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An Analysis on Mhd Natural Convection Flow In Open Square Cavity Containing Heated Circular Cylinder

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Abstract

The problem of MHD natural convection heat transfer in a square open cavity containing a heated circular cylinder at the centre has been investigated in this research. As boundary conditions the left vertical wall of the cavity is kept at a constant heat flux, bottom and top walls of the cavity are kept at different high and low temperature respectively. The right side wall is open. The software COMSOL Multiphysics is used to visualize the temperature distribution and fluid flow solving two-dimensional governing mass, momentum and energy equations for steady state, natural convection flow in presence of magnetic field in side an open square cavity. A uniformly heated circular cylinder is placed at the centre of the cavity. The objectives of this study is to describe the effects of Rayleigh number (Ra) on natural convection heat transfer and flow fields in a complicated domain like it in presence of magnetic field by visualization and line graphs. The investigations are conducted for different values of Rayleigh number and some fixed Hartmann numbers (Ha) with buoyancy effect. In the results it has been observed that an increase in Raleigh number of fluid corresponds to an increase in the total heat transfer when Hartmann number is fixed. Which is a good agreement with the existing Heat Transfer Theory.

Keywords: Natural Convection, MHD, Heated Cylinder, Open Cavity, Finite Element Analysis.

Introduction

The convective heat transfer and the natural convection flow with the presence of magnetic field of the fluid are of great importance in scientific and engineering research. Some numerical and experimental methods have been developed previously to investigate flow characteristics in side cavities with and without obstacle. Because these types of geometries have practical engineering and industrial application. This type of problems of heat transfer attracted significance attention of researchers since it's numerous application in the areas of energy conservations, cooling of electrical and electronic equipments, design of solar collectors, heat exchangers etc. Many scientists and engineers have recently studied heat transfer in enclosures with partitions, fins and block which influence the convection flow nature. It is difficult to solve natural convection problem in complicated bodies like it, which greatly influences the heat transfer process. An application is the use of MHD acceleration to shoot plasma into fusion devices or to produce high energy wind tunnels for simulating hypersonic flight. A uniformly heated circular cylinder is placed at the centre of the cavity. The objectives of this study is to describe the effects of Rayleigh number (Ra) on natural convection heat transfer and flow fields in a complicated domain like it in presence of magnetic field by visualization and line graphs. Also our main aim is to extend the work pf Hossain S. A. et al [12] for a different case. In the present work, we studied MHD natural convection heat transfer and flow in a square open cavity containing a heated circular cylinder. The left vertical wall is kept at a constant heat flux. Bottom and top walls are kept at different high and low temperature respectively. The numerical values of temperature at the bottom and top walls are selected randomly such that natural convection can be created in the cavity. The heat flux at left vertical wall is chosen randomly such that the system remain consistent. The remaining side wall is open. Finite element analysis based on Galerkin weighted Residual method is used to solve the problem. The Governing equations and boundary conditions are used in this study as S. Pervin, R. Nasrin [9] and Hossain S. A. et al [12] have been used successfully. For used boundary conditions and values of parameters in the governing equations it is observed that all isotherm lines are concentrated at right lower corner of the cavity and the heat transfer rate is suppressed in suppressed of Raleigh number in the cavity when Hartmann number is fixed. Which is a good agreement with existing Heat Transfer Theory.

Chan and Tien [1] investigated shallow open cavities and made a comparison study using a square cavity in an enlarged computational domain. In the result, they observed that for a square open cavity having an isothermal vertical side facing the opening and two adjoining adiabatic horizontal sides. Satisfactory heat transfer results could be obtained, especially at high Rayleigh numbers. Mohammad [2] investigated inclined open square cavities, by considering a restricted computational domain. The gradients of both velocity components were set to zero at the opening plane in that case which were different from that of Chan and Tien [1]. In the result, he found that heat transfer was not sensitive to inclination angle and the flow was unstable at high Rayleigh numbers and small inclination angles.

Ostrach [3], Davis [4], Hossain and Wilson [5], Hossain *et al*. [6] and Sarris *et al*. [7] studied MHD natural convection in a laterally and volumetrically heated square cavity. Their results show that the effect of increasing

Hartmann number was not found to be straight forward connected with the resulting flow patterns. Roy and Basak

[8] analyzed finite element method of natural convection flows in a square cavity with non-uniformly heated wall(s). S. Pervin and R. Nasrin [9], (2011) studied MHD free convection and heat transfer for different values of Rayleigh numbers *Ra* and Hartmann numbers *Ha* in

a rectangular enclosure. Their results show that the flow pattern and temperature field are significantly dependent on the used parameters. Sheikh Anwar Hossain and Alim [10] studied Effects of Natural Convection from an open square cavity containing a heated circular cylinder. S. Saha [11] studied thermo-magnetic convection and heat transfer of paramagnetic fluid in an open square cavity with different boundary conditions. His results show the Effects of Magnetic Rayleigh number, Prandtl number on the flow pattern and isotherm as well as on the heat absorption graphically. He found that the heat transfer rate is suppressed in decreased of the Magnetic Rayleigh number. Hossain, S. A. *et al* [12], (2015) studied MHD free convection heat transfer in an open square cavity containing a heated circular cylinder. In the result they found that heat transfer rate is increased in decreased Hartmann number.

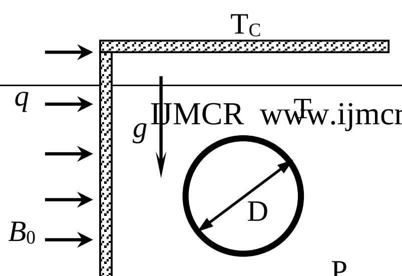
The study related to heat absorption or rejection in the confined rectangular enclosures has been well discussed in the literature C. Taylor and P. Hood [13], Chandrasekhar [14], Dechaumphai [15].

