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Analysis of Primes in Arithmetical Progressions 6*n* + *K* Up To A Trillion

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ABSTRACT

Prime numbers exhibit many mysteries one of which is their distribution amongst the positive integers, for which yet there is no regular looking pattern recognized.

The simplest form being arithmetical progression, there have been consistent efforts to track their occurrences in these. As part of continued contribution to theseefforts, in this work prime numbers are analyzed with view of their distribution in the arithmetical progressions $6n + k$.

Keywords: Arithmetical progressions,block-wise distribution,prime, prime density, prime spacing.

INTRODUCTION

Prime numbers are peculiar positive integers with minimum number of positive divisors with the exception of 1.The infinitude of primes is known to human race from more than two millenniums [1].

PRIMES DISTRIBUTIONS

These prime numbers are scattered in the list of integers in quite irregular-like fashion. There are arbitrarily many twin primes, those successive primes with spacing of 2 only and similarly there are also arbitrarily large gaps between successive primes. This poses the irregularity scenario.

The number of primes less than or equal to a positive real number x is expressed by using notation $\pi(x)$.

PRIMES DISTRIBUTIONS IN ARITHMETICAL PROGRESSIONS

An arithmetical progression is sequence of integers of form $an + b$, where a and b are fixed integers and n varies over all non-negative integers. If we fix a to be a positive integer and allow b to be take values from 0 to $a - 1$, then the resulting $a$number of arithmetical progressions $an + k$, for $0 \leq k < a$ cover all integers together.

Clearly for any fixed a, all primes will find their place in some or other arithmetical progression $an + b$; but the matter of interest lies in how many of them will be in each such progression and other related properties. Since there are infinitely many primes, for each fixed positive integer *a*, at least one of these is bound to contain infinitely many primes. Dirichlet [2] addressed this issue more concretely by proving classical result that every arithmetical progression $an + b$ with $\gcd(a, b) = 1$ contains infinitely many primes.

For notation purpose, the symbol $\pi_{a,b}(x)$ is used to represent the number of primes in a specific arithmetical progression $an + b$ that are less than or equal to x.

PRIMES DISTRIBUTIONS IN ARITHMETICAL PROGRESSIONS 6*n* + *k*

The possible values of remainders after division by 6 are 0, 1, 2, 3, 4 and 5. Every positive integer after dividing by 6 yields one and only one amongst these valuesas remainder. So it isin one of the arithmetical progressions $6n + 0 = 6n$ or $6n + 1$ or $6n + 2$ or $6n + 3$ or $6n + 4$ or $6n + 5$.

First few numbers of the form $6n$ are

$$6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, \cdots$$

Each of these is perfectly divisible by 6 and so none of these is a prime.

First few numbers of the form $6n + 1$ are

$$1, 7, 13, 19, 25, 31, 37, 43, 49, 55, 61, 67, \cdots$$

This contains infinitely many primes as $\gcd(6, 1) = 1$ as per requirement of Dirichlet's Theorem.

First few numbers of the form $6n + 2$ are

$$2, 8, 14, 20, 26, 32, 38, 44, 50, 56, 62, 68, \cdots$$

Each of these is even and hence divisible by 2. Except the first member, viz., 2, none of these is a prime.

First few numbers of the form $6n + 3$ are

$$3, 9, 15, 21, 27, 33, 39, 45, 51, 57, 63, 69, \cdots$$

Each of these is divisible by 3. Except the first member, viz., 3, none of these is a prime. Thus this sequence contains only one prime 3 and its all other members are composite numbers.

First few numbers of the form $6n + 4$ are

$$4, 10, 16, 22, 28, 34, 40, 46, 52, 58, 64, 70, \cdots$$

Each of these is even. None of these is prime.

First few numbers of the form $6n + 5$ are

$$5, 11, 17, 23, 29, 35, 41, 47, 53, 59, 65, 71, \cdots$$

This sequence does contain infinitely many primes as $\gcd(6, 5) = 1$ as per requirement of Dirichlet's Theorem.

There are independent proofs about infinitude of primes of both types $6n + 1$ and $6n + 5$ [3].

PRIMES NUMBER RACE

For a positive integer a and all b with $0 \leq b < a$, all the arithmetical progressions $an + b$ which contain infinitely many primes are compared for more number of primes in them. This is known as prime number race [4].

We compared the number of primes of form $6n + 1$ and $6n + 5$ till one trillion, i.e., 1,000,000,000,000 (10^{12}). The ambitiousprocedure could be worked out by using an efficient algorithm from those compared in [5]. Java Programming Language [6] was used on computer to execute this task.

Table 1.*Number of Primes of form 6n + k in First Blocks of 10 Powers.*

Sr. No.	Range 1-x (1 to x)	Ten Power (x)	Number of Primes of the form $6n + 1$ $\pi_{6,1}(x)$	Number of Primes of the form $6n + 5$ $\pi_{6,5}(x)$
	1-10	10^1	1	1
	1-100	10^2	11	12
	1-1,000	10^3	80	86
	1-10,000	10^4	611	616
	1-100,000	10^5	4,784	4,806
	1-1,000,000	10^6	39,231	39,265
	1-10,000,000	10^7	332,194	332,383
	1-100,000,000	10^8	2,880,517	2,880,936
	1-1,000,000,000	10^9	25,422,713	25,424,819
	1-10,000,000,000	10^{10}	227,523,123	227,529,386
	1-100,000,000,000	10^{11}	2,059,018,668	2,059,036,143
	1-1,000,000,000,000	10^{12}	18,803,933,520	18,803,978,496

