505
67.9	0.6317	0.1073	0.1882	0.2508	0.1839	0.2636	0.2274	0.0113	0.0135
69.3	3.0283	1.0725	0.8808	0.5264	0.6959	0.6076	0.8444	0.3076	0.4783
67.5	0.0520	0.0604	0.3166	0.3559	0.2036	0.3207	0.1524	0.0530	0.0163
68.7	0.8336	1.7577	1.5826	1.5683	2.2572	1.8026	2.3510	1.3764	1.7232
66.5	0.8947	0.7307	0.4007	0.6876	0.1449	0.3849	0.0859	0.1961	0.1252
67	1.0433	0.1030	0.4775	0.0259	0.0238	0.3505	0.0010	0.0884	0.0850
68.5	0.7418	0.3176	1.3479	0.5607	1.1094	1.2557	1.1238	1.6871	1.5715
69.3	1.2916	1.8698	0.8028	1.7506	1.7646	0.8330	1.4948	0.6877	0.5570
Total	40.4470	34.3289	31.3210	32.9453	33.0731	31.0651	32.7724	30.3520	30.4551
MSE	1.0932	0.9278	0.8465	0.8904	0.8939	0.8396	0.8857	0.8203	0.8231

From the above table, we get the error values of nine ARMA models i.e., ARMA (1, 1), ARMA (1, 2), ARMA (1, 3), ARMA (2, 1), ARMA (2, 2), ARMA (2, 3), ARMA (3, 1), ARMA (3, 2) and ARMA (3, 3). Computing mean square error values for nine ARMA models are 1.0931623 for ARMA (1, 1), 0.92780833 for ARMA (1, 2), 0.84651368 for ARMA (1, 3), 0.8904127 for ARMA (2, 1), 0.8938671 for ARMA (2, 2), 0.8395974 for ARMA (2, 3), 0.88574098 for ARMA (3, 1), 0.82032426 for ARMA (3, 2), 0.82311 for ARMA (3, 3). An autoregressive moving average model (3, 2) possess lowest mean square error value of 0.82032426. By using ARMA (3, 2) model, we estimating ‘α’ value.

An ARMA (3, 2) model possesses lowest mean square error value of 0.8203.

x_i	$\hat{x}_i$	**Error**	**abs error**
64.4	64.4846	-0.0846	0.0846
66.2	64.5741	1.6259	1.6259
64.4	64.8073	-0.4073	0.4073
63.9	64.5748	-0.6748	0.6748
66.8	64.2647	2.5353	2.5353
65.5	65.1889	0.3111	0.3111
65.4	64.5224	0.8776	0.8776
63.3	64.2982	-0.9982	0.9982
65.7	64.6453	1.0547	1.0547
65.1	65.2166	-0.1166	0.1166
66.2	65.7104	0.4896	0.4896
65.5	65.2631	0.2369	0.2369
65.7	65.7068	-0.0068	0.0068
64.5	65.1594	-0.6594	0.6594
66.8	65.9376	0.8624	0.8624
66.1	65.8632	0.2368	0.2368
65.9	66.3059	-0.4059	0.4059
65.4	65.4939	-0.0939	0.0939
66.1	66.4533	-0.3533	0.3533
66	66.0848	-0.0848	0.0848
65.8	66.9652	-1.1652	1.1652
64.5	66.3778	-1.8778	1.8778
68.3	67.3678	0.9322	0.9322
68.6	67.509	1.091	1.091
67.4	67.5476	-0.1476	0.1476
65.6	65.8788	-0.2788	0.2788
65.3	66.5558	-1.2558	1.2558
66	66.4854	-0.4854	0.4854
66.5	68.1012	-1.6012	1.6012
67.9	67.7935	0.1065	0.1065
69.3	68.7454	0.5546	0.5546
67.5	67.7303	-0.2303	0.2303
68.7	67.5268	1.1732	1.1732
66.5	66.9428	-0.4428	0.4428
67	67.2974	-0.2974	0.2974
68.5	67.2011	1.2989	1.2989
69.3	68.4707	0.8293	0.8293
	Total	2.5481	25.8839

The above table contains 4 columns, first column tells about original value, second column tells about estimated value of ARMA (3, 2), third column gives subtractions of estimated values from original values (errors) and fourth column says about modules of subtraction of estimated values from original values (absolute errors). α^* is estimated by using ratio of sum of errors of ARMA (3, 2) to sum of absolute errors of ARMA (3, 2).