Originality - However, comparatively a little work has been done in the case of open square cavities. The reason might be the complexity on using the boundary conditions at the open side. To the best knowledge of the authors no study with the same configuration and boundary conditions is available. So, comparative study cannot be done in this study.

The terms magneto hydrodynamic, hydrodynamics, magneto gas dynamics and magneto aerodynamics all are the branches of fluid dynamics that deals with the motion of electrically conducting fluids in presence of electric and magnetic fields. In a magnetic field the moving electric charge carried by a flowing fluid and acting in the opposite direction, it is also very small. So the influence of the magnetic field on the boundary layer is exerted only through induced forces within the boundary layer itself, with no additional effects arising from free stream pressure gradient.

Mathematical Formulation

The schematic diagram of the system considered in the present study is shown in fig.1. The system consists of an open square cavity with sides of length L and a heated circular cylinder of diameter D is located at the center of the cavity. The Cartesian co-ordinate system with origin at the lower left corner of the computational domain is considered here. A constant heat flux q is considered at the left wall of the cavity. The bottom wall is kept at high temperature T_h and top wall is kept at low temperature T_c. The remaining right side wall is open. The temperature at the cylinder T_{h1} is taken as the average temperature of the bottom and top walls . A magnetic force of strength B_0 is applied horizontally normal to the side walls.



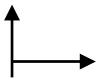


Fig. 1. Schematic diagram of the problem.

The system of governing equations of MHD natural convection can considered by the differential equation expressing conservation of mass or continuity equations, conservation of momentums and conservation of energy. Here, flow is considered as steady, laminar, incompressible, two-dimensional and with the buoyancy force. The Boussinesq approximation is used to relate density changes to temperature changes in the fluid properties and to couple in this way the temperature field to the flow field. The steady MHD natural convection can be governed by the following differential equations.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \qquad (2)$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+g\rho\beta(T_h-T_c)-\sigma B_0^{\,2}\upsilon \qquad (3)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \qquad (4)$$

Boundary conditions:

At Bottom wall: $u=v=0;\ T=T_h;$ $T_h(x,0)$ = 375ºK (say); 373ºK≤T_h≤375ºK

At Top wall: $u=v=0;\ T=T_c;\ T_c(x,\ t)=275^O K$; T_c = 275ºK

At the Left wall: u = v = 0, and heat flux q =100 w/m^2, p = 0.

At the right side & open side: Convective Boundary Condition (CBC), p = 0, u = v.

At the circular cylinder u(x, y) = v(x, y) = 0, T(x, y) = T_{h_1}, T_{h_1} = 324ºK.

$$\frac{\partial T(x,0)}{\partial y}=\frac{\partial T(x,L)}{\partial y}=\frac{\partial T(L,y)}{\partial x}=0$$

At the right side & open side: Convective Boundary Condition (CBC), p = 0,

Here x and y are the distances measured along the horizontal and vertical directions respectively from the origin; u and v are the velocity components in the x and y direction respectively; T denotes the temperature in Kelvin scale of measurement; γ and α are the kinematic viscosity and the thermal diffusivity respectively; p is the pressure and ρ is the density.

Governing Equations in Non- Dimensional Form

We can nondimensionalize the governing equations using the following scales.

Non-dimensional scales:

$$X = \frac{x}{L},\ Y = \frac{y}{L},\ U = \frac{u}{U_0},\ V = \frac{v}{U_0},\ P = \frac{pL^2}{\rho\alpha^2},\ \theta = \frac{T-T_c}{T_h-T_c},\ Pr = \frac{\nu}{\alpha}$$

$$Ra = \frac{gB(T_h-T_c)L^3}{\upsilon\alpha},\quad Ha = \sqrt{\frac{\sigma L^3 B^2{}_0}{\mu}},\quad dr = \frac{D}{L},\ \Delta T = (T_h\text{-}T_\infty),\quad \Delta T = \frac{qL}{K}$$

Non-dimensional governing equations:

Continuity equation:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad (5)$$

Momentum equations:

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \qquad (6)$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + Ra\ Pr\ \theta\ - H_a^2 Pr\ V \qquad (7)$$

Energy equation: $$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \qquad (8)$$

Where $Pr = \frac{\nu}{\alpha}$ is Prandtl number, $Ra = \frac{gB(T_h-T)L^3}{\upsilon\alpha}$ is Rayleigh number and $Ha = \sqrt{\frac{\sigma L^3 B^2{}_0}{\mu}}$ is Hartmann number, $Gr = \frac{gB(T_h-T)L^3}{\upsilon^2}$

BOUNDARY CONDITONS

At Bottom wall: $U = V = 0\ ;\ \theta = 1$. At Top wall: $U = V = 0\ ;\ \theta = 0$

At the left wall: U =V =0; Heat Flux q = 110 w/m^2, p = 0

At the right side & open side: Convective Boundary Condition (CBC), P = 0.

We can obtain the local Nusselt number Nu from the temperature field by applying the function $Nu = -\frac{1}{\theta(0,Y)}$.

The overall or average Nusselt number was calculated by integrating the temperature gradient over the heated wall as follows:

$$Nu_{av} = -\int_0^1 \frac{1}{\theta(0,Y)}\,dy$$

Pr is a heat transfer characteristic in the flow field of natural convection. Because the dimensionless Prandtl number Pr is the ratio of kinematic viscosity to thermal diffusivity.

Numerical Technique

The numerical technique based on the Galerkin weighted residual method of finite element method has been used in this study. This technique is well described by Tailor and Hood [13] and Dechaumphai [15]. In this case the solution domain is discretized into finite element meshes, which are composed of non-uniform triangular elements. Then the nonlinear governing partial differential equations (i.e. mass, momentum and energy equations) are transferred into a system of integral equations by applying the Galerkin weighted residual technique. Here the integration over each term of these equations is performed by using Gauss's quadrature method and then the nonlinear algebraic equations are obtained. These nonlinear algebraic equations are modified by imposing boundary conditions. These modified nonlinear equations are transferred into linear algebraic equations by using Newton's method. Finally, these linear equations are solved by using triangular factorization method.