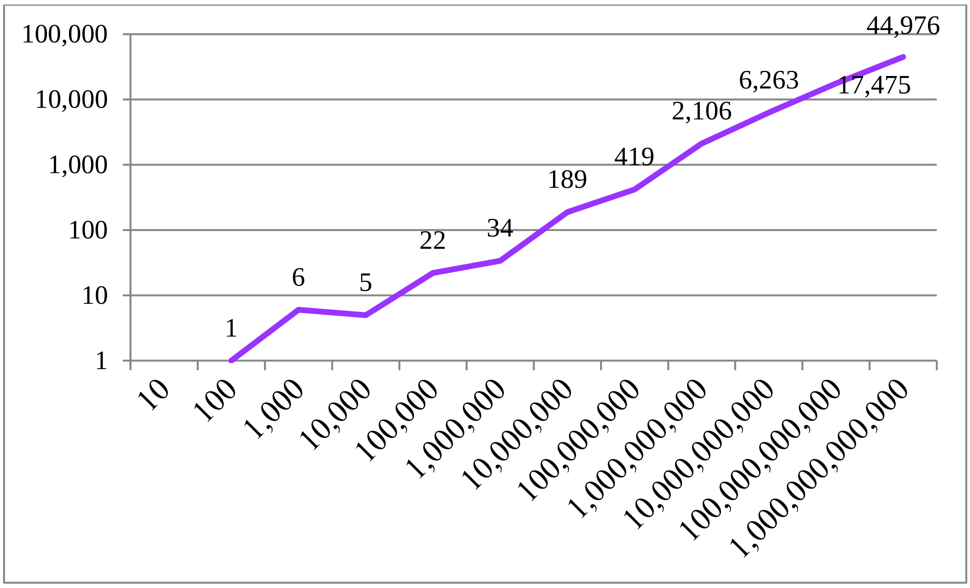


Figure 1.*Dominance of* $\pi_{6,5}(x)$ *over* $\pi_{6,1}(x)$

It is observed that the number of primes of the form $6n + 5$ is more than those of form $6n + 1$ in the initial ranges up to 10^{12} in discrete blocks of 10 powers. Whether this trend of $\pi_{6,5}(x) > \pi_{6,1}(x)$ continues ahead on majority is an area of future explorations.

BLOCK-WISE DISTRIBUTION OF PRIMES

Owing to both the facts that there is no simple formula to cover all primes and at the same time they are quite randomly distributed, we have considered all primes up to one trillion (10^{12}) and divided this range in blocks of powers of 10 each as :

0-9, 10-19, 20-29, 30-39, · · ·

0-99, 100-199, 200-299, 300-399, · · ·

0-999, 1000-1999, 2000-2999, 3000-3999, · · ·

⋮

Then analysis is performed for blocks of all sizes of 10^{12-i} for each $1 \leq i \leq 12$in our range of 10^{12}.

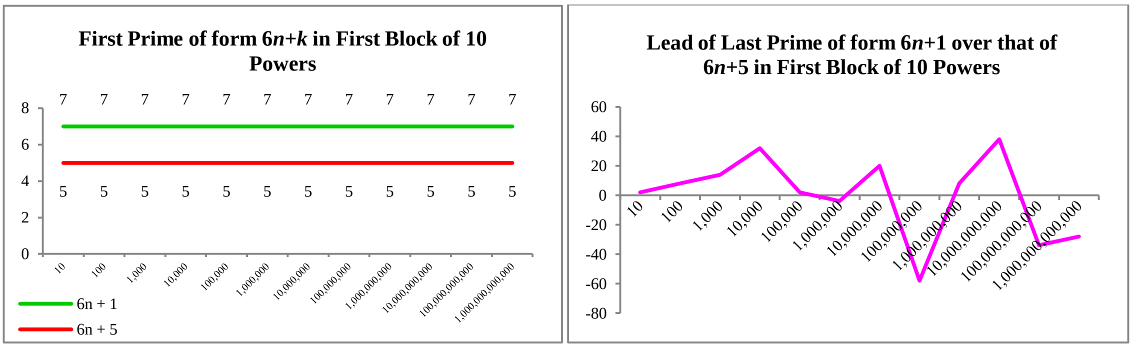
The First and the Last Primes in the First Blocks of 10 Powers

The first prime of first block continues for all higher-sized blocks ahead as their first prime also. The last prime of 10 power blocks naturally goes on increasing with increased block-size.

Table 2.*First and Last Primes of form 6n+k in First Blocks of 10 Powers.*

Sr No	Blocks of Size (of 10 Power)	First Prime in the First Block		Last Prime in the First Block	
		Form $6n + 1$	Form $6n + 5$	Form $6n + 1$	Form $6n + 5$
	10	7	5	7	5
	100	7	5	97	89
	1,000	7	5	997	983
	10,000	7	5	9,973	9,941
	100,000	7	5	99,991	99,989
	1,000,000	7	5	999,979	999,983
	10,000,000	7	5	9,999,991	9,999,971
	100,000,000	7	5	99,999,931	99,999,989
	1,000,000,000	7	5	999,999,937	999,999,929
	10,000,000,000	7	5	9,999,999,967	9,999,999,929
	100,000,000,000	7	5	99,999,999,943	99,999,999,977
	1,000,000,000,000	7	5	999,999,999,961	999,999,999,989

The difference in the last primes of form $6n + 1$ and $6n + 5$ in the first blocks has uncertain trend.



Figures2.*First &Last Primes of form 6n+k in First Blocks of 10 Powers.*

Minimum Number of Primes in Blocks of 10 Powers

Inspecting all blocks from 10^1 to 10^{12}, the minimum number of primes found in them has been determined for primes of forms 6*n* + 1 and 6*n* + 5.

Table 3.*Minimum Number of Primes of form 6n + k in Blocks of 10 Powers*

Sr. No.	Blocks of Size (of 10 Power)	Minimum No. of Primes of form 6*n* + 1 in Block	Minimum No. of Primes of form 6*n* + 5 in Block
	10	0	0
	100	0	0
	1,000	1	1
	10,000	126	124
	100,000	1,653	1,646
	1,000,000	17,756	17,619
	10,000,000	180,001	180,115
	100,000,000	1,808,103	1,808,105
	1,000,000,000	18,094,690	18,093,491
	10,000,000,000	180,988,251	180,989,170
	100,000,000,000	1,812,964,422	1,812,960,010
	1,000,000,000,000	18,803,933,520	18,803,978,496

There is fluctuation in difference in minimum number of primes of form 6*n* + 1 and 6*n* + 5.

