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Key Properties Related To Soft Connected Spaces

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Abstract

This paper defines soft connected space, soft connected subspace, soft component and some properties related to them are established.

Keywords

Soft set, soft topological space, soft subspace, soft continuous, soft connected space, soft connected subspace.

Introduction

In 1999 D.Molodtsov introduced the concept of soft set theory as a new mathematical modeling for uncertainties in economics, environment, social science, medical science and engineering. In this process he proposed a certain parameterization of a set which results in a soft structure. Many researchers work on soft set theory and its applications in different areas.In 2001 Maji has extended the soft set theory to fuzzy soft set theory and in 2003 [6] he gave some new definitions on soft sets.[5] Shabir and M.Naz formulated the notion of topological space for softsets in 2011 and derived some interesting results which are useful for further research.[4] In 2012 AAygunoglu, H Aygun defined soft continuity of soft mappings.They introduced soft product topology and studied properties of soft projection mappings.[3] J.Mahanta and PK.Das also studied soft set theory.They introduced semiopen, semiclosed soft sets and proved their properties. Further soft semicompactness, soft semiconnectedness and soft semiseparationaxims were introduced and studied.

Preliminaries

Throughout this paper Let X be the universal set and E be a collection of all possible parameters with respect to X, where parameters are the characteristics or properties of objects in X.We call E the universe set of parameters with respect to X.

Definition 2.1[11].A soft set (F,A) denoted by F_A is called a soft set over X if $A \subset E$ and F : $A \rightarrow$ (X) where (X) is the set of all subsets of X.

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Definition 2.2[12]. Let F_A and G_B be soft sets over a common universe set X and A, B$\subseteq$ E. Then F_A is a soft subset of G_B denoted by $F_A \subseteq G_B$ if A $\subseteq$ B and for all e $\in$E, F(e) $\subseteq$ G(e).

Definition 2.3[6].A soft set F_A over X is called a soft null set, denoted by ϕ if e $\in$A , F(e) = ϕ.

Definition 2.4 [6]. A soft set F_A over X is called an absolute soft set, denoted by Ã, if e $\in$A,F(e) = X.

Definition 2.5[6]. The union of two soft sets F_A and G_B over a common universe X is the soft set H_C where C= A $\cup$B and $\forall$ e $\in$C.

$$H(e)= \begin{cases} F(e), & \text{if } e \in A\text{-}B; \\ G(e), & e \in B - A; \\ F(e) \cup G(e), & e \in B \cap A. \end{cases}$$

We write $F_A \cup G_B = H_C$.

Definition 2.6 [12]. The intersection of two soft sets F_A and G_B over a common universe X is the soft set H_C where C= A $\cap$ B and $\forall$ e $\in$C, H(e)=F(e) $\cap$ G(e). We write $F_A \cap G_B = H_C$.

Definition 2.7: [13] Let τ be the collection of soft sets over X then τ is called a soft topology on X if τ satisfies the following axioms:

(1). ϕ, $\tilde{X}$ belong to τ.

(2). The union of any number of soft sets in τ belongs to τ

(3). The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E), denoted by (X_E,τ) is called a soft topological space over X.

Note: Here τ(e) is a topology on X.

Definition 2.8[14].The members of τ in a soft topological space are said to be soft open sets in X.

Definition 2.9 [14]. Let Y_F be a non-empty soft subset of X_E. Let τ^* be a collection of all intersections with Y_F of soft open sets in τ.Then Y_F equipped with its relative topology τ^* is called a soft subspace of (X_E,τ) and is denoted by $(Y_{F,}\tau^*)$.

$G_M \in \tau^*, \Rightarrow G_M = Y_F \cap H_A$ where $H_A \in \tau$, a soft open set over X. Here clearly M,F,A $\subseteq$ E and for all e $\in$ M, G(e)= Y(e) $\cap$ H(e).

Definition 2.10[14].A soft set F_A over X is said to be a soft closed set in X if its relative complement, F_A‘ belong to τ.

Definition 2.11[14]. Let F_A be a soft set over X and x $\in$X. We say that x $\in$ F_A read as x belongs to the soft set F_A , whenever x $\in$ F(e) for all e $\in$ E.

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Soft Connected Space

Definition 3.1 :A soft topological space (X_E,τ) is said to be soft connected space if for all $e \in E$,
$(X, \tau(e))$ is connected.