Results and Graphical Discussion

Computer simulation of Finite element method is applied to perform the analysis of laminar natural convection heat transfer and fluid flow in an open square cavity with a heated circular cylinder. Effects of the parameters Rayleigh number (Ra), Hartmann number (Ha) and Heat flux q on heat transfer and fluid flow inside the cavity has been studied. The visualization focused on temperature and flow fields, which contains isotherms and streamlines for different cases. The range of Ra and Ha for this investigation vary from $1e^5$ to $5e^6$ and 0 to 150 respectively while Pr = 0.71 & heat flux q = 110.

The flow with all Ra in this work have been affected by the buoyancy force. Figures from Fig. 2(a) to 8(a) illustrate temperature field in the flow region and in figures from Fig.2(b) to 8(b) illustrate the recirculation around the cylinder as streamlines in the flow field. In figures from Fig. 2. (a) to 6. (a) the highest temperature region remain at the lower part of the open side of the cavity and in figures Fig. 7(a) & Fig.8(a) the highest temperature remain at the upper part of the open side of the cavity. The isothermal lines are nonlinear for all Ra used in this work and they occupied more than right half of the region in the cavity. The isothermal lines are concentrated at the right lower corner of the cavity for all cases in this work. The isothermal lines & recirculation's region are increasing with increasing Ra in the same Ha. One small vortex is formed above the cylinder for Ra = $5*e^6$ for every Ha in this study.

Fig. 2. Shows the effects of Ra = e^5, $5*e^5$, e^6, $5*e^6$, on isotherms as well as on streamlines for the present configuration at Ha = 0 , q = 110. The recirculation's region are increased for Ra = $5*e^5$, e^6, $5*e^6$.

Fig.3 shows the effects for Ra = e^5, $5*e^5$, e^6, $5*e^6$ on isotherms as well as on streamlines for the present configuration at Ha = 25, q = 110. In this case the isothermal lines are concentrated at the right lower corner of the cavity. The recirculation's region are increased for Ra = $5*e^5$,e^6, $5*e^6$.

Fig.4 shows the effects of Ra = e^5 , $5*e^5$,e^6, $5*e^6$ on isotherms as well as on streamlines for the present configuration at Ha = 50, q = 110. The recirculation's region are increased for Ra = $5*e^5$,e^6, $5*e^6$. Fig.5 shows the effects for Ra = e^5 , $5*e^5$,e^6, $5*e^6$ on isotherms as well as on streamlines for the present configuration at Ha = 75, q = 110. The recirculation's regions are increased for Ra = $5*e^5$,e^6, $5*e^6$.

Fig.6 shows the effects for Ra = e^5 , $5*e^5$,e^6, $5*e^6$ on isotherms as well as on streamlines for the present configuration at Ha = 100, q = 110. The recirculation's region are increased for Ra = $5*e^5$,e^6, $5*e^6$ .The isothermal lines are concentrated at the lower right corner of the cavity for every Ra.

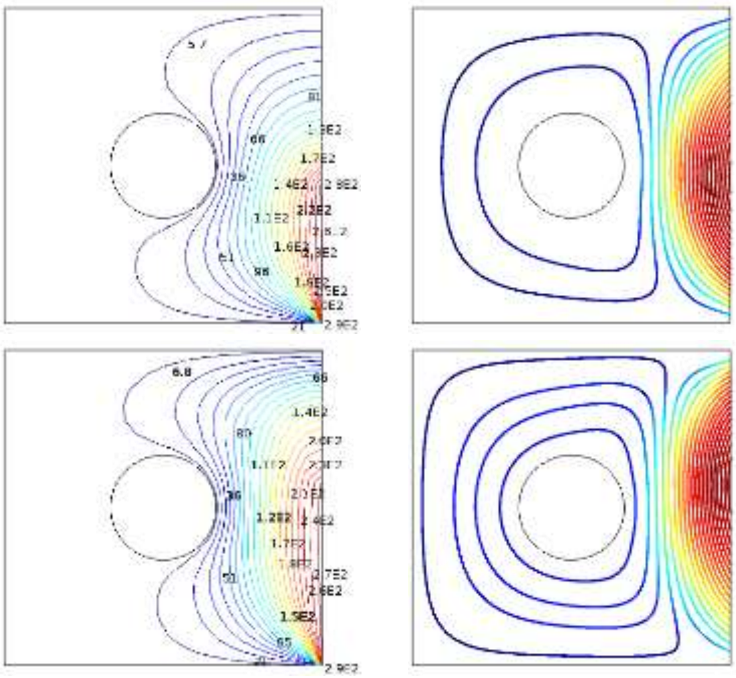
Fig. 7 shows the effects of Ra=e^5, $5*e^5$, e^6, $5*e^6$ at Ha= 125 and heat flux q = 110. In this case the isotherm lines are concentrated at right lower corner of the cavity for every Ra and the isotherm lines are located in the right half of the cavity . The recirculation's region are increased for Ra = $5*e^5$, e^6, $5*e^6$. Here the highest temperature remains at upper half of the open side for Ra = $5*e^6$ & Ha =125.

Fig.8 Shows the effects for Ra = e^5 , $5*e^5$,e^6, $5*e^6$ on isotherms as well as on streamlines for the present configuration at Ha = 150, q = 110. Here isotherm lines concentrate at lower right corner of the cavity for every Ra . In fig.8. one small vortex is formed above the cylinder for Ra = $5*e^6$. The recirculation's region are increased for Ra = $5*e^5$, e^6, $5*e^6$. Here the highest temperature remains at upper half of the open side for Ra = $5*e^6$ & Ha =150.

The various Raleigh numbers and magnetic field affect the heat flux along the heated surface. These are observed in figures from Fig.9 to Fig. 10 that Raleigh number proportionately affect on Heat flux. That is heat flux increasing with increasing Ra. That is if Ra rises then heat flux increases at the open side and upper side of the cavity. But Fig.11. shows that heat flux at the cylinder is highest when Ra = $5e^5$ and heat flux is lowest for Ra = $5e^6$. Which agree with the existing Heat Transfer Theory.

