[Volume 2 issue 3 March 2014]
Page No.353-358 ISSN :2320-7167

INTERNATIONAL JOURNAL OF MATHEMATICS AND COMPUTER RESEARCH

Bold signed total domination

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Abstract

A set D is a subset of V (G) is called dominating (or total dominating) set in G, if $D \cap N[v] \neq \phi$ (or $D \cap N(v) \neq \phi$, respectively) for every vertex $v \in V(G)$. The minimum number of vertices of a dominating set (or of a total dominating set) in G is called the domination number $\gamma(G)$ (or the total domination number $\gamma_t(G)$, respectively) of G. If v is a vertex of a graph G, then N(v) is its open neighbourhood, (ie) the set of all vertices adjacent to v in G. A mapping $f : V(G) \rightarrow \{ -2,1 \}$, where V(G) is the vertex set of G, is called a Bold Signed Total Dominating Function (BSTDF) on G, if $w(f) = \sum_{x \in N(v)} f(x) \geq 1$ for each $v \in V(G)$. $\min_f\{\sum_{x \in V(G)} f(x)$: f is a BSTDF } is called the bold signed total domination number of G and is denoted by $\gamma_{bst}(G)$. The bold signed total domination number of a graph is a certain variant of the domination number. The lower bounds of $\gamma_{bst}(G)$ are found for the case of regular graphs, and $\gamma_{bst}(G)$ are found for complete graphs, circuits and complete bipatite graphs. The independent proofs are seen.

AMS subject classification (2000): 05C69,05C35,05C22.

Keywords: Dominating function; Domination number; Bold signed total dominating function; Bold signed total domination number.

Title: Bold signed total domination.

1 Introduction

In this paper we study the bold signed total domination number of a graph and using the notation as in [2]. We consider finite undirected graphs without loops and multiple edges [1]. The vertex set of a graph G is denoted by V (G). If $v \in V(G)$, then the open neighbourhood N(v) of v in G is the set of all vertices which are adjacent to v in G. Further, the closed neighbourhood of v in G is defined as N[v] = N(v) U {v}. Let f be a mapping of V (G) into set of real numbers, let S is a subset of V(G). Then we denote $f(S) = \sum_{x \in S} f(x)$. Futher, the weight of f is $w(f) = f(V(G)) = \sum_{x \in V(G)} f(x)$. We will study the concept, from the definition. A function $f : V(G) \rightarrow \{ -2,1 \}$ is called a Bold Signed dominating function (shortly BSDF) of G, if $f(N[v]) \geq 1$ for each $v \in V(G)$. The minimum of $w(f) = f(V(G)) = \sum_{x \in V(G)} f(x)$, taken over all BSDF of G, is the bold signed domination number $\gamma_{bs}(G)$ of G. Similarly, a function $f : V(G) \rightarrow \{ -2,1 \}$ is called a bold signed total dominating function (shortly BSTDF) of G, if

$f(N(v)) \geq 1$ for each $v \in V(G)$. The minimum of $w(f) = f(V(G)) = \sum_{x \in V(G)} f(x)$, taken over all BSTDF of G, is the bold signed total domination number $\gamma_{bst}(G)$ of G.

Lemma 1.1 Let $f : V(G) \rightarrow \{-2,1\}$ and S is a subset of V(G). Then $f(S) \equiv |S| \pmod 3$.

Proof: Let $S^+ = \{x \in S : f(x) = 1\}$, $S^- = \{x \in S : f(x) = -2\}$. Then $|S| = |S^+| + |S^-|$. Therefore $f(S) = \sum_{x \in S} f(x) = |S^+|-2|S^-|$. Therefore $|S|- f(S) = 3|S^-|$ (i.e.) $f(S) \equiv |S| \pmod 3$.

Theorem 1.2 For a circuit C_n of length $n \geq 3$ we have $\gamma_{bst}(C_n) = n$.

Proof: Let C_n be a circuit of length n. Let r be the number of vertices assigned with -2. (ie) n-r vertices assigned with 1. Now $f(N(v)) = (2-r)-2r \geq 1$ (since N(v) contains only 2 vertices in C_n). (i.e.) $2-3r \geq 1$ implies $3r \leq 1$ (i.e.) $r \leq (1/3)$.

Since r is an integer, r=0. Therefore all the vertices are assigned with 1.

Hence $\gamma_{bst}(C_n) = \min w(f) = \Sigma_{v \in V(G)} f(v) = n$.