Example: Let X={s1, s2, s3, s4, s5, s6, s7, s8, s9, s10}.
Let E = {Very intelligent, Intelligent, Average, Active, Well mannered}.
Consider a soft set F_A which describes the "intelligence of a student" and the soft set G_B which describes the "behavior of the student".

Let A={Very intelligent, Intelligent, Average}.
B={Active, Well mannered, Average}.
F(Very intelligent) ={ s2, s4, s7, s8},
F(Intelligent) = {s1, s3, s5},
F(Average) = {s6, s9},
G(Active) = {s5, s6, s8},
G(Well mannered) = {s2,s3,s7},
G(Average) = {s4, s8}.
Then $A \cap B = C$ ={Average}, H(Average) = ϕ.
$\therefore F_A \cap G_B = H_C = \phi$.
And $A \cup B = E$ = {Very intelligent, Intelligent, Average, Active, Well mannered}.
Also $F_A \cup G_B = X_E$.
$\therefore X_E$ is soft connected and hence (X_E,τ) is soft connected space.

Definition 3.3 :[2]. Let (X_E,τ) and (Y_F, γ) be two soft topological spaces. $(\psi,\varphi) : (X_E,\tau) \rightarrow (Y_F, \gamma)$ is called soft continuous if for all $e \in E$ and $\psi(e)=f \in F$, $\varphi : (X, \tau(e)) \rightarrow (Y,\gamma(f))$ is continuous.
Note : Here $\tau(e)$ and $\gamma(f)$ are topologies on X and Y respectively.

Theorem 3.4: The soft continuous image of a soft connected space is soft connected.
Proof:
Let (X_E,τ) and(Y_F, γ) be two soft topological spaces and (X_E,τ) be a soft connected space
And $(\psi,\varphi) : (X_E,\tau) \rightarrow (Y_F, \gamma)$ be soft continuous mapping.
Then by definition,3.2 for all $e \in E$ and $\psi(e)=f \in F$, $\varphi : (X, \tau(e)) \rightarrow (Y,\gamma(f))$ is continuous.
Also $(X, \tau(e))$ is connected for all $e \in E$, by definition 3.1
Which implies that $(Y,\gamma(f))$ is continuous for all
$f \in F$ (since the continuous image of a connected space is connected).
Hence (Y_F, γ) is soft connected space .
Definition 3.5:A soft connected subspace of a soft connected space (X_E,τ) is a soft subspace (Y_F, γ) which is soft connected as a soft topological space in its own right.

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Theorem 3.6: Let $\{(A_{iEi}, \tau_i)\}i\in J$ be a nonempty class of soft connected subspaces of a soft topological space (X_E,τ) such that $\cap_i(A_{iEi}, \tau_i,)$ is nonempty , then $(A_F,\gamma) = \cup_i(A_{iEi}, \tau_i)$is also a soft connected subspace of(X_E,τ).

Proof :

In a contrary way suppose that (A_F,γ) is soft disconnected space

Where $(A_F,\gamma) = \cup_i(A_{iEi}, \tau_i)$; each τ_i is a collection of soft sets over A_i

$E_i \subseteq E$ for each i,$F = \cup E_i$,$F \subseteq E$.

$(A,\gamma(e))$ is disconnected for some $e \in E$.

Then there exists two nonempty soft open subsets $(G,\gamma(e))$ and $(H,\gamma(e))$ of $(A,\gamma(e))$ such that

$(A,\gamma(e)) = (G,\gamma(e)) \cup (H,\gamma(e))$.

$\Rightarrow(A_i,\tau_i(e)) \subseteq (G,\gamma(e))$ or $(A_i,\tau_i(e)) \subseteq (H,\gamma(e))$ for each i as each $(A_i,\tau_i(e))$ is soft connected set.

Suppose that $(A_i,\tau_i(e)) \subseteq (G,\gamma(e))$ for each i.

$\Rightarrow \cap_i(A_i,\tau_i(e)) \subseteq (G,\gamma(e))$

$\Rightarrow \cap_i (A_i,\tau_i(e)) \cap (H,\gamma(e)) = \phi$

$\Rightarrow \cap_i(A_{iEi}, \tau_i,) = \phi$ which is a contradiction to the hypothesis.