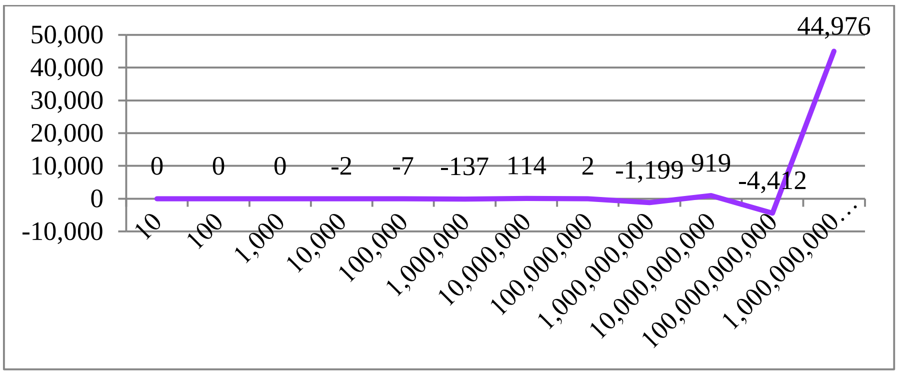


Figure 3.*Minimality Lead of Number of Primes of form 6n+1 over 6n+5 in 10 Power Blocks*

The first and last blocks in range of 10^{12} with minimum number of primes of forms $6n + 1$ and $6n + 5$ in them are also determined.

Table 4.*First and last blocks of 10 powers with minimum number of primes of form 6n + k.*

Sr. No	Blocks of Size (of 10 Power)	First Block with Minimum Number of Primes		Last Block with Minimum Number of Primes	
		Form $6n + 1$	Form $6n + 5$	Form $6n + 1$	Form $6n + 5$
	10	20	30	999,999,999,990	999,999,999,990
	100	69,500	103,100	999,999,999,700	999,999,999,000
	1,000	208,627,276,000	682,833,699,000	946,441,029,000	949,672,786,000
	10,000	991,093,580,000	772,787,800,000	991,093,580,000	772,787,800,000
	100,000	844,002,100,000	930,488,800,000	844,002,100,000	930,488,800,000
	1,000,000	970,693,000,000	997,040,000,000	970,693,000,000	997,040,000,000
	10,000,000	970,280,000,000	998,020,000,000	970,280,000,000	998,020,000,000
	100,000,000	995,400,000,000	999,300,000,000	995,400,000,000	999,300,000,000
	1,000,000,000	997,000,000,000	998,000,000,000	997,000,000,000	998,000,000,000
	10,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000	990,000,000,000
	100,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000	900,000,000,000

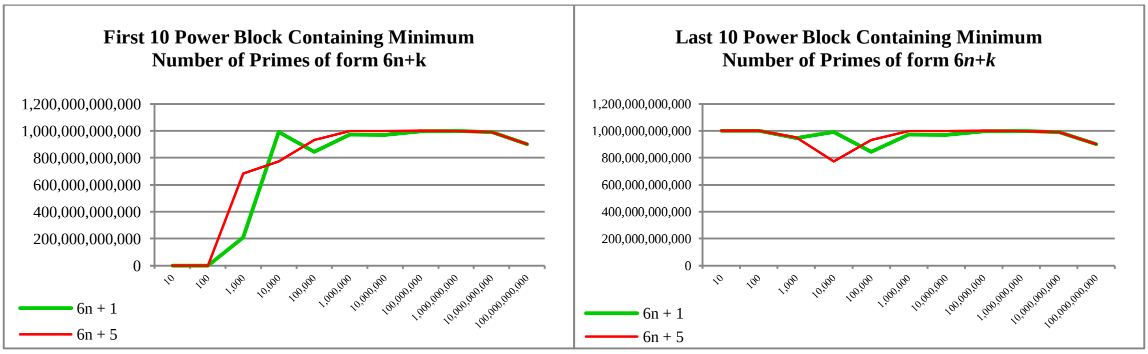


Figure 4.*First and Last Blocks of 10 Powers with Minimum Number of Primes of form 6n + k.*

The frequencies of minimum occurrences of primes of forms 6*n* + *k* decrease.

Table 5. *Number of 10 Power Blocks with Minimum Number of Primes of form 6n + k.*

Sr. No.	Blocks of Size (of 10 Power)	Occurrence Frequency of Minimum No. of Primes of form 6*n* + 1	Occurrence Frequency of Minimum No. of Primes of form 6*n* + 5
	10	82,443,117,633	82,443,091,281
	100	1,227,978,147	1,228,005,131
	1,000	8	5
	10,000	1	1
	100,000	1	1
	1,000,000	1	1
	10,000,000	1	1
	100,000,000	1	1
	1,000,000,000	1	1
	10,000,000,000	1	1
	100,000,000,000	1	1
	1,000,000,000,000	1	1

Maximum Number of Primes in Blocks of 10 Powers

Like the minimum number of primes of form 6*n* + *k* in blocks of 10^i, the maximum number of them is also determined.

Table 6.*Maximum Number of Primes of form 6n + k in Blocks of 10 Powers.*

Sr. No.	Blocks of Size (of 10 Power)	Maximum No. of Primes of form 6*n* + 1 in Block	Maximum No. of Primes of form 6*n* + 5 in Block
	10	2	2
	100	11	12
	1,000	80	86
	10,000	611	616
	100,000	4,784	4,806
	1,000,000	39,231	39,265
	10,000,000	332,194	332,383
	100,000,000	2,880,517	2,880,936
	1,000,000,000	25,422,713	25,424,819
	10,000,000,000	227,523,123	227,529,386
	100,000,000,000	2,059,018,668	2,059,036,143
	1,000,000,000,000	18,803,933,520	18,803,978,496

Except for the first block size of 10, primes of form 6*n* + 5 dictate in all blocks.

