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Some Identities of Fibonacci Like Sequences

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Abstract

The Fibonacci sequence has been studied extensively and generalized in many ways.Hordam[5] has considered a generalized Fibonacci sequence $w_0, w_1, w_2 ...$ .defined by $w_n = pw_{n-1} - qw_{n-2}$, $n \geq 2$ with initial condition $w_0 = a, w_1 = b$. In this paper ,We present some identities of Fibonacci like sequence.
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Introduction: sequence have been fascinating topic for mathematicians for centuries.The Fibonacci Sequences is a source of many nice and intresting identities identities.It is well known that the Fibonacci numbers and Lucas numbers are closely related.

Horadam [5] has considerd a generalized Fibonacci Sequences (w_n) defined b $w_n = pw_{n-1} - qw_{n-2}$, $n \geq 2$ with initial condition $w_0 = a, w_1 = b$. (1.1)

Where p and q are arbitrary integers, Although the sequence (w_n) has been studied extensively for years. For example as in [2-4].] B.Singh,Pooja Bhadouria and O.P. Sikhwal [1]present Some Identities involving common factor of Fibonacci and lucas numbers .

Here is some special cases of the sequence (w_n), namely the following Fibonacci Like and Lucas Like sequences.

$$S_n = mS_{n-1} + S_{n-2},\quad S_0 = 0, S_1 = 1 \qquad (1.2)$$

$$T_n = mT_{n-1} + T_{n-2},\quad T_0 = 2, T_1 = m \qquad (1.3)$$

Where m is positive integer.

By (1.1) ,the Binets forms for the sequences $(S_n),(T_n)$ can be easily obtained as follows

$$S_n = \frac{1}{\sqrt{m^2+4}}\left\{\left(\frac{m+\sqrt{m^2+4}}{2}\right)^n - \left(\frac{m-\sqrt{m^2+4}}{2}\right)^n\right\}$$

$$T_n = \left(\frac{m+\sqrt{m^2+4}}{2}\right)^n + \left(\frac{m-\sqrt{m^2+4}}{2}\right)^n$$

Let

$$\alpha = \left(\frac{m+\sqrt{m^2+4}}{2}\right) and$$

$$\beta = \left(\frac{m-\sqrt{m^2-4}}{2}\right)$$

Then $S_n = \left(\dfrac{\alpha^n - \beta^n}{\alpha-\beta}\right) \qquad (1.4)$

$$T_n = \alpha^n + \beta^n \qquad (1.5)$$

Which gives

$$\alpha + \beta = m$$

$$\alpha.\beta = -1$$

$$\alpha - \beta = \sqrt{m^2+4}$$

Now we present some Identities involving Binets formula of Fibonacci like sequence.

2.Some Identities:

Theorem2.1 $S_{2n+p}-(-1)^n S_p = S_n.T_{n+p}$, *Where* n ≥ 1, p ≥ 0.

Proof: $S_n.T_{n+p}=\left[\frac{\alpha^n-\beta^n}{\alpha-\beta}\right].\left[\alpha^{n+p}-\beta^{n+p}\right]$ By (1.4) and (1.5)

$$=\left[\frac{\alpha^{2n+p}-\beta^{2n+p}}{\alpha-\beta}\right]-\left[\frac{\beta^n.\alpha^{n+p}-\alpha^n.\beta^{n+p}}{\alpha-\beta}\right]$$

$$=S_{2n+p}-\frac{1}{\alpha-\beta}\left[(-1)^n.\left(\alpha^p-\beta^p\right)\right]$$ By(1.4)

$$=S_{2n+p}-(-1)^n.S_p$$

Corollary 2.2 For different values of p,(2.1) can be expressed for even and odd numbers.

If p=0, then $S_{2n}=S_n.T_n$.

Theorem 2.3 $S_{2n+p}+S_p=S_{n+p}.T_n$ Where n ≥ 1 and p ≥ 0.

Proof: $S_{n+p}.T_n=\left[\frac{\alpha^{n+p}-\beta^{n+p}}{\alpha-\beta}\right].\left(\alpha^n+\beta^n\right)$ By(1.4) and (1.5)

$$=\left[\left(\frac{\alpha^{2n+p}-\beta^{2n+p}}{\alpha-\beta}\right)+\left(\frac{\alpha^{n+p}.\beta^n-\beta^{n+p}.\alpha^p}{\alpha-\beta}\right)\right]$$

$$=S_{2n+p}+(-1)^n\left(\frac{\alpha^p-\beta^p}{\alpha-\beta}\right)$$ By (1.4)

$$=S_{2n+p}+(-1)^n S_p$$

Corollary 2.4: Ifp=0 then (2.3) can be expressed in the following way.

$$S_{2n}=S_n.T_n \, .$$

Theorem 2.5 $T_{2n+p}-(-1)^n T_p=5S_n.S_{n+p}$ Where n ≥ 1, p ≥ 0.

Theorem 2.6 $S_{4n+p}+(-1)^n S_{2n+p}=S_{3n+p}.L_n$,n ≥ 1,p ≥ 0.

Theorem 2.7 $S_{4n+p}+(-1)^n S_{2n+p}=S_{3n+p}.S_n$ Where n ≥ 1, p ≥ 0.

Theorem 2.8 $S_{2n+p}.T_{2n+p}=S_{4n+p}$,n ≥ 1,p ≥ 0.

Theorem 2..9 $T_{8n+3}-m=T_{4n+1}.T_{4n+2}$, Where m is positive integer and m,n ≥ 1.

3. Conclusion: This paper describes some identities of Fibonacci like sequences. Many similar identities can be developed for higher order Fibonacci like sequence.

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