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Hyers-Ulam Stability of AQ Functional Equation

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ABSTRACT

In this paper authors investigate the general solution of the functional of the for

$$f\left(3x+9y+27z\right)+f\left(3x-9y+27z\right)+f\left(3x+9y-27z\right)+f\left(-3x+9y+27z\right)=3\left[f\left(x\right)-f\left(-x\right)\right.$$
$$\left.+27\left[f\left(z\right)-f\left(-z\right)\right]+18\left[f\left(x\right)+f\left(-x\right)\right]+162\left[f\left(y\right)+f\left(-y\right)\right]+1458\left[f\left(z\right)+f\left(-z\right)\right]\right.$$

in Banach space using direct and fixed point methods and also odd and even case discussed the above functional equation.

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KEYWORDS: ***Additive functional equation, Quadratic functional equation, fixed point theory***

1. Introduction

One of the most famous functional equations is the additive functional equation

$$f\left(x+y\right)=f\left(x\right)+f\left(y\right)\quad(1.1)$$

In 1821, it was first solved by A.L.Cauchy in the class of continuous real-valued functions. It is often called Cauchy additive functional equation in honor of A.L.Cauchy.The theory of additive functional equations is frequently applied to the development of theories of other functional equations. Moreover, the properties of additive functional equations are powerful tools in almost every field of natural and social sciences. Every solution of the additive functional equation (1.1) is called an additive function.

The quadratic function $f\left(x\right)=cx^2$ satisfies the functional equation

$$f\left(x+y\right)+f\left(x-y\right)=2f\left(x\right)+2f\left(y\right)\quad(1.2)$$

And therefore, the equation (1.2) is called quadratic functional equation. The Hyers-Ulam stability theorem for the quadratic functional equation (1.2) was proved by F.Skof for the functions $f:E_1\to E_2$ where E_1 a normed space and E_2 be a Banach space. The result of Skof is still true if the relevant domain E_1 is replaced by an Abelian group, and it was delta by P.W. Cholewa [9]. S. Czerwik [10], [11] proved the Hyers-Ulam-Rassias stability of the quadratic functional equation (1.2).This result was further generalized by Th. M. Rassais, C. Borelli and G.L. Forti [6].

The solution and stability of the following mixed type additive-quadratic functional equations are,

$$f\left(2x+y\right)+f\left(2x-y\right)=2f\left(x+y\right)+2f\left(x-y\right)-2f\left(2x\right)-4f\left(x\right)$$

$$\begin{aligned}&f\left(x+2y+3z\right)+f\left(x-2y+3z\right)+f\left(x+2y-3z\right)+f\left(x-2y-3z\right)\\&=4f\left(x\right)+8\left[f\left(y\right)+f\left(-y\right)\right]+18\left[f\left(z\right)+f\left(-z\right)\right]\end{aligned}\quad(1.3)$$

Were discussed by A. Najati and M.B. Moghimi [22], M. Arunkumar, P. Agilan [3].Motivated by the above findings in this project, we introduce the general solution and generalized Ulam-Hyers stability of a generalized Additive Quadratic functional equation of the form

$$\begin{aligned}&f\left(3x+9y+27z\right)+f\left(3x-9y+27z\right)+f\left(3x+9y-27z\right)+f\left(-3x+9y+27z\right)\\&=3\left[f\left(x\right)-f\left(-x\right)\right]+9\left[f\left(y\right)-f\left(-y\right)\right]+27\left[f\left(z\right)-f\left(-z\right)\right]+18\left[f\left(x\right)+f\left(-x\right)\right]\\&+162\left[f\left(y\right)+f\left(-y\right)\right]+1458\left[f\left(z\right)+f\left(-z\right)\right]\end{aligned}\quad(1.4)$$

2. General Solution of Additive Functional Equation

In this section, the general solution of the functional equation (1.4) , for the odd case is discussed. Throughout this section, let us consider X and Y to be real vector spaces.