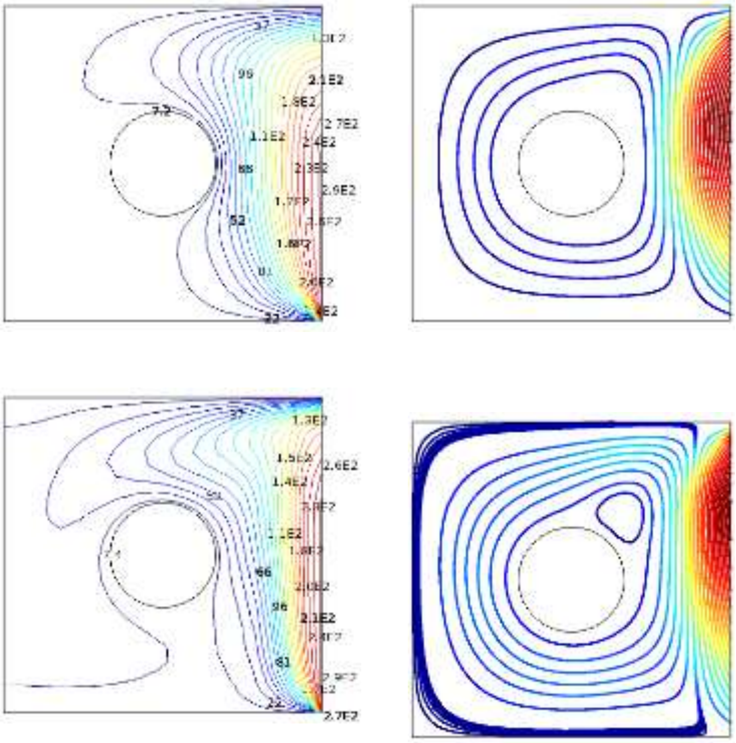
Ra=1e5

Ra= 5e5



Ra=1e6

Ra=5e6

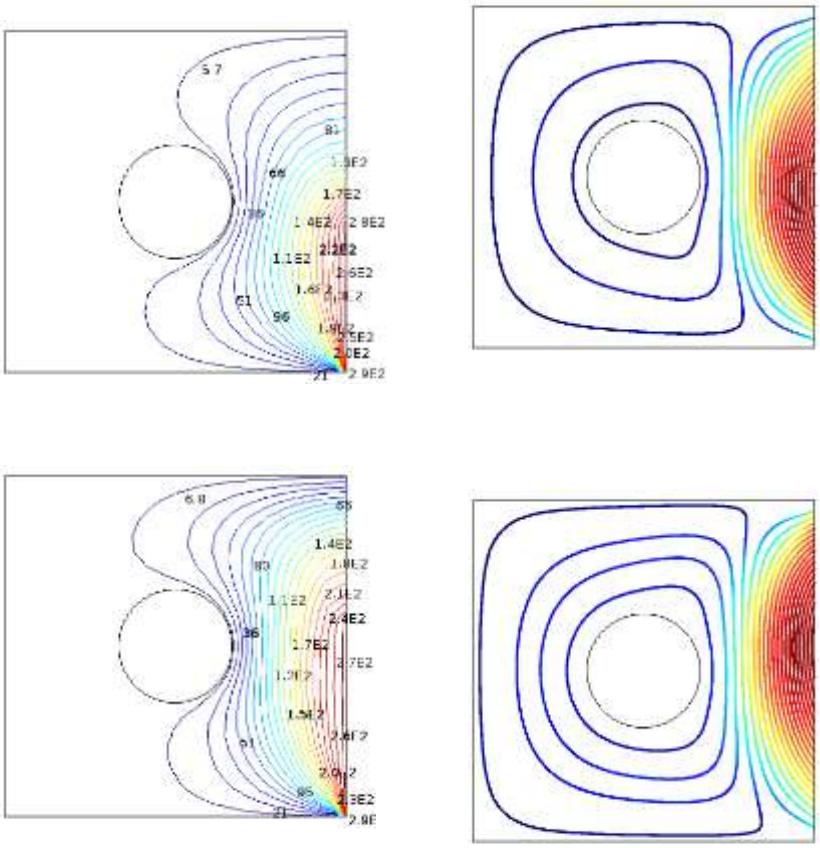


(a) (b)

Fig: 2 Isotherms (a) & Streamlines (b) for various Ra and Ha = 0, q = 110 in the cavity.