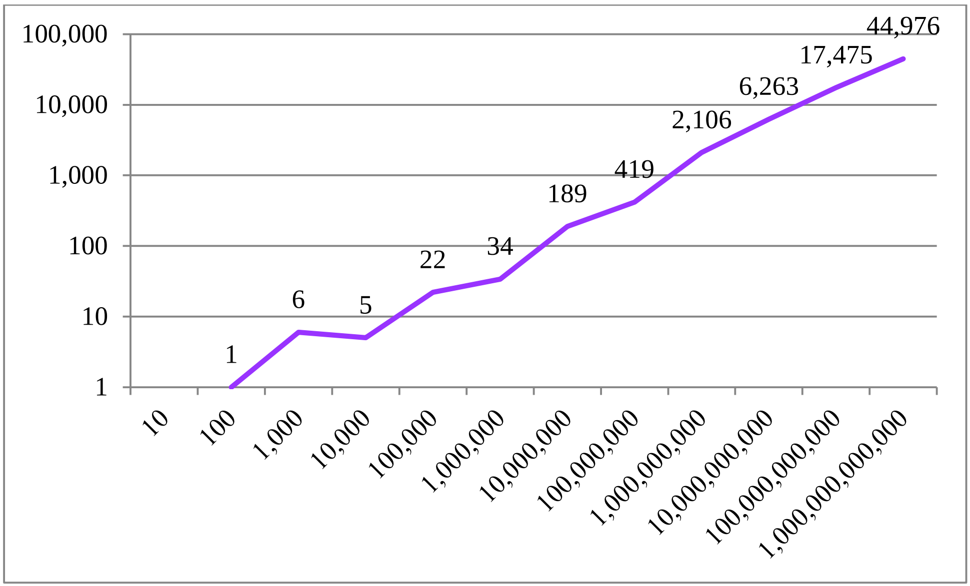


Figure 5.*Maximality Lead of Number of Primes of form 6n+5 over 6n+1 in 10 Power Blocks.* The first and last blocks of 10^i till one trillion with maximum number of primes of forms 6*n* + 1 and 6*n* + 5 are determined.

Table 7.*First and last blocks of 10 powers with maximum number of primes of form 6n + k.*

Sr. No.	Blocks of Size (of 10 Power)	First Block with Max No. of Primes		Last Block with Max No. of Primes	
		Form 6*n* + 1	Form 6*n* + 5	Form 6*n* + 1	Form 6*n* + 5
	10	10	10	999,999,999,570	999,999,999,610
	100	0	0	977,727,538,300	0
	1,000	0	0	0	0
	10,000	0	0	0	0
	100,000	0	0	0	0
	1,000,000	0	0	0	0
	10,000,000	0	0	0	0
	100,000,000	0	0	0	0
	1,000,000,000	0	0	0	0
	10,000,000,000	0	0	0	0
	100,000,000,000	0	0	0	0

In general, the prime density shows a decreasing trend with increasing range of numbers. So it is natural that for higher block sizes, the first as well as the last occurrences of maximum number of primes in them starts in the first block after 0.

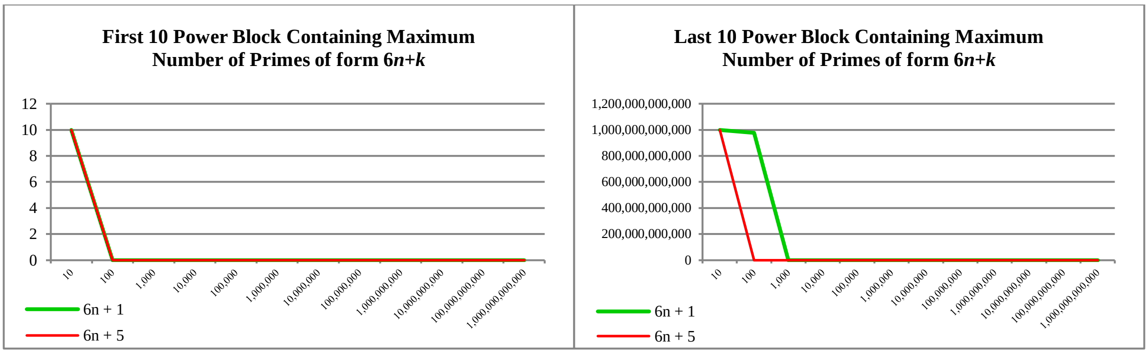


Figure 6.*First and last blocks of 10 powers with maximum number of primes of form 6n + k.* Due to reduction in the prime density, the maximum number of primes cannot occur frequently in blocks.

Table 8.*Number of 10 power blocks with maximum number of primes of form 6n + k.*

Sr. No.	Blocks of Size (of 10 Power)	Number of Times the Maximum Number of Primes of form 6*n* + 1 Occur in Blocks	Number of Times the Maximum Number of Primes of form 6*n* + 5 Occur in Blocks
	10	1,247,051,153	1,247,069,777
	100	40	1
	1,000& Higher Sized Blocks till 10^{12}	1	1

For block of 10, the frequency of occurrence of maximum primes of form 6*n* + 5 is more than that of form 6*n* + 1, then 6*n* + 1 has taken a marginal lead and then both have the same unit value for higher blocks.

SPACINGS BETWEEN PRIMES OF FORM 6*n* + *k* IN BLOCKS OF 10*i*

Minimum Spacings between Primes of Form 6n + k in Blocks of 10 Powers

Exempting prime-empty blocks, the minimum spacing between primes of form 6*n* + 1 and 6*n* + 5 in blocks of 10 powers are determined to be 6 each beginning with the first block 10^1 = 10. Since,found once, for larger block sizes, the minimum spacing value cannot exceed, it remains same for all blocks of all higher powers of 10.