Theorem 2.1: If an odd function $f : x \to y$ satisfies the functional equation

$$f\left(x+y\right)=f\left(x+y\right) \quad (2.1)$$

For all $x, y \in X$,if and only if $f : x \to y$ satisfies the functional equation

$$\begin{aligned} &f\left(3x+9y+27z\right)+f\left(3x-9y+27z\right)+f\left(3x+9y-27z\right)+f\left(-3x+9y+27z\right) \\ &=3\left[f\left(x\right)-f\left(-x\right)\right]+9\left[f\left(y\right)-f\left(-y\right)\right]+27\left[f\left(z\right)-f\left(-z\right)\right]+18\left[f\left(x\right)+f\left(-x\right)\right] \quad (2.2) \\ &+162\left[f\left(y\right)+f\left(-y\right)\right]+1458\left[f\left(z\right)+f\left(-z\right)\right] \end{aligned}$$

For all $x, y, z \in X$.

Proof Let $f : X \to Y$ satisfies the functional equation (2.1) .Setting $\left(x, y\right)$ by $\left(0,0\right)$ in (2.1), we get $f\left(0\right)=0$. Replacing $\left(x, y\right)$ by $\left(x, x\right)$ and $\left(x, 2x\right)$ respectively in (2.1),we obtain

$$f\left(2x\right)=2f\left(x\right) \text{and } f\left(3x\right)=3f\left(x\right) \quad (2.3)$$

for all $x \in X$ .In general for any positive integer a , we have

$$f\left(ax\right)=af\left(x\right) \quad (2.4)$$

for all $x \in X$ .It is easy to verify from (2.4) that

$$f\left(a^2x\right)=a^2f\left(x\right) \text{ And } f\left(a^3x\right)=a^3f\left(x\right) \quad (2.5)$$

for all $x \in X$ .Replacing $\left(x, y\right)$ by $\left(3x+9y, 27z\right)$ in (2.1) and using (2.1), (2.4) and (2.5), we get

$$f\left(3x+9y+27z\right)=3f\left(x\right)+9f\left(y\right)+27f\left(27z\right) \quad (2.6)$$

for all $x, y, z \in X$ .Again replacing z by –z in equation (2.6) and using oddness of f , we obtain

$$f\left(3x+9y-27z\right)=3f\left(x\right)+9f\left(y\right)-27f(z) \quad (2.7)$$

for all $x, y, z \in X$ .Also replacing y by –y in (2.6) and using oddness of f, we get

$$f\left(3x-9y+27z\right)=3f\left(x\right)-9f\left(y\right)+27f(z) \quad (2.8)$$

for all $x, y, z \in X$ .Finally replacing x by –x in (2.6) and using oddness of f , we obtain

$$f\left(-3x+9y+27z\right)=-3f\left(x\right)+9f\left(y\right)+27f(z) \quad (2.9)$$

for all $x, y, z \in X$ .adding the equations (2.6),(2.7),(2.8) and (2.9), we have

$$f\left(3x+9y+27z\right)+f\left(3x-9y+27z\right)+f\left(-3x+9y+27z\right)=6f\left(x\right)+18f\left(y\right)+54f(z) \quad (2.10)$$

for all $x, y, z \in X$ .using oddness of f in (2.10) and remodifying, we arrive

$$\begin{aligned} &f\left(3x+9y+27z\right)+f\left(3x-9y+27z\right)+f\left(3x+9y-27z\right)+f\left(-3x+9y+27z\right) \\ &=3\left[f\left(x\right)-f\left(-x\right)\right]+9\left[f\left(y\right)-f\left(-y\right)\right]+27\left[f\left(z\right)-f\left(-z\right)\right] \end{aligned} \quad (2.11)$$

for all $x, y, z \in X$. Adding $18f\left(x\right)+162f\left(y\right)+1458f\left(z\right)$ on both sides, modifying and using oddness of f , we reach (2.2) as desired.Conversely $f : X \to Y$ satisfies the functional equation (2.2) using oddness of f in (2.2), we arrive

$$\begin{aligned} &f\left(3x+9y+27z\right)+f\left(3x-9y+27z\right)+f\left(3x+9y-27z\right)+f\left(-3x+9y+27z\right) \\ &=6f\left(x\right)+18f\left(y\right)+54f\left(z\right) \end{aligned}$$

(2.12)for all $x, y, z \in X$ .Replacing $\left(x, y, z\right)$ by $\left(x,0,0\right)$ $\left(0, x,0\right)$ and $\left(0,0, x\right)$, respectively in (2.12), we obtain

$$f\left(3x\right)=3f\left(x\right), \ f\left(9x\right)=9f\left(x\right) \text{ and } f\left(27x\right)=27f\left(x\right) \quad (2.13)$$

for all $x \in X$ .One can easy to verify from (2.13) that

$$f\left(\frac{x}{3^i}\right)=\frac{1}{3^i}f\left(x\right), i=1,2,3 \quad (2.14)$$

for all $x \in X$. Replacing (x,y,z) by $\left(\frac{x}{3},\frac{y}{9},0\right)$ in equation (2.12) and using oddness of f and (2.14), we arrive our result.

