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Q- B Continuous Function In Quad Topological Spaces

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Abstract

The purpose of this paper is to study the properties of q-b open sets and q-b closed sets and introduce q-continous function in quad topological spaces (q-topological spaces).

Keywords- Quad topological spaces, q-b open sets, q-b interiror , q-b closure ,q-b continuous function.

1.INTRODUCTION

J .C. Kelly [1] introduced bitopological spaces in 1963. The study of tri-topological spaces was first initiated by Martin M. Kovar [2] in 2000,where a non empty set X with three topologies is called tri-topolgical spaces.Tri α Continuous Functions and tri βcontinuous functions introduced by S. Palaniammal [4] in 2011. D.V. Mukundan [3] introduced the concept on topological structures with four topologies, quad topology (4-tuple topology) and defined new types of open (closed) set . In year 2011 Luay Al-Sweedy and A.F.Hassan defined δ^{**}-continuous function in tritopolgical space. In this paper, we study the properties of q-b open sets and q-b closed sets and q-b continuous function in quad topological space (q-topological spaces).

2. PRILIMINARIES

Definition 2.1 [3] :Let X be a nonempty set and $T_1, T_2, T_3\ and T_4$ are general topologies on X.Then a subset A of space X is said to be quad-open(q-open) set if $A \subset T_1 \cup T_2 \cup T_3 \cup T_4$ and its complement is said to be q-closed and set X with four topologies called q-topological spaces $(X, T_1, T_2, T_3, T_4\ )$ .q-open sets satisfy all the axioms of topology.

Definition 2.2 [3] : A subset A of a space X is said to be q-b open set if

$$A \subset q - cl(q - intA) \cup q - int(q - clA).$$

Note 2.3[3] : We will denote the q-b interior (resp. q-b closure) of any subset ,say of A by q- b intA (q-b clA),where q-b intA is the union of all q-b open sets contained in A, and q-b clA is the intersection of all q-b closed sets containing A.

3.1 q-b open & q-b closed sets:

Theorem3.1.1: Arbitrary union of q-b open sets is q-b open.

Proof: Let $\{A_\alpha \,/\, \alpha \in I\}$ be a family of q-b open sets in X.

For each $\alpha \in I, A \subset q - cl(q - intA) \cup q - int(q - clA)$.

Therefore $\cup\, A \subset [\cup \{q - cl(q - intA)\}] \cup [\cup \{q - int(q - clA)\}]$.

$$\cup\, A \subset \{q - cl(q - \cup\, intA)\} \cup \{q - int(q - \cup\, clA)\}.$$

(by definition of q-b open sets).Therefore $\cup\, A_\alpha$ is q-b open .

Theorem3.1.2:Arbitrary intersection of q-b closed sets is q-b closed.

Proof: Let $\{B_\alpha \,/\, \alpha \in I\}$ be a family of q-b closed sets in X.

Let $A_\alpha = B_\alpha{}^c$. $\{A_\alpha \,/\, \alpha \in I\}$ be a family of q-b open sets in X.

Arbitrary union of q-b open sets is q-b open .Hence $\cup\, A_\alpha$ is q-b open and hence $(\cup\, A_\alpha)^c$ is q-b closed i.e $\cap\, A_\alpha{}^c$ is q-closed i.e $\cap\, B_\alpha$ is q-b closed. Hence arbitrary intersection of q-b closed sets is q-b closed.

Note 3.1.3: 1.$q - b\ \ int\ A\ \subset A$.

2. $q - b\ int\ A$ is q- b open.

3. q- b int A is the largest q-b open set contained in A.

Theorem 3.1.4: A is q- b open iff $A = q - b\ int\ A$.

Proof: A is q-b open and $A \subset A$. Therefore $A \in \{B \,/\, B \subset A, B$ is q-b open$\}$

A is in the collection and every other member in the collection is a subset of A and hence the union of this collection is A. Hence $\cup \{B \,/\, B \subset A, B$ is q-b open$\} = A$

and hence $q - b\ int\ A = A$.

Conversely since $q - b\ int\ A$ is q-b open,

$A = q - b\ int\ A$ implies that A is q- b open.

Theorem 3.1.5: $q - b\ int\ (A \cup B) \supset\ q - b\ int\ A\ \cup q - b\ int\ B$

Proof : $q - b\ int\ A\ \subset A$ and q-b int A is q- b open.

$q - b\ int\ B\ \subset B$ and q-b int B is q-b open.

Union of two q-b open sets is q-b open and hence q$-b\ int\ A\ \cup q - b\ int\ B$ is a q-b open set.

Also $q - b\ int\ A\ \cup q - b\ int\ B\ \subset A \cup B$.

$q - b\ int\ A\ \cup q - b\ int\ B$ is one q-b open subset of $A \cup B$ and $q - b\ int\ (A \cup B)$ is the largest q-b open subset of $A \cup B$.

Hence $q - b\ int\ (A \cup B) \supset\ q - b\ int\ A\ \cup q - b\ int\ B$.