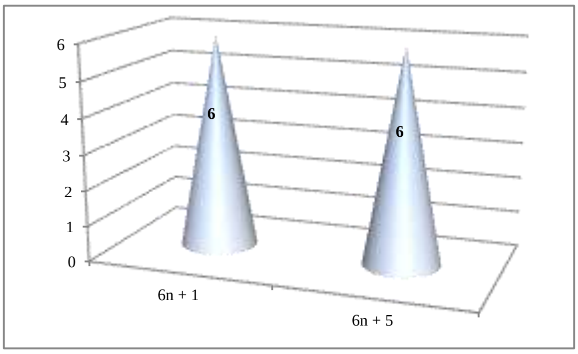


Figure 7.*Minimum Block Spacing*

This minimum block spacing of 6 occurs for primes of form 6*n* + 1 first at 13 for blocks of 10 and for higher power blocks at 7. For blocks of 10, it is not in first block at 7 as the next prime of this form 13 with spacing of 6 occurs in next block. The minimum block spacing of 6 occurs for primes of form 6*n* + 5 first at 11 for blocks of 10 and for higher power blocks at 5. The variation is due to same reason for the other form.

The minimum block spacing of 6 for primes of form 6*n* + 1 occurs last in our range at 999,999,999,571 for all 10 power blocks. This last occurrence for primes of form 6*n* + 5 is at 999,999,999,611 for block of 10 and at 999,999,999,857 for all higher blocks in our range. The reason for change in higher blocks is same block crossing for 10.

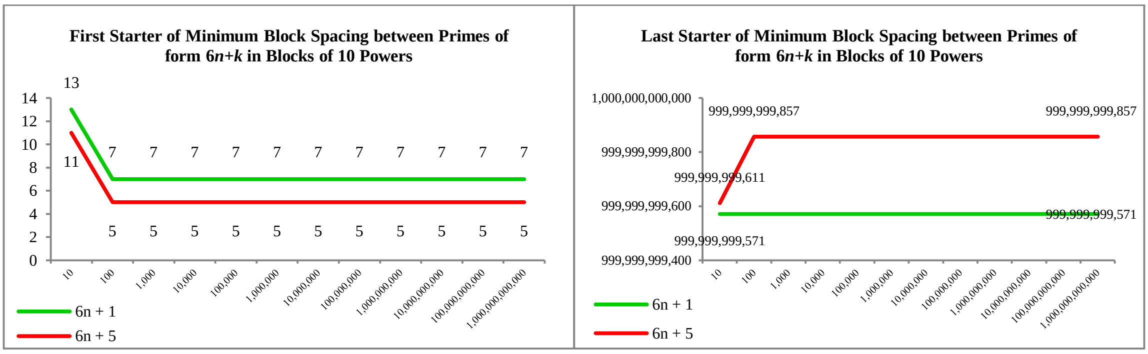


Figure 8.*First & Last Starters of Minimum Block Spacing between Primes of form 6n+k.*

It is important to note the number times this minimum block spacing occurs.

Table 9.*Frequency of Occurrence of Minimum Block Spacing Occurring for 6n+k form Primes*

Sr. No.	Blocks of Size (of 10 Power)	Number of Times the Minimum Block Spacing Occurring for Primes of form 6*n* + 1	Number of Times the Minimum Block Spacing Occurring for Primes of form 6*n* + 5
	10	1,247,051,153	1,247,069,777
	100	1,808,234,686	1,808,281,094
	1,000	1,864,352,043	1,864,395,871
	10,000	1,869,963,048	1,870,006,890
	100,000	1,870,524,725	1,870,568,132
	1,000,000	1,870,580,790	1,870,624,398
	10,000,000	1,870,586,258	1,870,629,961
	100,000,000	1,870,586,799	1,870,630,570
	1,000,000,000	1,870,586,847	1,870,630,632
	10,000,000,000	1,870,586,853	1,870,630,642
	100,000,000,000	1,870,586,855	1,870,630,643
	1,000,000,000,000	1,870,586,855	1,870,630,643

The increase in the number of times the minimum spacing occurs for primes of both form is because ofthose primes with desired spacing occurring at the crossing of earlier blocks finding themselves in same larger blocks raising the count. Of course, this rate of increase gradually decreases.

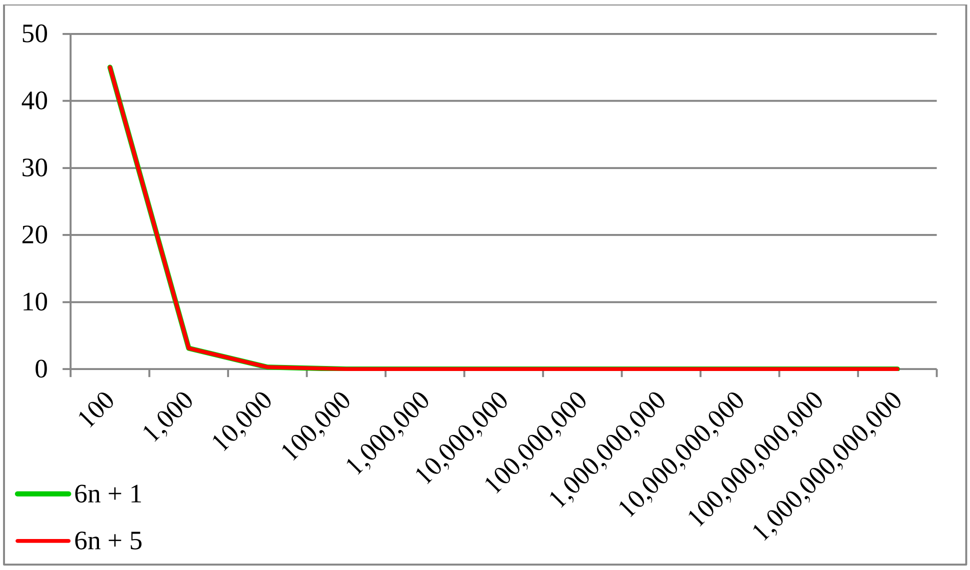


Figure 9.*% Increase in Occurences of Minimum Block Spacing between Primes of form 6n+k.*

Maximum Spacings between Primes of form 6n + k in Blocks of 10 Powers

The maximum spacing in 10 power blocks increases with increase in block size.