3. General Solution of the Quadratic Functional Equation

In this section, the general solution of the functional equation (1.4) for even case is given. Throughout this section, let us consider X and Y to be real vector spaces.

Theorem: 3.1If an even mapping $f: X \to Y$ satisfies the functional equation

$$f(x+y)+f(x-y)=2f(x)+2f(y) \quad (3.1)$$

For all $x, y \in X$ if and only if $f: X \to Y$ satisfies the functional equation

$$\begin{aligned} & f(3x+9y+27z)+f(3x-9y+27z)+f(3x+9y-27z)+f(-3x+9y+27z) \\ & =3\left[f(x)-f(-x)\right]+9\left[f(y)-f(-y)\right]+27\left[f(z)-f(z)\right]+18\left[f(x)+f(-x)\right] \quad (3.2) \\ & +162\left[f(y)+f(-y)\right]+1458\left[f(z)+f(-z)\right] \end{aligned}$$

for all $x, y, z \in X$.

Proof: Let $f: X \to Y$ satisfies the functional equation (3.1).Setting (x,y) by $(0,0)$ in (3.1), we get $f(0)=0$ .Replacing y by x and y by $2x$ in (3.1), we obtain

$$f(2x)=4f(x) \text{ And } f(3x)=9f(x) \quad (3.3)$$

for all $x \in X$ .In general for any positive integer b, such that

$$f(bx)=b^2 f(x) \quad (3.4)$$

for all $x \in X$ .It is easy to verify from (3.4) that

$$f\left(b^2x\right)=b^4 f(x) \text{ And } f\left(b^3x\right)=b^6 f(x) \quad (3.5)$$

for all $x \in X$ .Replacing (x,y) by $(3x,9y)$ in (3.1) and using (3.1), we get

$$f(3x+9y)+f(3x-9y)=18f(x)+162f(y) \quad (3.6)$$

for all $x, y \in X$. Setting (x,y) by $(3x,27z)$ in (3.1) and using (3.1), we obtain

$$f(3x+27z)+f(3x-27z)=18f(x)+1458f(z) \quad (3.7)$$

for all $x, z \in X$ .Replacing (x,y) by $(9y,27z)$ in (3.1) and using (3.1) ,we have

$$f(9y+27z)+f(9y-27z)=162f(y)+1458f(z) \quad (3.8)$$

for all $y, z \in X$ .Adding equations (3.6),(3.7) and (3.8), we arrive

$$\begin{aligned} & f(3x+9y)+f(3x-9y)+f(3x+27z)+f(3x-27z)+f(9y+27z)+f(9y-27z) \\ & =36f(x)+324f(y)+2916f(z) \end{aligned} \quad (3.9)$$