Theorem 1.3 Let G be a regular graph of degree r. Then for all $n \geq 3$,

$$\gamma_{bst}(G) \geq \begin{cases} n/r & \text{if } r \equiv 1 \pmod 3. \\ 2n/r & \text{if } r \equiv 2 \pmod 3. \\ 3n/r & \text{if } r \equiv 0 \pmod 3. \end{cases}$$

Proof: Let G be a regular graph of degree r and n be the number of vertices. If $r = 1$, then $\gamma_{bst}(G) = 2$. If $r = 2$, then $\gamma_{bst}(G) = n$ (since $G = C_n$). For $r \geq 3$. Let f be a BSTDF of G such that min $w(f) = \gamma_{bst}(G)$. Let $V^+ = \{v \in V(G) : f(v) = 1\}$ and $V^- = \{v \in V(G) : f(v) = -2\}$. Let E_0 be the set of all edges joining a vertex of V^+ with a vertex of V^- in G. Let $u \in V^+$ and let u be adjacent to exactly s vertices of V^- . Hence s vertices assign values -2. Then u is adjacent to r- s vertices of V^+,since deg u = r. r - s vertices are assigned with value 1. Now $f(N(u)) = (r-s)-2s = r-3s \geq 1$. (since f is BSTDF, $f(N(u)) \geq 1$). $3s \leq r - 1$, $s \leq (r-1)/3$. Therefore u is adjacent to atmost $(r-1)/3$ vertices ov V^-.

Since s is an integer,

$$s \leq \begin{cases} (r-1)/3 & \text{if } r \equiv 1 \pmod 3. \\ (r-1)/3-(1/3) & \text{if } r \equiv 2 \pmod 3. \\ (r-1)/3-(2/3) & \text{if } r \equiv 0 \pmod 3. \end{cases}$$

Now let $v \in V^-$ and let v be adjacent to exactly t vertices of V^+. Then v is adjacent to (r-t) vertices of V^-. Therefore $f(N(v)) = t-2(r-t) = 3t-2r \geq 1$(since $f(N(v)) \geq 1$). (i.e.) $t \geq (1+2r)/3$.

$$\text{Therefore} \quad t \geq \begin{cases} (1+2r)/3 & \text{if } r \equiv 1 \pmod 3. \\ (1+2r)/3+(1/3) & \text{if } r \equiv 2 \pmod 3. \\ (1+2r)/3+(2/3) & \text{if } r \equiv 0 \pmod 3. \end{cases}$$

If $n^+ = |V^+|$ and $n^- = |V^-|$, then $|E_0| \leq n^+ s$ and $|E_0| \geq n^- t$.

Case (i) For $r \equiv 1 \mod 3$.

$|E_0| \leq n^+ (r-1)/3$ and $|E_0| \geq n^- (1+2r)/3$.

$$n^- (1+2r)/3 \leq n^+ (r-1)/3$$
$$n^+ + n^- \leq (n^+ - 2n^-)\, r$$
$$n \leq w(f)\, r$$
$$n \leq \gamma_{bst}(G)\, r$$
$$\gamma_{bst}(G) \geq n/r$$

Hence $\gamma_{bst}(G) \geq n/r$ if $r \equiv 1 \mod 3$.

Case (ii) For $r \equiv 2 \mod 3$.

$|E_0| \leq n^+ [(r-1)/3 – (1/3)]$ and $|E_0| \geq n^- [(1+2r)/3 + (1/3)]$.

$$n^- [(1+2r)/3 + (1/3)] \leq n^+ [(r-1)/3 – (1/3)]$$
$$n^- (2+2r) \leq n^+ (r-2)$$
$$2(n^+ + n^-) \leq (n^+ - 2n^-)\, r$$
$$2n \leq w(f)\, r$$
$$2n \leq \gamma_{bst}(G)\, r$$
$$\gamma_{bst}(G) \geq 2n/r$$

Hence $\gamma_{bst}(G) \geq 2n/r$ if $r \equiv 2 \mod 3$.

Case (iii) For $r \equiv 0 \mod 3$.

$|E_0| \leq n^+ [(r-1)/3 – (2/3)]$ and $|E_0| \geq n^- [(1+2r)/3 + (2/3)]$.

$$n^- [(1+2r)/3 + (2/3)] \leq n^+ [(r-1)/3 – (2/3)]$$
$$n^- (3+2r) \leq n^+ (r-3)$$
$$3(n^+ + n^-) \leq (n^+ - 2n^-)\, r$$
$$3n \leq w(f)\, r$$
$$3n \leq \gamma_{bst}(G)\, r$$
$$\gamma_{bst}(G) \geq 3n/r$$

Hence $\gamma_{bst}(G) \geq 3n/r$ if $r \equiv 0 \mod 3$.