Table 10.*Maximum Block Spacing between Primes of form 6n + k.*

Sr. No.	Blocks of Size (of 10 Power)	Maximum Block Spacing Occurring for Primes of form 6*n* + 1	Maximum Block Spacing Occurring for Primes of form 6*n* + 5
	10	6	6
	100	96	96
	1,000	960	942
	10,000	1,068	1,068
	100,000	1,068	1,068
	1,000,000	1,068	1,068
	10,000,000	1,068	1,068
	100,000,000	1,068	1,068
	1,000,000,000	1,068	1,068
	10,000,000,000	1,068	1,068
	100,000,000,000	1,068	1,068
	1,000,000,000,000	1,068	1,068

In the range of 1 - 10^{12}, with the exception for the block of 1000, in-block maximum spacing for primes of both forms is same.

The first and last primes of forms 6*n* + 1 and 6*n* + 5 with these maximum in-block spacings are determined this range.

Table 11.*First & Last Primes of form 6n + k with Maximum Block Spacings.*

Sr. No.	Blocks of Size (of 10 Power)	First Prime with Respective Maximum Block Spacing		Last Prime with Respective Maximum Block Spacing	
		Form 6*n* + 1	Form 6*n* + 5	Form 6*n* + 1	Form 6*n* + 5
	10	13	11	999,999,999,571	999,999,999,611
	100	93,001	144,203	999,999,994,801	999,999,981,503
	1,000	653,064,334,009	596,580,025,049	653,064,334,009	596,580,025,049
	10,000	759,345,224,761	423,034,793,273	759,345,224,761	423,034,793,273
	100,000	759,345,224,761	423,034,793,273	759,345,224,761	423,034,793,273
	1,000,000	759,345,224,761	423,034,793,273	759,345,224,761	423,034,793,273
	10,000,000	759,345,224,761	423,034,793,273	759,345,224,761	423,034,793,273
	100,000,000	759,345,224,761	423,034,793,273	759,345,224,761	423,034,793,273
	1,000,000,000	759,345,224,761	423,034,793,273	759,345,224,761	423,034,793,273
	10,000,000,000	759,345,224,761	423,034,793,273	759,345,224,761	423,034,793,273
	100,000,000,000	759,345,224,761	423,034,793,273	759,345,224,761	423,034,793,273
	1,000,000,000,000	759,345,224,761	423,034,793,273	759,345,224,761	423,034,793,273

We present graphical representation for comparison.

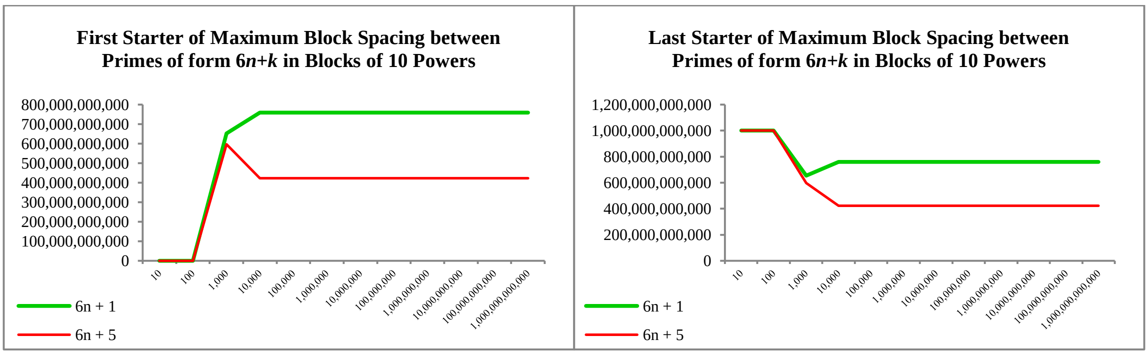


Figure 10.*First & Last Primes of form 6n + k with Maximum Block Spacings.*

The frequencies of occurrences of these maximum block spacing are determined.

Table 12.*Frequency of maximum block spacings between primes of form 6n + k.*

Sr. No.	Blocks of Size (of 10 Power)	Number of Times the Maximum Block Spacing Occurs for Primes of form 6*n* + 1	Number of Times the Maximum Block Spacing Occurs for Primes of form 6*n* + 5
	10	1,247,051,153	1,247,069,777
	100	21,217,945	21,205,830
	1,000& Higher Sized Blocks till 10^{12}	1	1

END DIGIST OF PRIMES OF FORM 6*n* + *k*

UNITS PLACE DIGITS IN PRIMES FORM 6n + k

Out of six possible digits in units place, the primes of form 6*n* + *k* exhibit following trends.

Table 13.*Number of Primes of form 6n + k with Different Units Place Digits till* 10^{12}.

Sr. No.	Digit in Units Place	Number of Primes of form	
		6n + 1	6n + 5
	1	4,700,968,833	4,700,992,147
	2	0	0
	3	4,700,984,929	4,700,994,974
	5	0	1
	7	4,701,002,681	4,700,994,319
	9	4,700,977,077	4,700,997,055

As the only even prime 2 and only prime with its unit place digit 5 are exceptional cases for units place digits of primes, they are generally ignored.

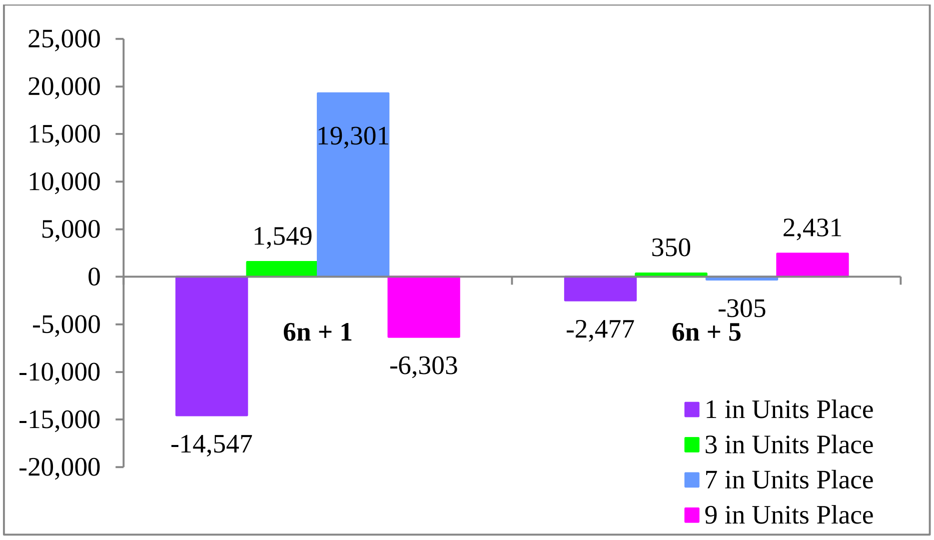


Figure 11.*Deviation of Number of Unit Place Digits of Primes of form 6n+k from Average.*