for all $x, y, z \in X$ .Replacing (x,y) by $(3x+9y,27z)$ in (3.1), we get

$$f(3x+9y+27z)+f(3x+9y-27z)=2f(3x+9y)+2f(27z) \quad (3.10)$$

for all $x, y, z \in X$. Setting (x,y) by $(27z,3x-9y)$ in (3.1), we obtain

$$f(3x-9y+27z)+f(-3x+9y+27z)=2f(27z)+2f(3x-9y) \quad (3.11)$$

for all $x, y, z \in X$. Replacing (x,y) by $(9y,3x+27z)$ in (3.1), we obtain

$$f(3x+9y+27z)+f(-3x+9y+27z)=2f(9y)+2f(3x+27z) \quad (3.12)$$

for all $x, y, z \in X$. Setting (x,y) by $(9y,3x-27z)$ in (3.1), we obtain

$$f(3x+9y-27z)+f(-3x+9y+27z)=2f(9y)+2f(3x-27z) \quad (3.13)$$

for all $x, y, z \in X$. Replacing (x,y) by $(3x,9y+27z)$ in (3.1), we get

$$f(3x+9y+27z)+f(3x-9y-27z)=2f(3x)+2f(9y+27z) \quad (3.14)$$

for all $x, y, z \in X$. Setting (x, y) by $(3x, 9y-27z)$ in (3.1), we have

$$f(3x+9y-27z)+f(3x-9y+27z)=2f(3x)+2f(9y-27z) \quad (3.15)$$

for all $x, y, z \in X$. Now multiply by 2 on both sides of (3.9), we obtain

$$\begin{aligned}&2f(3x+9y)+2f(3x-9y)+2f(3x+27z)+2f(3x-27z)+2f(9y+27z)\\&+2f(9y-27z)=72f(x)+648f(y)+5832f(z)\end{aligned} \quad (3.16)$$

for all $x, y, z \in X$. Adding $2f(27z)$ on both sides of (3.16), we get

$$\begin{aligned}&2f(3x+9y)+2f(3x-9y)+2f(3x+27z)+2f(3x-27z)+2f(9y+27z)\\&+2f(9y-27z)+2f(27z)=72f(x)+648f(y)+5832f(z)+2f(27z)\end{aligned} \quad (3.17)$$

for all $x, y, z \in X$. Using (3.10),(3.15) in (3.17), we arrive

$$\begin{aligned}&f(3x+9y+27z)+f(3x+9y-27z)+2f(3x-9y)+2f(3x+27z)+2f(3x-27z)\\&+2f(9y+27z)+2f(9y-27z)=72f(x)+648f(y)+5832f(z)+1458f(z)\end{aligned} \quad (3.18)$$

for all $x, y, z \in X$. Adding $2f(27z)$ on both sides of (3.18), we get

$$\begin{aligned}&f(3x+9y+27z)+f(3x+9y-27z)+2f(27z)+2f(3x-9y)\\&+2f(3x+27z)+2f(3x-27z)+2f(9y+27z)+2f(9y-27z)\\&=72f(x)+648f(y)+7290f(z)+2f(27z)\end{aligned} \quad (3.19)$$

for all $x, y, z \in X$. Using (3.11) in (3.19), we arrive

$$\begin{aligned}&f(3x+9y+27z)+f(3x+9y-27z)+f(27z+3x-9y)+f(27z-3x+9y)\\&+2f(3x+27z)+2f(3x-27z)+2f(9y+27z)+2f(9y-27z)\\&=72f(x)+648f(y)+8748f(27z)\end{aligned} \quad (3.20)$$

for all $x, y, z \in X$. Adding $2f(9y)$ on both sides of (3.20), we get

$$\begin{aligned}&f(3x+9y+27z)+f(3x+9y-27z)+f(3x-9y+27z)+f(-3x+9y+27z)\\&+2f(9y)+2f(3x+27z)+2f(3x-27z)+2f(9y+27z)+2f(9y-27z)\\&=72f(x)+648f(y)+8748f(27z)+2f(9y)\end{aligned} \quad (3.21)$$

for all $x, y, z \in X$. Using (3.12) in (3.21), we arrive

$$\begin{aligned}&f(3x+9y+27z)+f(3x+9y-27z)+f(3x-9y+27z)+f(-3x+9y+27z)\\&+f(3x+9y+27z)+f(-3x+9y-27z)+2f(3x-27z)+2f(9y+27z)\\&+2f(9y-27z)=72f(x)+648f(y)+8748f(27z)+2f(9y)\end{aligned} \quad (3.22)$$

for all $x, y, z \in X$. Adding $2f(9y)$ on both sides of (3.22), we obtain

$$\begin{aligned}&f(3x+9y+27z)+f(3x+9y-27z)+f(3x-9y+27z)+f(-3x+9y+27z)\\&+f(3x+9y+27z)+f(-3x+9y-27z)+2f(3x-27z)+2f(9y)+2f(9y+27z)\\&+2f(9y-27z)=72f(x)+810f(y)+2f(9y)+8748f(27z)\end{aligned} \quad (3.23)$$

for all $x, y, z \in X$. Using (3.13) in (3.23), we arrive

$$\begin{aligned}&f(3x+9y+27z)+f(3x+9y-27z)+f(3x-9y+27z)+f(-3x+9y+27z)\\&+f(3x+9y+27z)+f(-3x+9y-27z)+f(3x+9y-27z)+f(-3x+9y+27z)\\&+2f(9y+27z)+2f(9y-27z)=72f(x)+972f(y)+8748f(27z)\end{aligned} \quad (3.24)$$