Hence (A_F,γ) is soft connected.

Theorem 3.7: Let (X_E,τ) be a soft topological space and (A_F, γ) be a soft connected subspace of (X_E,τ).

If (B_G,ν) is a soft subspace of (X_E,τ) such that $(A_F, \gamma) \subseteq (B_G,\nu) \subseteq (\tilde{A}_F, \gamma)$ then (B_G,ν) is soft connected space, in particular $(\tilde{A}_F, \gamma)$.

Proof : Assume that(B_G,ν) is soft disconnected space.

Then $(B,\nu(e))$ is disconnected for each $e \in G$.

There exists two nonempty soft open sets

$(B_1, \nu_1(e)), (B_2, \nu_2(e))$ such that

$(B, \nu(e)) = (B_1, \nu_1(e)) \cup (B_2, \nu_2(e))$ ....(**)

Where $\nu_1(e),\nu_2(e) \subseteq \nu(e)$ for each $e \in E$.

Since $(A_F,\gamma) \subseteq (B_G,\nu)$

we have $(A, \gamma (e)) \subseteq (B_1, \nu_1(e))$

or

$(A, \gamma (e)) \subseteq (B_2, \nu_2(e))$ for each $e \in E$

[Since (A_F,γ) is a soft connected space.].

Suppose $(A, \gamma (e)) \subseteq (B_1, \nu_1(e))$

then $(A, \gamma (e)) \cap (B_2, \nu_2(e)) = \phi$.

$\Rightarrow (\tilde{A}, \gamma (e)) \cap (B_2, \nu_2(e)) = \phi$. Since $(\tilde{A}, \gamma (e))$ is the smallest soft closed super set of $(A, \gamma (e))$.

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$\Rightarrow (B,\nu(e)) \cap (B_2, \nu_2(e)) = \phi$ which contradicts (**)

Hence (B_G,ν) is soft connected space.

In particular $(\tilde{A}_F, \gamma)$ is also soft connected

because $(\tilde{A}_F, \gamma) \subseteq (\tilde{A}_F, \gamma)$.

3.8 Soft Component: A maximal soft connected subspace of a soft topological space i.e. a soft connected subspace which is not properly contained in any larger soft connected subspace is called soft component of the space.

Theorem 3.9: If X is an arbitrary soft topological space, then we have the following:

(1). Each soft point in X is contained in exactly one soft component of X;

(2). Each soft connected subspace of X is contained in a soft component of X;

(3). A soft connected subspace of X which is both soft open and soft closed is a component of X; and

(4). Each component of X is soft closed.

Proof: (1). Let x be a point in X.

Consider the class $\{F_{Ci}\}$of all soft connected subspaces of X which contain x.

This class is nonempty because x itself is soft connected.

By theorem3.6 $F_C = \cup_i F_{Ci}$ is a soft connected subspace of X which contains x.

F_C is clearly maximal and therefore a component of X because any soft connected subspace of X which contains x is one of the F_{Ci}'s ans is thus contained in F_C.

Uniqueness: Let F_C* be another soft component of X containing x.

Then it is clearly one among the F_{Ci}'s and hence is contained in F_C and since F_C* is maximal as a soft connected subspace of X, we must have F_C*= F_C.

(2). It is a direct consequence of the above part.

(3). Let F_A be a soft connected subspace of X which is both open and closed.

By (2) F_A is contained in some soft component F_C.

If F_A is a proper subset of F_C then F_C can be written as $F_C = (F_C \cap F_A) \cup (F_C \cap F_A')$, a disjoint union of two nonempty soft open sets.

$\therefore F_C$ is soft disconnected space which is a contradiction.

$\therefore F_A = F_C$

And hence each soft connected subspace which is both soft open and soft closed is a soft component.

(4). Let F_C be a soft component of X.

Now we show that F_C is soft closed.

In a contrary way suppose that it is soft open.

Then its closure $\overline{F_C}$ is a soft connected subspace of X which properly contains F_C and this contradicts the maximality of F_C as a soft connected subspace of X.

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Totally soft disconnected space: It is a Soft topological space (X_E,τ) in which every pair of soft distinct points can be separated by a soft disconnection of (X_E,τ) .

Conclusion:

This paper investigates properties of soft connected space and few properties of Soft component. Other concepts can be studied further.

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