TENS &UNITS PLACE DIGITS IN PRIMES FORM 6n + k

Table 14. *Number of Primes of form 6n + k with Different Tens and Units Place Digits till 10^{12}.*

Sr. No.	Digits in Tens & Units Place	Number of Primes of form	
		$6n + 1$	$6n + 5$
	01	470,091,333	470,109,891
	02	0	0
	03	470,094,770	470,104,271
	05	0	1
	07	470,097,248	470,104,276
	09	470,094,613	470,103,424
	11	470,102,397	470,089,234
	13	470,100,789	470,099,915
	17	470,091,448	470,097,857
	19	470,106,050	470,118,517
	21	470,098,988	470,108,463
	23	470,102,820	470,102,293
	27	470,103,643	470,103,729
	29	470,102,782	470,094,647
	31	470,103,390	470,097,906
	33	470,101,752	470,095,882
	37	470,104,142	470,094,694
	39	470,101,627	470,093,736
	41	470,093,947	470,096,059
	43	470,095,523	470,102,070
	47	470,095,217	470,102,515
	49	470,105,420	470,095,356
	51	470,107,468	470,097,412
	53	470,094,341	470,101,246
	57	470,099,603	470,093,392
	59	470,103,290	470,096,232
	61	470,093,061	470,103,049
	63	470,102,739	470,092,627
	67	470,104,073	470,099,284
	69	470,085,723	470,086,721
	71	470,100,170	470,096,319
	73	470,094,450	470,102,497

Sr. No.	Digits in Tens & Units Place	Number of Primes of form	
		$6n + 1$	$6n + 5$
	77	470,097,789	470,098,854
	79	470,091,636	470,097,190
	81	470,087,306	470,092,697
	83	470,090,913	470,100,987
	87	470,105,175	470,093,879
	89	470,093,415	470,108,593
	91	470,090,773	470,101,117
	93	470,106,832	470,093,186
	97	470,104,343	470,105,839
	99	470,092,521	470,102,639

Again neglecting the cases 02 and 05, these numbers can be compared graphically.

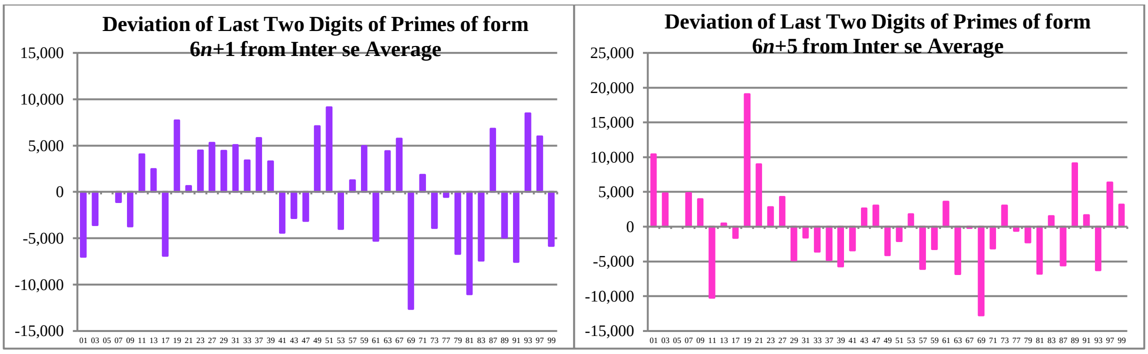


Figure 12.*Deviation of Last Two Digits of Primes of form 6n+k from Inter se Average.*

Analysis of Successive Primes of form 6*n* + 1 and 6*n* + 5

The case when two successive primes are of same form; either $6n + 1$ or $6n + 5$; is interesting. The number of successive primes of desired forms is as follows.