Definition 3.1.6[3]: Let (X, T_1, T_2, T_3, T_4) be a quad topological space and let $A\ \subset X$. The intersection of all q-b closed sets containing A is called the q-b closure of A & denoted by $q - b\ cl\ A$. $q - b\ cl\ A\ = \cap\{\ B\ /\ B\ \supset A\ , B$ is q-b closed$\}$.

Note 3.1.7: Since intersection of q-closed sets is q-b closed, q-b cl A is a q-b closed set.

Note 3.1.8: q-b cl A is the smallest q-b closed set containing A.

Theorem 3.1.9: A is q-b closed iff $A\ =\ q - b\ cl\ A$.

Proof: $q - b\ cl\ A\ = \cap\{\ B\ /\ B\ \supset A\ , B$ is q-b closed$\}$.

If A is a q-closed then A is a member of the above collection and each member contains A. Hence their intersection is A. Hence $q - cl\ A\ =\ A$. Conversely if $A\ =\ q - b\ cl\ A$, then A is q-closed because q-b cl A is a q-b closed set.

Definition 3.1.10: Let $A\ \subset X$, be a quad topological space. $x\ \in X$ is called a q-b limit point of A, if every q-b open set U containing x, intersects $A - \{x\}$.(ie) every q-b open set containing x,contains a point of A other than x.

3.2: q-b continuous function

Definition 3.2.1: Let (X, T_1, T_2, T_3, T_4) and $(Y, T_1{'}, T_2{'}, T_3{'}, T_4{'})$ be two quad topological spaces. A function $f: X\ \rightarrow Y$ is called a q-b continuous function if $f^{-1}(V)$ is q-b open in X, for every q-b open set V in Y.

Example 3.2.2:Let $X\ =\ \{1, 2, 3, 4\}$, $T_1\ =\ \{\emptyset, \{1\}, X\}$, $T_2 = \{\emptyset,\ \{1\}, \{1,3\}, X\}$

$$T_3\ =\ \{\emptyset, \{1\}, \{1,2\}, X\},\ T_4\ =\ \{\emptyset, \{4\}, \{1,4\}, X\}$$

$$Let\ Y\ =\ \{a, b, c, d\},\ T_1{'}\ =\ \{\emptyset, \{a\}, Y\},\ T_2{'} = \{\emptyset, \{a\}, \{a, c\}, Y\},$$

$$T_{3'} = \{\emptyset, \{a\}, \{a,b\}, Y\}, T_{4'} = \{\emptyset, \{d\}, \{a,d\}, Y\}$$

Let f : X ⟶ Y be a function defined as $f(1) = a; f(2) = b; f(3) = c;$

$f(4) = d$

q-open sets in (X, T_1, T_2, T_3, T_4) are $\emptyset, \{1\}, \{1,2\}, \{1,3\}, \{4\}, \{1,4\}, X$.

q-open sets in $(Y, T_{1'}, T_{2'}, T_{3'}, T_{4'})$ are $\emptyset, \{a\}, \{a,b\}, \{a,c\}, \{d\}, \{a,d\}, Y$.

q-b open sets in (X, T_1, T_2, T_3, T_4) are $X, \emptyset, \{1\}, \{4\}, \{1,2\}, \{1,3\}, \{1,4\}, \{2,3\}, \{1,2,3\}$.

q-b open sets in $(Y, T_{1'}, T_{2'}, T_{3'}, T_{4'})$ are Y, $\emptyset, \{a\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{a,b,c\}$.

Since $f^{-1}(V)$ is q-b open in X for every q-b open set V in Y,

f is q-b continuous.

Definition 3.2.3 :Let X and Y be two q-topological spaces. A function f: X ⟶Y is said to be q-bcontinuous at a point $a \in X$ if for every q-b open set V containing f(a), ∃ a q-b open set U containing a , such that $f(U) \subset V$.

Theorem 3.2.4: $f: X \to Y$ is q-b continuous iff f is q-b continuous at each point of X.

Proof: Let $f: X \to Y$ be q-b continuous.

Take any $a \in X$. Let V be a q-b open set containing f(a).

$f: X \to Y$ is q-b continuous,Since $f^{-1}(V)$ is q-b open set containing a.

Let U = $f^{-1}(V)$ .Then $f(U) \subset V \Rightarrow \exists$ a q-b open set U containing a and $f(U) \subset V$

Hence f is q-b continuous at a.

Conversely, Suppose f is q-b continuous at each point of X.

Let V be a q-b open set of Y. If $f^{-1}(V) = \emptyset$ then it is q-b open.

Take any $a \in f^{-1}(V)$ f is q-b continuous at a.

Hence ∃ Ua ,q-b open set containing a and $f($ Ua$) \subset V$.

Let $U = \cup \{$ Ua $/ a \in f^{-1}(V) \}$.

Claim: U = $f^{-1}(V)$.

$a \in f^{-1}(V) \Rightarrow$ Ua $\subset U \Rightarrow a \in U$.

$x \in U \Rightarrow x \in$ Uafor some $a \Rightarrow f(x) \in V \Rightarrow x \in f^{-1}(V)$. Hence $U = f^{-1}(V)$

Each Ua is q-b open. Hence U is q-b open. $\Rightarrow f^{-1}(V)$ is q-b open in X.