For all $x, y, z \in X$. Adding $2f(3x)$ on both sides of (3.24), we get

$$\begin{aligned}&f(3x+9y+27z)+f(3x+9y-27z)+f(3x-9y+27z)+f(-3x+9y+27z)\\&+f(3x+9y+27z)+f(-3x+9y-27z)+f(3x+9y-27z)+f(-3x+9y+27z)\\&+2f(3x)+2f(9y+27z)+2f(9y-27z)=72f(x)+972f(y)+8748f(27z)+2f(3x)\end{aligned}\qquad(3.25)$$

for all $x,y,z\in X$. Using (3.14) in (3.25), we arrive

$$\begin{aligned}&f(3x+9y+27z)+f(3x+9y-27z)+f(3x-9y+27z)\\&+f(-3x+9y+27z)+f(3x+9y+27z)+f(-3x+9y-27z)\\&+f(3x+9y-27z)+f(-3x+9y+27z)+f(3x+9y+27z)\\&+f(3x-9y-27z)+2f(9y-27z)=90f(x)+972f(y)+8748f(27z)\end{aligned}\qquad(3.26)$$

for all $x,y,z\in X$. Adding $2f(3x)$ on both sides of (3.26), we have

$$\begin{aligned}&f(3x+9y+27z)+f(3x+9y-27z)+f(3x-9y+27z)+f(-3x+9y+27z)\\&+f(3x+9y+27z)+f(-3x+9y-27z)+f(3x+9y-27z)+f(-3x+9y+27z)\\&+f(3x+9y+27z)+f(3x-9y-27z)+2f(3x)+2f(9y-27z)\\&=90f(x)+972f(y)+8748f(27z)+2f(3x)\end{aligned}\qquad(3.27)$$

for all $x,y,z\in X$. Using (3.15)in (3.27), we arrive

$$\begin{aligned}&f(3x+9y+27z)+f(3x+9y-27z)+f(3x-9y+27z)+f(-3x+9y+27z)\\&+f(3x+9y+27z)+f(-3x+9y-27z)+f(3x+9y-27z)+f(-3x+9y+27z)\\&+f(3x+9y+27z)+f(3x-9y-27z)+f(3x+9y-27z)+f(3x-9y+27z)\\&=108f(x)+972f(y)+8748f(27z)\end{aligned}\qquad(3.28)$$

for all $x,y,z\in X$. Using evenness of f in (3.28), we have

$$\begin{aligned}&f(3x+9y+27z)+f(3x+9y-27z)+f(3x-9y+27z)+f(-3x+9y+27z)\\&=36f(x)+324f(y)+2916f(27z)\end{aligned}\qquad(3.29)$$

for all $x,y,z\in X$. Using evenness of f in (3.29) one can get,

$$\begin{aligned}&f(3x+9y+27z)+f(3x+9y-27z)+f(3x-9y+27z)+f(-3x+9y+27z)\\&=18[f(x)+f(-x)]+162[f(y)+f(-y)]+1458[f(z)+f(-z)]\end{aligned}\qquad(3.30)$$

for all $x,y,z\in X$ .Adding $3f(x)+9f(y)+27f(z)$ on both sides of (3.30) and using evenness of f,we desired our result.

Conversely, $f:X\to Y$ satisfies the functional equation (3.2). Using evenness of f in (3.2), we have

$$\begin{aligned}&f(3x+9y+27z)+f(3x-9y+27z)+f(3x+9y-27z)+f(-3x+9y+27z)\\&=36f(x)+324f(y)+2916f(z)\end{aligned}\qquad(3.31)$$

for all $x,y,z\in X$ .Setting (x,y,z) by $(x,0,0)$, $(0,x,0)$ and $(0,0,x)$ in (3.31), we obtain

$f(3x)=9f(x)$; $f(9x)=81f(x)$ and $f(27x)=729f(x)$ (3.32)

for all $x\in X$. It is easy to verify from (3.32), that

$$f\left\{\frac{x}{n^i}\right\}=\frac{1}{n^i}f(x),\ i=1,2,3\ (3.33)$$

For all $x\in X$. Replacing (x,y,z) by $\left(\frac{x}{3},\frac{y}{9},0\right)$ in (3.31) and using evenness of f and (3.33), we desired our result.