Theorem 1.4 If K_n $(n \geq 2)$ is a complete graph with n vertices, then

$$\gamma_{bst}(K_n) = \begin{cases} 3 & \text{if } n = 3s \\ 4 & \text{if } n = 3s + 1 \\ 2 & \text{if } n = 3s + 2 \qquad \text{for all } n \geq 3 \end{cases}$$

Proof: Let K_n be a complete graph with n vertices.Therefore N(v) contains (n-1) vertices. Let r be the number of vertices assign -2.Then (n-1)-r vertices assign 1.We know that $f(N(v)) \geq 1$.Therefore $(n-1-r)-2r \geq 1$. (i.e.) $n-1-3r \geq 1$. (i.e.) $n-2 \geq 3r$. Therefore $r \leq (n-2)/3$.
Since r is an integer,

$$\begin{cases} (n-2)/3 & \text{if } n = 3s + 2. \end{cases}$$

$r \leq \quad (n-2)/3-(2/3) \quad$ if $\quad n = 3s + 1.$

$\quad (n-2)/3-(1/3) \quad$ if $\quad n = 3s.$

Therefore $w(f) = \sum_{v \epsilon V(G)} f(v) = n-r-2r = n-3r.$

$$\text{(i.e.)} \quad w(f) \geq \begin{cases} n-3[(n-2)/3] = 2 & \text{if } r \leq (n-2)/3, \ \ n = 3s+2. \\ n-3[(n-2)/3-(2/3)] = 4 & \text{if } r \leq (n-2)/3-(2/3), \ \ n = 3s+1. \\ n-3[(n-2)/3-(1/3)] = 3 & \text{if } r \leq (n-2)/3-(1/3), \ \ n = 3s. \end{cases}$$

$$\text{Therefore} \quad \gamma_{bst}(K_n) = \min w(f) = \begin{cases} 3 \ \text{ if } \ n = 3s \\ 4 \ \text{ if } \ n = 3s+1 \\ 2 \ \text{ if } \ n = 3s+2 \qquad \text{for all } n \geq 3. \end{cases}$$

Theorem 1.5 For a complete bipartite graph $K_{m,n}$ we have

$$\gamma_{bst}(K_{m,n}) = \begin{cases} 2 \ \text{ if both m and n are } 3s_i + 1, \ i = 1 \text{ or } 2. \\ 3 \ \text{ if one of m, n is } 3s_i + 1 \text{ and another is } 3s_j + 2, \ i \neq j, \ i, j = 1, 2. \\ 4 \ \text{ if both m and n are } 3s_i + 2 \text{ or one of m, n is } 3s_i \text{ and another is } 3s_j + 1, \ i \neq j, \ i, j = 1, 2. \\ 5 \ \text{ if one of m, n is } 3s_i \text{ and another is } 3s_j + 2, \ i \neq j, \ i, j = 1, 2. \\ 6 \ \text{ if both m and n are } 3s_i, \ i = 1, 2 \qquad \text{for m, n} \geq 3 \end{cases}$$

Proof: Let $K_{m,n}$ be a complete bipartite graph. Let V_1 be a vertex set containing m vertices and V_2 be a vertex set containing n vertices. Let r_1 vertices assigned with -2 in V_1 and r_2 vertices assigned with -2 in V_2.
Therefore

$$f(N(v)) = \begin{cases} n-r_2-2r_2 \geq 1 & \text{if } v \epsilon V_1 \\ m-r_1-2r_1 \geq 1 & \text{if } v \epsilon V_2 . \end{cases}$$

$$= \begin{cases} n-3r_2 \geq 1 & \text{if } v \epsilon V_1 \\ m-3r_1 \geq 1 & \text{if } v \epsilon V_2 . \end{cases}$$

$$= \begin{cases} 3r_2 \leq n-1 & \text{if } v \epsilon V_1 \\ 3r_1 \leq m-1 & \text{if } v \epsilon V_2 . \end{cases}$$

$$= \begin{cases} r_2 \leq (n-1)/3 & \text{if } v \epsilon V_1 \\ r_1 \leq (m-1)/3 & \text{if } v \epsilon V_2 . \end{cases}$$

Since r_1 and r_2 are integers, $m = 3s_1, 3s_1 + 1$ or $3s_1 + 2$ and $n = 3s_2, 3s_2 + 1$ or $3s_2 + 2$.

If $m = 3s_1$, $r_1 \leq (m-1)/3 - (2/3)$ and $n = 3s_2$, $r_2 \leq (n-1)/3 - (2/3)$.

If $m = 3s_1 + 1$, $r_1 \leq (m-1)/3$ and $n = 3s_2 + 1$, $r_2 \leq (n-1)/3$.

If $m = 3s_1 + 2$, $r_1 \leq (m-1)/3 - (1/3)$ and $n = 3s_2 + 2$, $r_2 \leq (n-1)/3 - (1/3)$.