$$\alpha^* = \frac{\sum(x_i - \hat{x}_i)}{\sum|x_i - \hat{x}_i|}$$

$$= 0.9$$

$$\alpha^* + \beta^* = 1$$

$$\Rightarrow \beta^* = 0.1$$

Upon substituting estimated values of α^*, β^* in an equation, we get

$$F_{t+1} = \alpha^* Y_t + \beta^* F_t$$

$$= 0.9\, Y_t + 0.1\, S_t$$

Simple exponential smoothing model is also fitted for data by taking $\alpha = 0.1$ and $\beta = 0.9$, is of the form

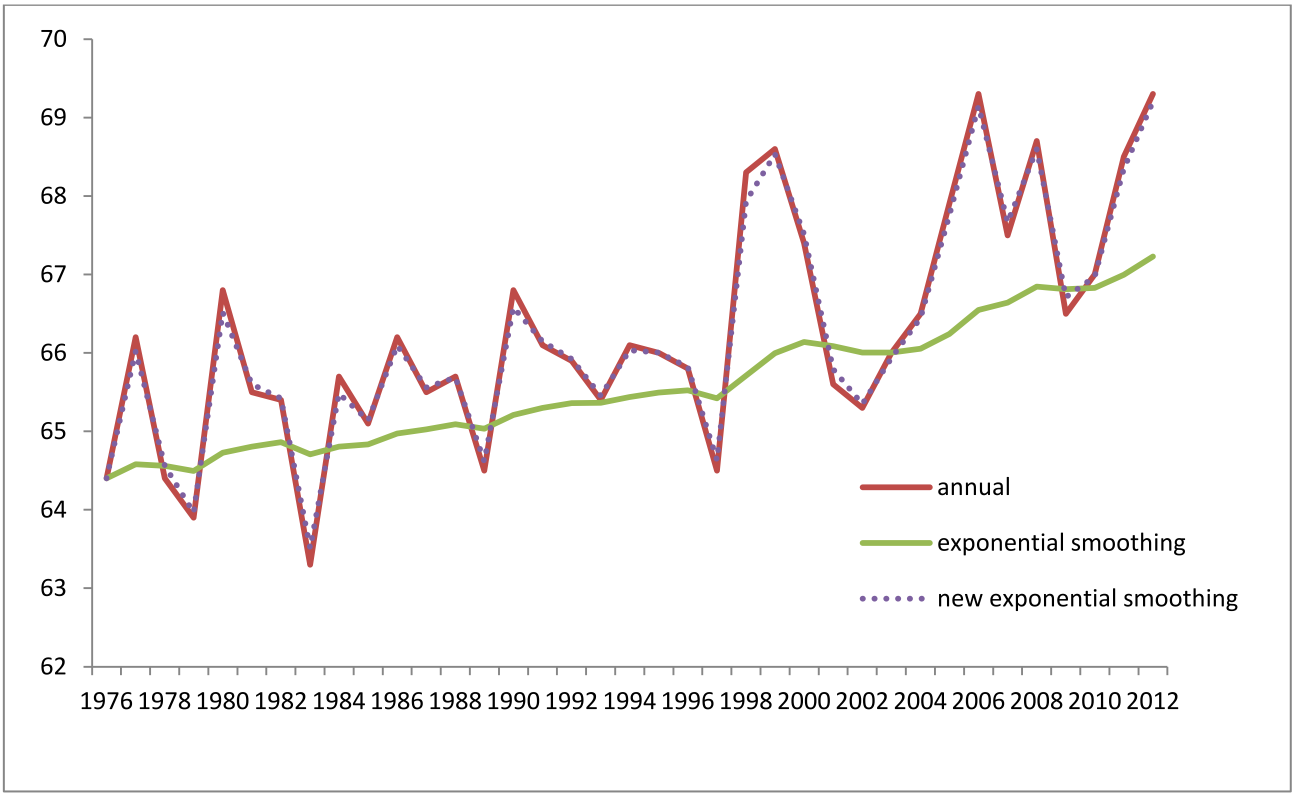
$$F_{t+1} = 0.1Y_t + 0.9F_t$$

A mean square error criterion is used for obtaining which model is best between simple exponential smoothing and new simple exponential smoothing. For that purpose the following table explains forecasted values using exponential smoothing model & new exponential smoothing model and their error squares are also discussed.