3. Stability of Additive Quadratic Functional Equation

In this section, we present the generalized Ulam – Hyers stability of the functional equation (1.4).

Theorem: 3.1[14] Let $j\in\{-1,1\}$ and $\alpha:X^3\to[0,\infty)$ be a function such that

$$\sum_{k=0}^{\infty} \frac{\alpha\left(3^{kj} x, 3^{kj} y, 3^{kj} z\right)}{3^{kj}} \text{ Converges in R and } \lim_{k\to\infty} \frac{\alpha\left(3^{kj} x, 3^{kjy}, 3^{kj} z\right)}{3^{kj}} = 0 \quad (3.1)$$

For all $x, y, z \in X$ .Let $f_a : X \to Y$ be an odd function satisfying the inequality

$$\left\|Df_a\left(x, y, z\right)\right\| \le \alpha\left(x, y, z\right) \qquad (3.2)$$

For all $x, y, z \in X$ .There exists a unique additive mapping $A: X \to Y$ which satisfies the functional equation (1.4) and

$$\left\|f_a\left(x\right) - A\left(x\right)\right\| \le \frac{1}{6} \sum_{k=\frac{1-j}{2}}^{\infty} \frac{\alpha\left(3^{kj} x, 0, 0\right)}{3^{kj}} \quad (3.3)$$

For all $x \in X$ .The mapping $A\left(x\right)$ is defined by

$$A\left(x\right) = \lim_{k\to\infty} \frac{f_a\left(3^{kj} x\right)}{3^{kj}} \qquad (3.4)$$

For all $x \in X$.

Corollary: 3.2 Let λ and s be a nonnegative real numbers. Let an odd function $f_a : X \to Y$ satisfying the inequality

$$\left\|Df_a\left(x, y, z\right)\right\| \le \begin{cases} \lambda \\ \lambda\left(\|x\|^s + \|y\|^s + \|z\|^s\right), s \ne 1 \\ \lambda\left(\|x\|^s \|y\|^s \|z\|^s + \left\{\|x\|^{3s} + \|y\|^{3s} + \|z\|^{3s}\right\}\right), s \ne \frac{1}{3}; \end{cases} \qquad (3.5)$$

For all $x, y, z \in X$. Then there exists unique additive function $A: X \to Y$ such that

$$\left\|f_a\left(x\right) - A\left(x\right)\right\| \le \begin{cases} \frac{\lambda}{2|3-1|}, \\ \frac{\lambda\|x\|^s}{2\left|3-3^s\right|}, \\ \frac{\lambda\|x\|^{3s}}{2\left|3-3^{3s}\right|} \end{cases} \quad (3.6)$$

Proof: If we replace

$$\alpha\left(x, y, z\right) = \begin{cases} \lambda; \\ \lambda\left(\|x\|^s + \|y\|^s + \|z\|^s\right); \\ \lambda\left(\|x\|^s \|y\|^s \|z\|^s + \left\{\|x\|^{3s} + \|y\|^{3s} + \|z\|^{3s}\right\}\right) \end{cases} \qquad (3.7)$$

In theorem 3.1, we arrive (3.6).

Theorem: 3.3 Let $j \in \{-1, 1\}$ and $\alpha : X^3 \to [0, \infty)$ be a function such that,

$$\sum_{k=0}^{\infty} \frac{\alpha\left(3^{kj} x, 3^{kj} y, 3^{kj} z\right)}{3^{kj}} \text{ Converges to R and } \lim_{k\to\infty} \frac{\alpha\left(3^{kj} x, 3^{kjy}, 3^{kj} z\right)}{3^{kj}} = 0 \quad (3.8)$$

For all $x, y, z \in X$ .Let $f_q : X \to Y$ be an even function satisfying the inequality,

$$\left\|Df_q\left(x, y, z\right)\right\| \le \alpha\left(x, y, z\right) \quad (3.9)$$

For all $x, y, z \in X$ .There exists unique quadratic mapping $Q: X \to Y$ which satisfies the functional equation (1.4) and

$$\left\| f_q(x) - Q(x) \right\| \leq \frac{1}{36} \sum_{k=\frac{1-j}{2}}^{\infty} \frac{\alpha\left(3^{kj}x, 0, 0\right)}{3^{2kj}} \quad (3.10)$$

For all $x \in X$. The mapping $Q(x)$ is defined by

$$Q(x) = \lim_{k\to\infty} \frac{f_q\left(3^{kj}x\right)}{3^{2kj}} \quad (3.11)$$

For all $x \in X$.