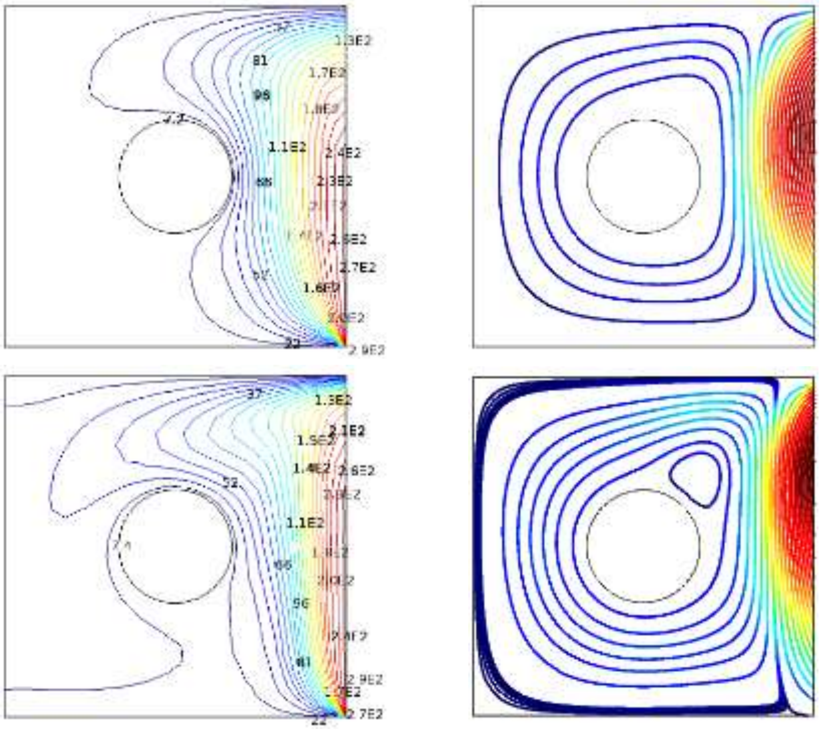
Ra=1e5

Ra=5e5



Ra=1e6

Ra= 5e6

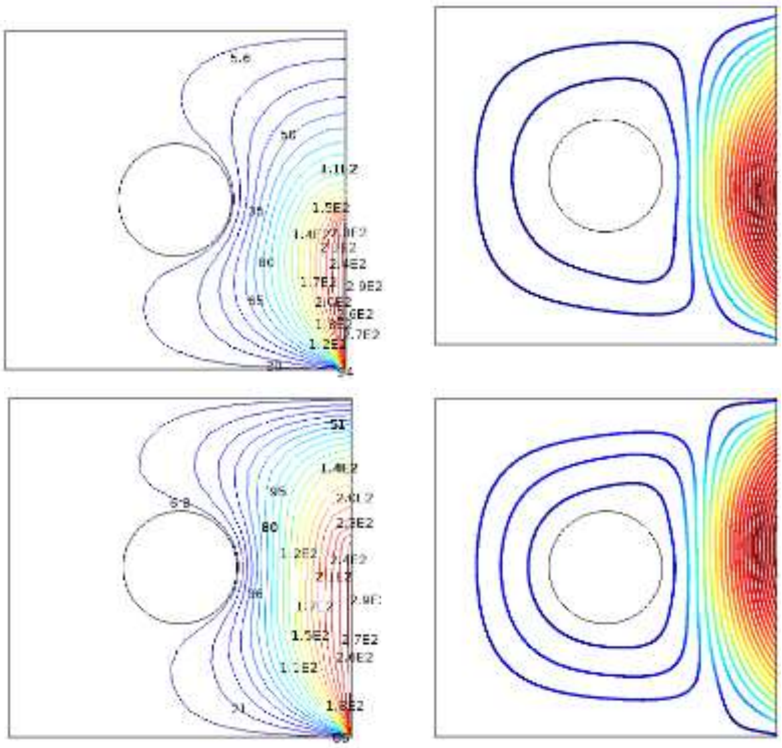


(a) (b)

Fig: 3 (a) Isotherms & (b) Streamlines in the cavity for various Ra and Ha = 25, q =110

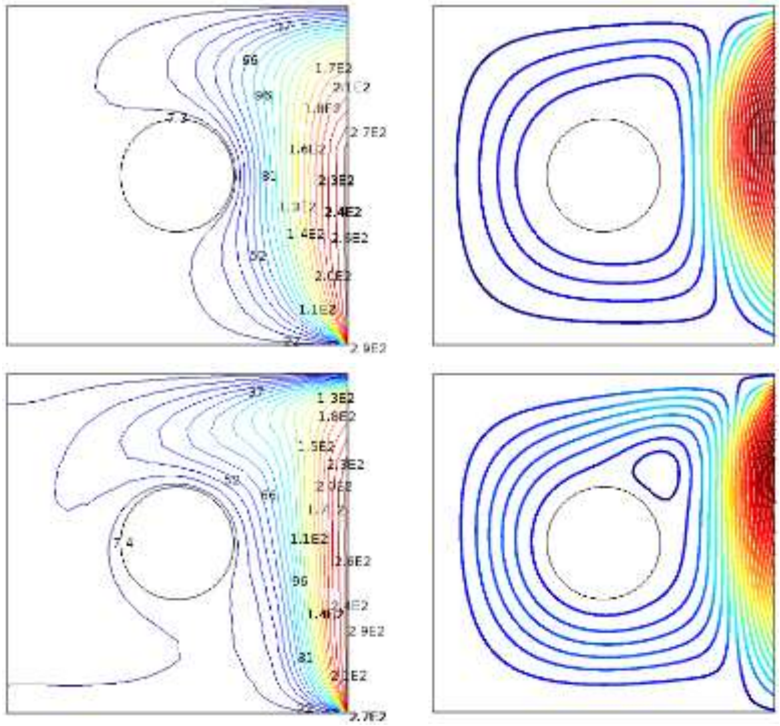
Ra=1e5

Ra=5e5



Ra=1e6

Ra=5e6

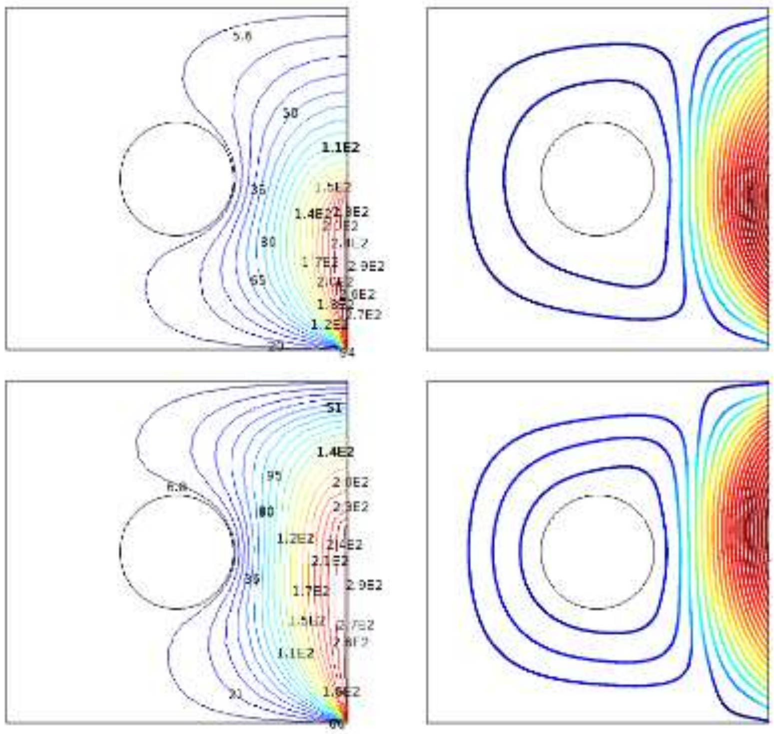


(a) (b)