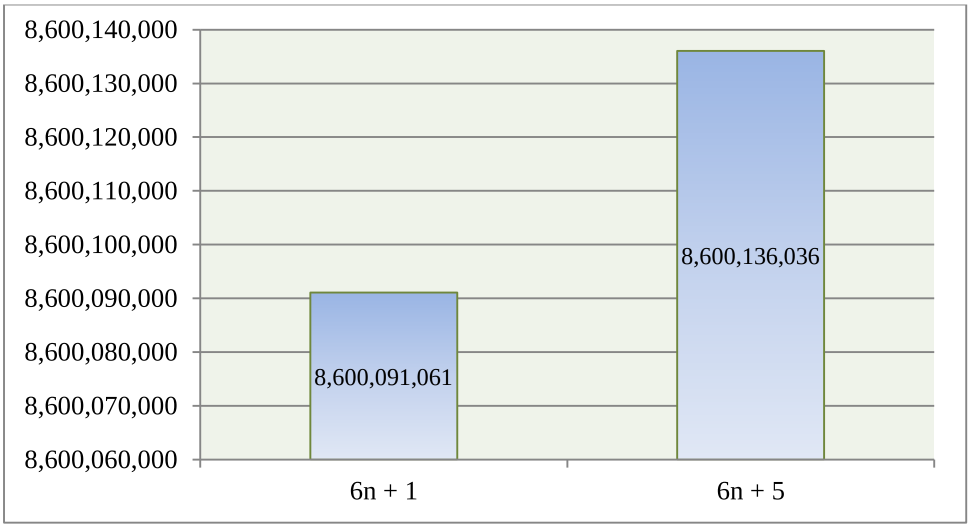
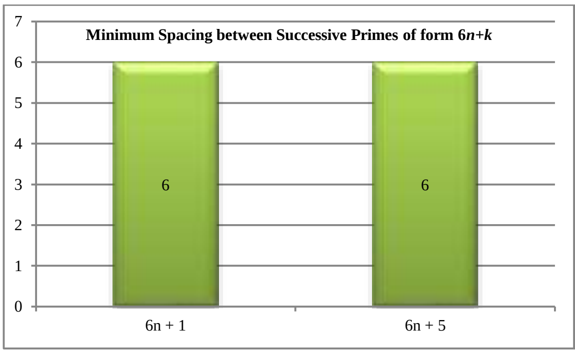
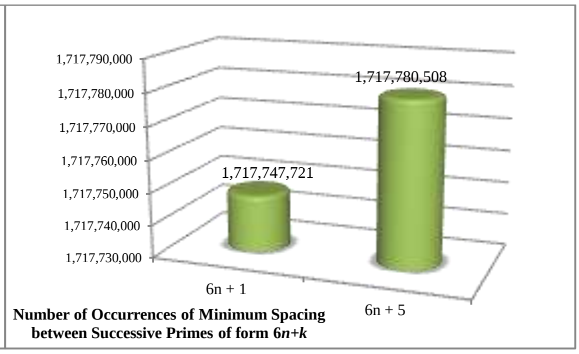
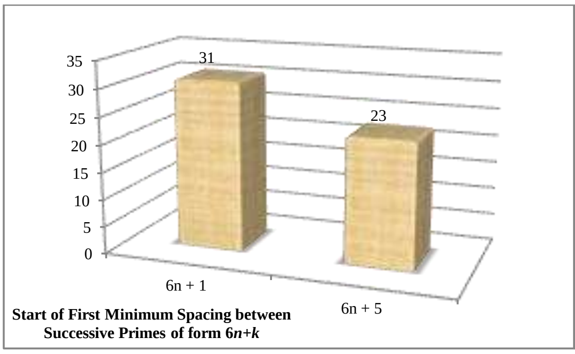


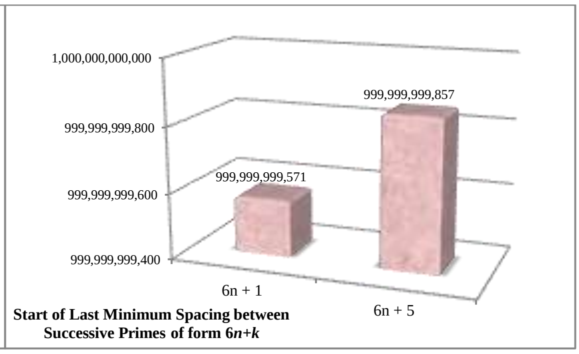
Figure 13.*Number of Successive Primes of form* $6n + k \leq 10^{12}$.

We have rigorously analyzed the successive primes of these specific forms. The minimum spacing between successive primes of forms 6n + k has following properties.

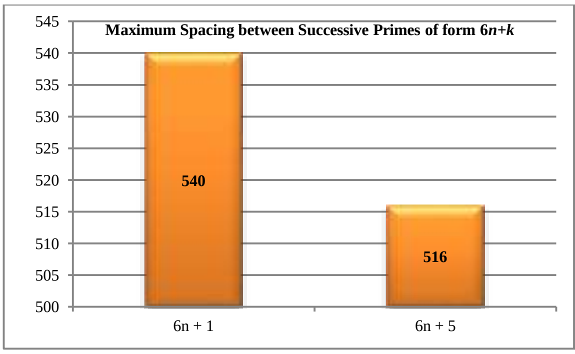


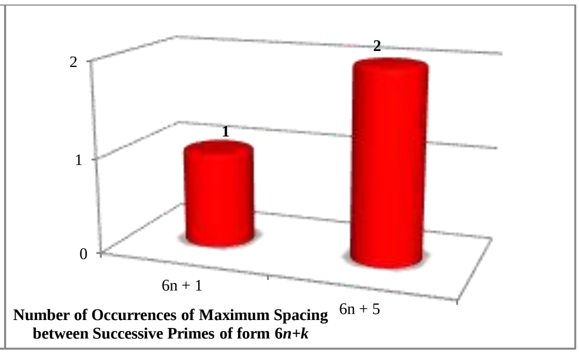


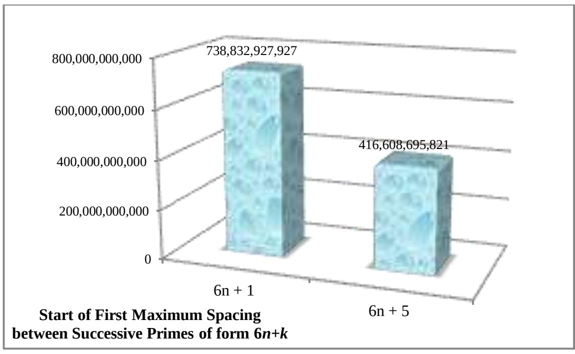


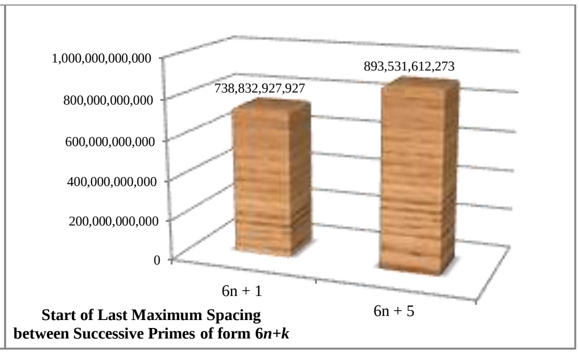


The maximum spacing between successive primes of forms 6n + k has following properties.









Owing to aforementioned irregularity in distribution, efforts, data and analysis are needed to study prime occurrence patterns. The work presented here is also in same direction with respect to a specific linear pattern of 6n + k. The availability of rigorous data like can help give a deeper insight into prime distribution.

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