Hence f is q-b continuous.

Theorem 3.2.5: Let $(X, T_1, T_2, T_3, T_4)$and$(Y, T_1', T_2', T_3', T_4')$ be two q-topological spaces. Then

f: X → Y is q-b continuous function iff $f^{-1}(V)$ is q-b closed in X whenever V is q-b closed in Y.

Proof: Let f : X → Y be q-b continuous function.

Let V be any q-b closed in Y.

⇒ V^c is q-b open in Y ⇒$f^{-1}(V^c)$ is q-b open in X.

⇒ $[f^{-1}(V)]^c$ is q-b open in X.

⇒ $f^{-1}(V)$is q-b closed in X.

Hence $f^{-1}(V)$is q- b closed in X whenever V is q-b closed in Y.

Conversely, suppose $f^{-1}(V)$is q-b closed in X whenever V is q-b closed in Y.

V is a q-b open set in Y.

⇒ V^c is q-b closed in Y.

⇒ $f^{-1}(V^c)$is q-b closed in X.

⇒ $[f^{-1}(V)]^c$ is q-b closed in X.

⇒ $f^{-1}(V)$ is q- b open in X.

Hence f is q-b continuous.

Theorem 3.2.6: : Let $(X, T_1, T_2, T_3, T_4)$and$(Y, T_1', T_2', T_3', T_4')$ be two q-topological spaces..Then,f : X → Y is q-b continuous iff $f[q - cl\ A] \subset q - cl\ [f(A)]\ \ \forall A\ \subset X$.

Proof: Suppose f: X → Y is q-b continuous. Since $q - b\ cl\ [f(A)]$ is q-b closed in Y.Then by theorem (3.2.5) $f^{-1}(\ q - cl\ [f(A)]\ )$ is q-b closed in X,

$$q - b\ cl\ [f^{-1}(q - b\ cl(f(A))] = f^{-1}(q - b\ cl(f(A)) - - - -(1).$$

Now :$f(A) \subset q - b\ cl\ [f(A)],\ A \subset f^{-1}\big(f(A)\big) \subset f^{-1}(q - b\ cl(f(A)).$

Then $q - b\ cl(A) \subset\ q - bcl\ [f^{-1}(q - b\ cl(f(A))] = f^{-1}(q - b\ cl(f(A))$ by (1).

Then $f(q - b\ cl(f(A)) \subset\ q - b\ cl(f(A)).$

Conversely, Let $f\big(q - b\ cl(A)\big) \subset q - b\ cl\big(f(A)\big)\forall A\ \subset X.$

Let F be q-b closed set in Y ,so that $q - b\ cl(F) = F$. Now $f^{-1}(F) \subset X$,by hypothesis,

$$f(q - b\ cl(f^{-1}(F)) \subset\ q - b\ cl\left(f\big(f^{-1}(F)\big)\right) \subset q - b\ cl(F) = F\ .$$

Therefore $q - b\ cl(f^{-1}(F)) \subset f^{-1}(F)$. But $f^{-1}(F) \subset q - b\ cl(f^{-1}(F))$ always.

Hence $q - b\ cl(f^{-1}(F)) = f^{-1}(F)$ and so $f^{-1}(F)$ is q-b closed in X.

Hence by theorem (3.2.5) f is q- b continuous.

3.3: q- b Homomorphism

Definition 3.3.1: Let $(X, T_1, T_2, T_3, T_4)$and $(Y, T_1', T_2', T_3', T_4')$ be two q-topological spaces. A function f: X → Y is called q-b open map if f (V) q-b open in Y for every q-b open set V in X.

Example 3.3.2: In example 3.2.2 f is q-b open map also.

Definition 3.3.3: Let $(X, T_1, T_2, T_3, T_4)$and $(Y, T_1', T_2', T_3', T_4')$ be two q-topological spaces .Let f: X → Y be a mapping . f is called q- b closed map if f(F) is q-b closed in Y for every q-b closed set F in X.

Example 3.3.4:The function f defined in the example 3.2.2 is q-b closed map.

Result 3.3..5:Let X & Y be two q-topological spaces. Let f: X → Y be a mapping.f is q-b continuous iff f^{-1} : Y → X is q-b open map.

Definition 3.3.6:Let $(X, T_1, T_2, T_3, T_4)$and $(Y, T_1', T_2', T_3', T_4')$ be two q-b topological spaces.Let f: X → Y be a mapping . f is called a q-b homeomorphism.

If (i) f is a bijection.

(ii) f is q-b continuous.

(iii) f^{-1} is q-b continuous.

Example 3.3.7:The function f defined in the example 3.2.2 is

(i) a bijection. (ii) f is q-b continuous. (iii) f^{-1} is q-b continuous.

Therefore f is a q-b homeomorphism.

CONCLUSION:

In this paper the idea of q-b continous function in quad topological spaces were introduced and studied ,Also properties of q-b open and q-b closed sets were studied .

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