Fig. 4. (a) Isotherms and (b) Streamlines for various Ra while Ha =50 & Heat flux =11

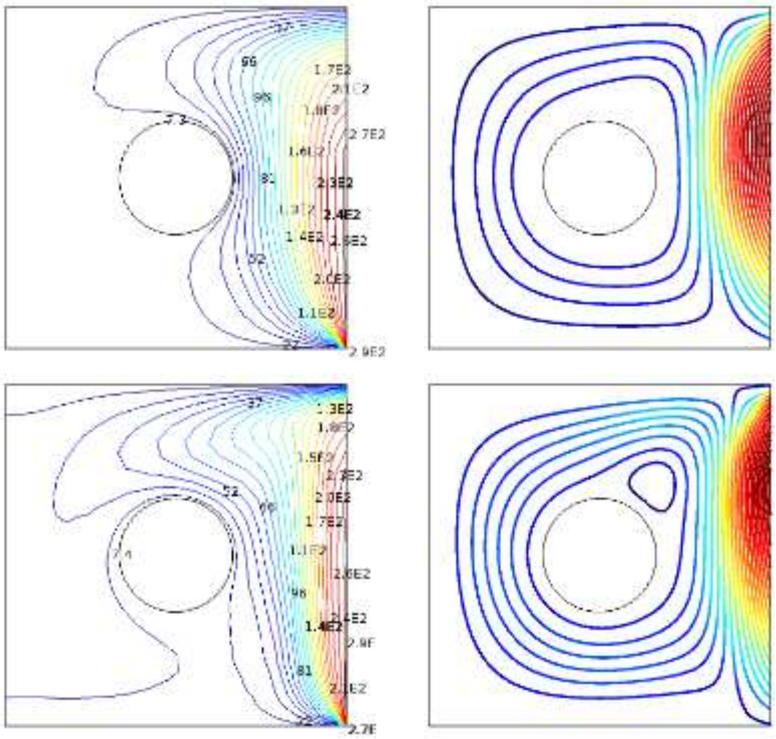
Ra= e5

Ra=5e5



Ra= 10e5

Ra=5e6

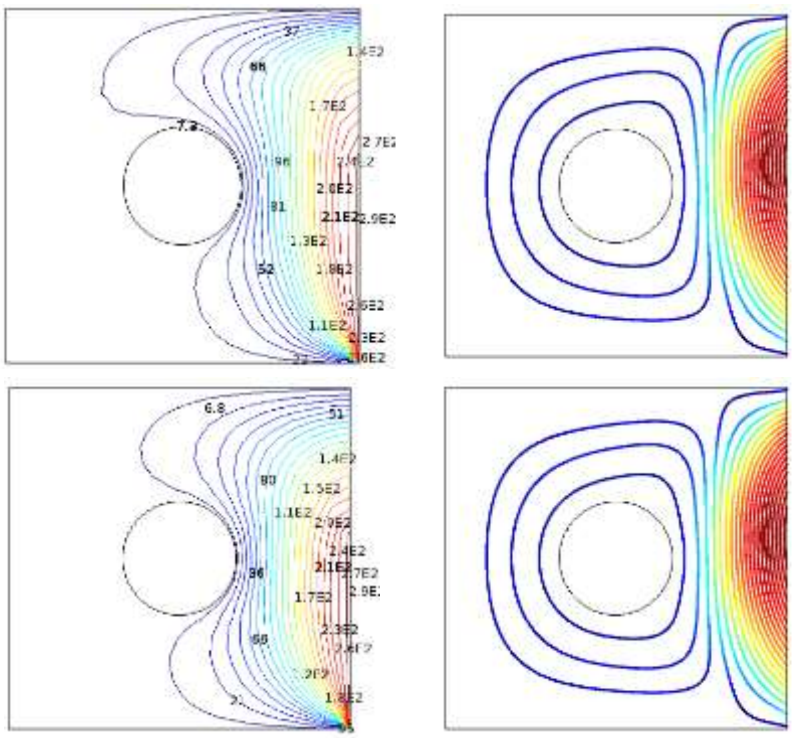


(a) (b)

Fig. 5. (a) Isotherms and (b) Streamlines for various Ra while Ha = 75 & Heat flux =110