Proof: The rest of the proof is similar to that Theorem 4.1.
The following corollary is an immediate consequence of Theorem 4.2 concerning the stability of (1.4).

Corollary: 3.4 Let λ and s be a nonnegative real numbers. Let an even function $f_q : X \to Y$ satisfying the inequality

$$\left\| Df_q(x,y,z) \right\| \leq \begin{cases} \lambda; \\ \lambda\left(\|x\|^s + \|y\|^s + \|z\|^s\right); s \neq 2; \\ \lambda\left(\|x\|^s\|y\|^s\|z\|^s + \left\{\|x\|^{3s} + \|y\|^{3s} + \|z\|^{3s}\right\}\right); s \neq \frac{2}{3}; \end{cases} \quad (3.12)$$

For all $x, y, z \in X$. Then there exists unique quadratic function $Q : X \to Y$ such that,

$$\left\| f_q(x) - Q(x) \right\| \leq \begin{cases} \frac{\lambda}{4\left|3^2 - 1\right|}, \\ \frac{\lambda\|x\|^s}{4\left|3^2 - 3^s\right|}, \\ \frac{\lambda\|x\|^{3s}}{4\left|3^2 - 3^{3s}\right|}, \end{cases} \quad (3.13)$$

For all $x \in X$.

Theorem: 3.5 Let $j \in \{-1, 1\}$ and $\alpha : X^3 \to [0, \infty)$ be a function satisfying (3.1) and (3.8) for all $x, y, z \in X$ .Let $f : X \to Y$ be a function satisfying the inequality,

$$\left\| Df(x,y,z) \right\| \leq \alpha(x,y,z) \quad (3.14)$$

for all $x, y, z \in X$ .Then there exists a unique additive mapping and a unique quadratic mapping $Q : X \to Y$ which satisfies the functional equation (1.4) and

$$\left\| f(x) - A(x) - Q(x) \right\|$$
$$\leq \frac{1}{2}\left[\frac{1}{6} \sum_{k=\frac{1-j}{2}}^{\infty} \left(\frac{\alpha\left(3^{kj}x,0,0\right)}{3^{kj}} + \frac{\alpha\left(-3^{kj}x,0,0\right)}{3^{kj}} \right) + \frac{1}{36} \sum_{k=\frac{1-j}{2}}^{\infty} \left(\frac{\alpha\left(3^{kj}x,0,0\right)}{3^{2kj}} + \frac{\alpha\left(-3^{kj}x,0,0\right)}{3^{2kj}} \right) \right] \quad (3.15)$$

for all $x \in X$. The mapping $A(x)$ and $Q(x)$ is defined in (3.3) and (3.10) respectively for all $x \in X$.

Corollary: 3.6 Let λ and s be a nonnegative real numbers. Let a function $f : X \to Y$ satisfying the inequality

$$\left\| Df(x,y,z) \right\| \leq \begin{cases} \lambda; \\ \lambda\left(\|x\|^s + \|y\|^s + \|z\|^s\right); s \neq 1, 2; \\ \lambda\left(\|x\|^s\|y\|^s\|z\|^s + \left\{\|x\|^{3s} + \|y\|^{3s} + \|z\|^{3s}\right\}\right); s \neq \frac{1}{3}, \frac{2}{3}; \end{cases} \quad (3.16)$$

for all $x, y, z \in X$. Then there exists unique additive mapping $A : X \to Y$ and a unique quadratic mapping $Q : X \to Y$ such that,

$$\left\| f\left(x\right)-A\left(x\right)-Q\left(x\right)\right\| \le \begin{cases} \frac{\lambda}{2}\left[\frac{1}{|3-1|}+\frac{1}{2\left|3^2-1\right|}\right], \\ \frac{\lambda\|x\|^s}{2}\left[\frac{1}{3-3^s}+\frac{1}{2\left|3^2-3^s\right|}\right], \\ \frac{\lambda\|x\|^{3s}}{2}\left[\frac{1}{3-3^{3s}}+\frac{1}{2\left|3^2-3^{3s}\right|}\right], \end{cases} \quad (3.17)$$

for all $x \in X$

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