Year	Annual	Exponential Smoothing	$(\text{Error})^2$	New Exponential Smoothing	$(\text{Error})^2$
1976	64.4	64.4000	0.0000	64.4000	0.0000
1977	66.2	64.5800	2.6244	66.0200	0.0324
1978	64.4	64.5620	0.0262	64.5620	0.0262
1979	63.9	64.4958	0.3550	63.9662	0.0044
1980	66.8	64.7262	4.3006	66.5166	0.0803
1981	65.5	64.8036	0.4850	65.6017	0.0103
1982	65.4	64.8632	0.2881	65.4202	0.0004
1983	63.3	64.7069	1.9794	63.5120	0.0450
1984	65.7	64.8062	0.7988	65.4812	0.0479
1985	65.1	64.8356	0.0699	65.1381	0.0015
1986	66.2	64.9720	1.5079	66.0938	0.0113
1987	65.5	65.0248	0.2258	65.5594	0.0035
1988	65.7	65.0924	0.3692	65.6859	0.0002
1989	64.5	65.0331	0.2842	64.6186	0.0141
1990	66.8	65.2098	2.5287	66.5819	0.0476
1991	66.1	65.2988	0.6419	66.1482	0.0023
1992	65.9	65.3589	0.2927	65.9248	0.0006
1993	65.4	65.3630	0.0014	65.4525	0.0028
1994	66.1	65.4367	0.4399	66.0353	0.0042
1995	66	65.4931	0.2570	66.0035	0.0000
1996	65.8	65.5238	0.0763	65.8204	0.0004
1997	64.5	65.4214	0.8490	64.6320	0.0174
1998	68.3	65.7092	6.7120	67.9332	0.1345

1999	68.6	65.9983	6.7687	68.5333	0.0044
2000	67.4	66.1385	1.5914	67.5133	0.0128
2001	65.6	66.0846	0.2349	65.7913	0.0366
2002	65.3	66.0062	0.4987	65.3491	0.0024
2003	66	66.0056	0.0000	65.9349	0.0042
2004	66.5	66.0550	0.1980	66.4435	0.0032
2005	67.9	66.2395	2.7573	67.7544	0.0212
2006	69.3	66.5456	7.5870	69.1454	0.0239
2007	67.5	66.6410	0.7379	67.6645	0.0271
2008	68.7	66.8469	3.4340	68.5965	0.0107
2009	66.5	66.8122	0.0975	66.7097	0.0440
2010	67	66.8310	0.0286	66.9710	0.0008
2011	68.5	66.9979	2.2563	68.3471	0.0234
2012	69.3	67.2281	4.2928	69.2047	0.0091
		Total	55.5964	Total	0.7112
		MSE	1.5026	MSE	0.0192

MSE of simple exponential smoothing is 1.5026; MSE of new simple exponential smoothing is 0.0192; Mean square error of new exponential smoothing model is less than the mean square error of simple exponential smoothing model. 0.0192 < 1.5026. Therefore, we conclude that the new single exponential smoothing model is the best model compared with general single exponential smoothing model and the parameter α= 0.1.These two models for the original annual data are shown in the figure-1.



Figer-1

SUMMARY AND CONCLUSIONS

We fitted nine ARMA models by changing autoregressive and moving average parameters. The mean square error of nine ARMA models are 1.0932 for ARMA (1, 1), 0.9278 for ARMA (1, 2), 0.8465 for ARMA (1, 3), 0.8904 for ARMA (2, 1), 0.8939 for ARMA (2, 2), 0.8396 for ARMA (2, 3), 0.8857 for ARMA (3, 1), 0.8203 for ARMA (3, 2), and 0.8231 for ARMA (3, 3). MSE of ARMA (3, 2) is smaller than all other ARMA models. So, we use ARMA (3, 2) model for estimating the parameter in simple exponential smoothing.

The fitted ARMA (3, 2) model is

$Y_t + 0.131Y_{t-1} - 0.407Y_{t-2} + 0.461Y_{t-3} - \varepsilon_t - 0.180\,\varepsilon_{t-1} + 0.815\,\varepsilon_t - 2 + 122.774 = 0$

α* is estimated by the ratio of average of error to average of absolute error.

$$\alpha^* = \frac{mean\,error}{mean\,absoluteerror}.$$

The estimated parameter $\alpha^* = 0.9$

The forecast equation for simple exponential smoothing is

$$F_{t+1} = 0.1Y_t + 0.9F_t$$

Fitted new exponential smoothing by using α^* is

$$F_{t+1} = 0.9\,Y_t + 0.1F_t$$

A mean square error criterion is used for choosing best model between simple exponential smoothing and new simple exponential smoothing. MSE for simple exponential smoothing is 1.5026 and MSE for new exponential smoothing is 0.0192. MSE of new exponential smoothing is less than MSE of simple exponential smoothing. So we conclude that the new simple exponential smoothing is better than the simple exponential smoothing model.

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