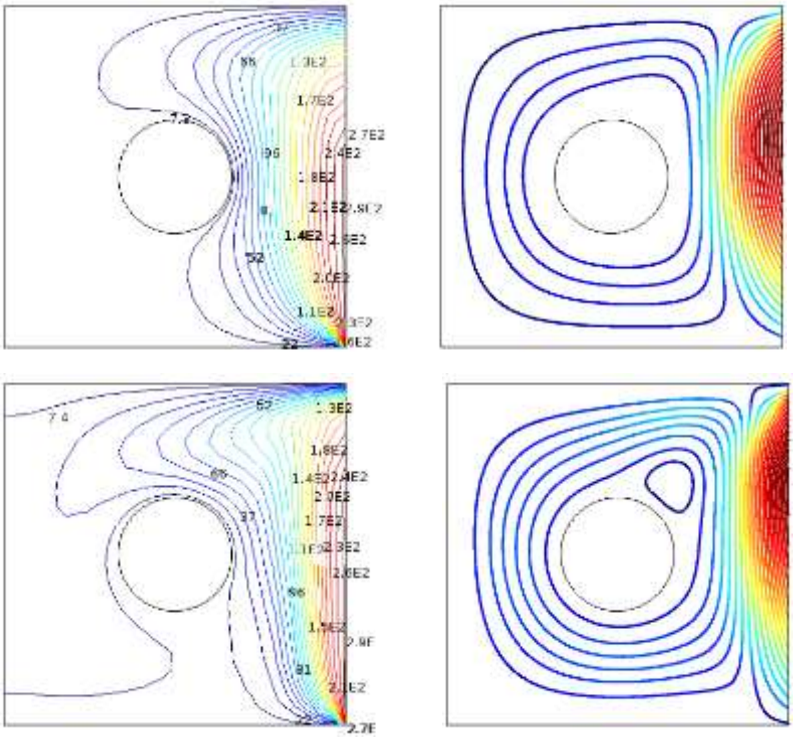
Ra= 1e5

Ra=5e5



Ha=
10e5

Ra=5e6

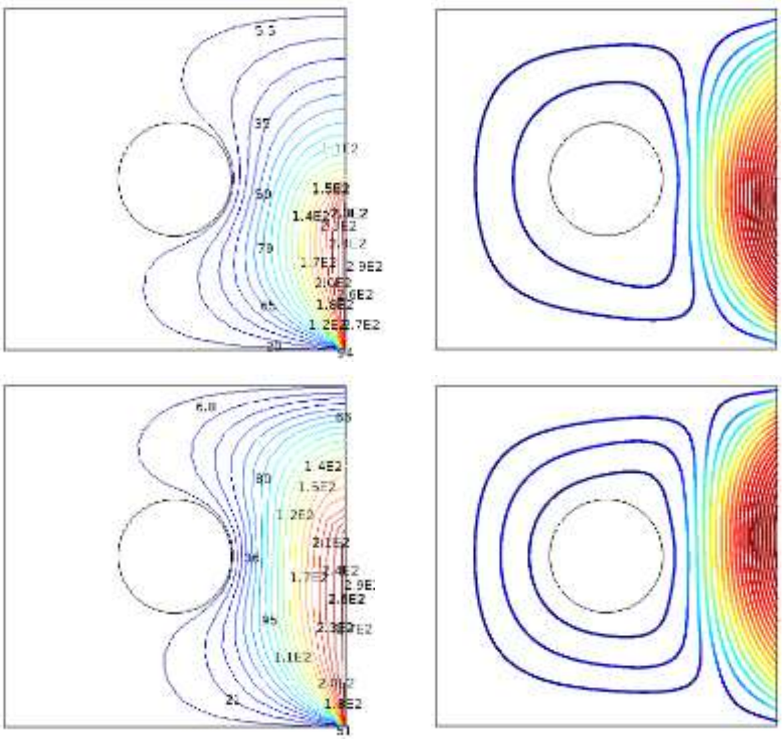


(a) (b)

Fig. 6. (a) Isotherms and (b) Streamlines for various Ra while Ha =100 & Heat flux =110.

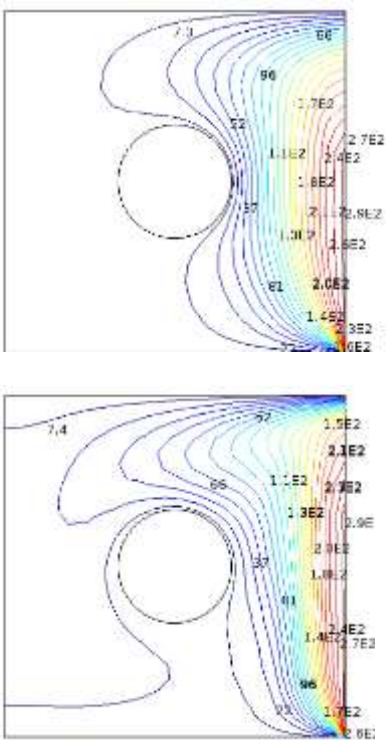
Ra=e5

Ra=5e5



Ra=e6

Ra=5e6

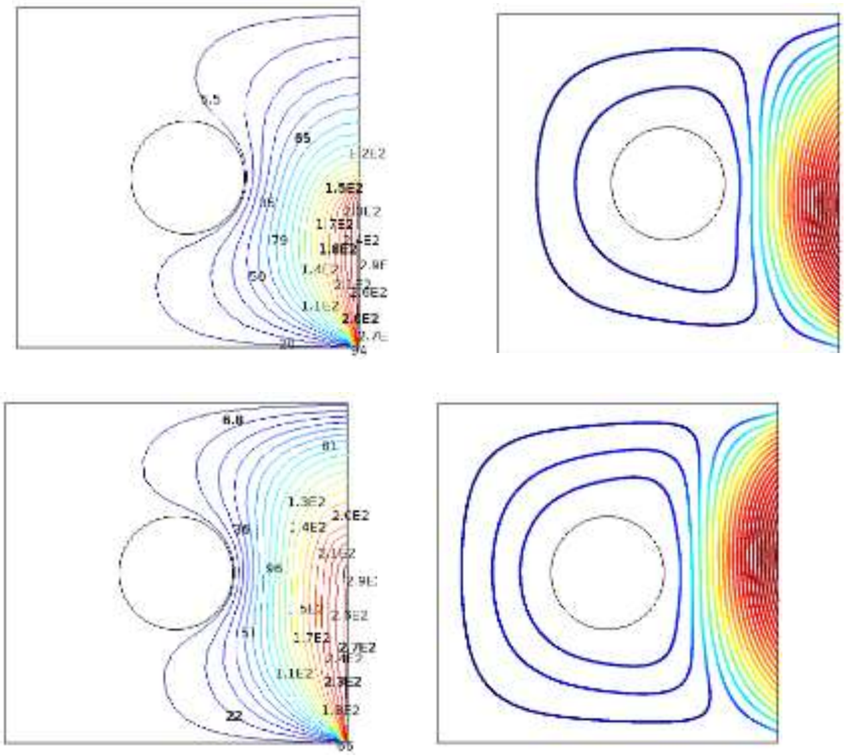


(a) (b)

Fig. 7. (a) Isotherms and (b) Streamlines for various Ra while Ha =125 & Heat flux =110.