Case (i):
If $m = 3s_1 + 1$, $n = 3s_2 + 1$.
(i.e.) $r_1 \leq (m-1)/3$ and $r_2 \leq (n-1)/3$.
Therefore

$$\begin{aligned} w(f) &= \sum_{v \in V(G)} f(v). \\ &= m-r_1-2r_1+n-r_2-2r_2 \\ &= m-3r_1+n-3r_2 \\ &\geq m+n-3[(m-1)/3]-3[(n-1)/3] \\ &= m+n-m+1-n+1 \\ &= 2. \end{aligned}$$

Therefore $\gamma_{bst}(G) = \gamma_{bst}(K_{m,n}) = \min w(f) = 2$.

Case (ii):
If $m = 3s_1 + 1$, $n = 3s_2 + 2$.
(i.e.) $r_1 \leq (m-1)/3$ and $r_2 \leq (n-1)/3 - (1/3)$.
Therefore

$$\begin{aligned} w(f) &= \sum_{v \in V(G)} f(v). \\ &= m-3r_1+n-3r_2 \\ &\geq m+n-3[(m-1)/3]-3[(n-1)/3-(1/3)] \\ &= 3. \end{aligned}$$

Therefore $\gamma_{bst}(K_{m,n}) = \min w(f) = 3$.

Case (iii):
If $m = 3s_1 + 2$, $n = 3s_2 + 2$.
(i.e.) $r_1 \leq (m-1)/3-(1/3)$ and $r_2 \leq (n-1)/3 - (1/3)$.
Therefore

$$\begin{aligned} w(f) &= \sum_{v \in V(G)} f(v). \\ &= m-3r_1+n-3r_2 \\ &\geq m+n-3[(m-1)/3-(1/3)]-3[(n-1)/3-(1/3)] \\ &= 4. \end{aligned}$$

Therefore $\gamma_{bst}(K_{m,n}) = \min w(f) = 4$.

Case (iv):
If $m = 3s_1$, $n = 3s_2 + 1$.
(i.e.) $r_1 \leq (m-1)/3-(2/3)$ and $r_2 \leq (n-1)/3$.
Therefore

$$\begin{aligned} w(f) &= \sum_{v \in V(G)} f(v). \\ &= m-3r_1+n-3r_2 \\ &\geq m+n-3[(m-1)/3-(2/3)]-3[(n-1)/3] \\ &= 4. \end{aligned}$$

Therefore $\gamma_{bst}(K_{m,n})$ = min w(f) = 4.

Case (v):

If $m = 3s_1$, $n = 3s_2 + 2$.

(i.e.) $r_1 \leq (m-1)/3-(2/3)$ and $r_2 \leq (n-1)/3 - (1/3)$.

Therefore

$$\begin{aligned} w(f) &= \sum_{v \in V(G)} f(v). \\ &= m-3r_1+n-3r_2 \\ &\geq m+n-3[(m-1)/3-(2/3)]-3[(n-1)/3-(1/3)] \\ &= 5. \end{aligned}$$

Therefore $\gamma_{bst}(K_{m,n})$ = min w(f) = 5.

Case (vi):

If $m = 3s_1$, $n = 3s_2$.

(i.e.) $r_1 \leq (m-1)/3-(2/3)$ and $r_2 \leq (n-1)/3 - (2/3)$.

Therefore

$$\begin{aligned} w(f) &= \sum_{v \in V(G)} f(v). \\ &= m-3r_1+n-3r_2 \\ &\geq m+n-3[(m-1)/3-(2/3)]-3[(n-1)/3-(2/3)] \\ &= 6. \end{aligned}$$

Therefore $\gamma_{bst}(K_{m,n})$ = min w(f) = 6.

$$\gamma_{bst}(K_{m,n}) = \begin{cases} 2 & \text{if both m and n are } 3s_i + 1,\ i = 1 \text{ or } 2. \\ 3 & \text{if one of m, n is } 3s_i + 1 \text{ and another is } 3s_j + 2,\ i \neq j,\ i, j = 1, 2. \\ 4 & \text{if both m and n are } 3s_i + 2 \text{ or one of m, n is } 3s_i \text{ and another is } 3s_j + 1,\ i \neq j,\ i, j = 1, 2. \\ 5 & \text{if one of m, n is } 3s_i \text{ and another is } 3s_j + 2,\ i \neq j,\ i, j = 1, 2. \\ 6 & \text{if both m and n are } 3s_i,\ i = 1, 2 \qquad \text{for } m, n \geq 3. \end{cases}$$

References

[1] J.A.Bondy and U.S.R.Murty, Graph Theory with Applications,The MACMILLAN Press Ltd., London and Basingstoke.

[2] Bohdan Zelinka, Liberec, Signed Total Domination Number of a Graph, Czechoslovak Mathematical Journal,51 (126)(2001), 225-229.

[3] V.R.Kulli,Theory of Domination in Graphs, Vishwa International Publications, Gulbarga, India.