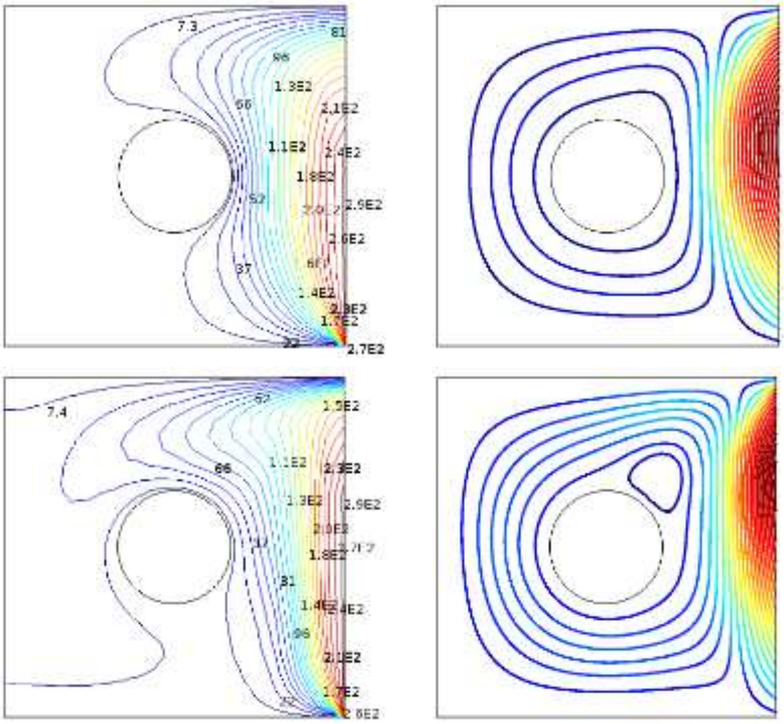
Ra= e5

Ra=5e5



Ra=e6

Ra=5e6



(a) (b)

Fig.8. Isotherms and streamlines (b) for various Ra & Ha=150, q= 110

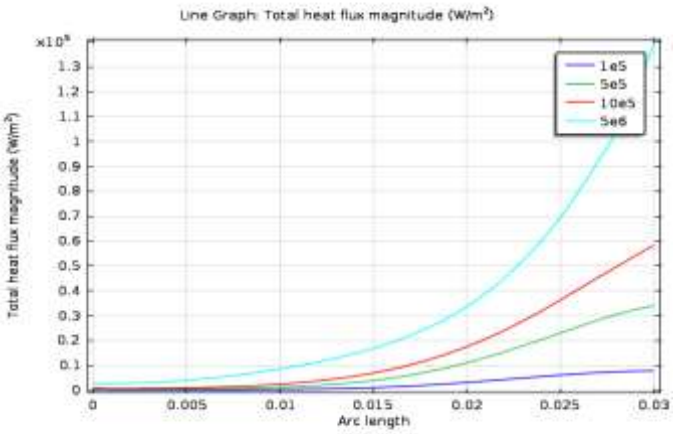


Fig. 9. Line graph of Heat flux at upper wall for Ra= 1e5,5e5,10e5,5e6 & Ha =50, q =110.

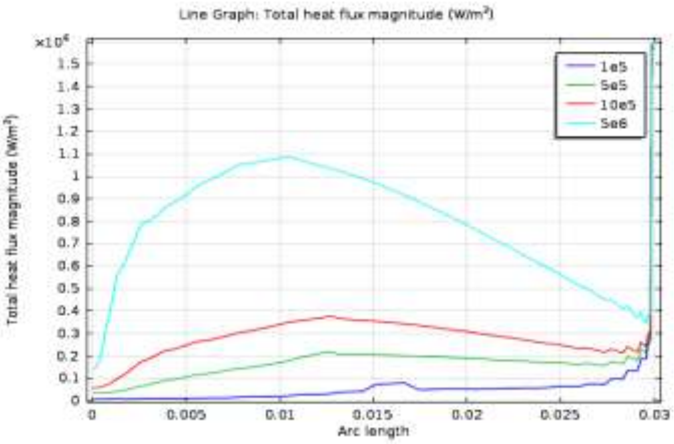


Fig.10. Line graph of Heat flux at the open side for Ha= 100 & Ra=1e5, 5e5, e6, 5e6, q =110

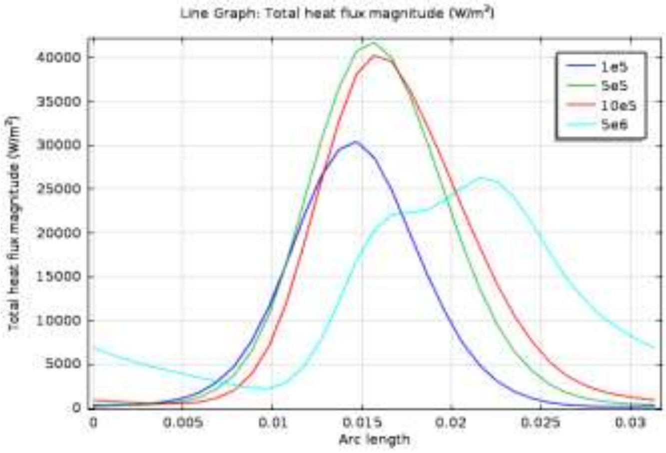


Fig.11. Line graph of Heat flux at the cylinder for Ha=100 & Ra=1e5, 5e5, e6, 5e6, q =110

Conclusion

Finite element analysis based on Galerkin weighted Residual approach is used to visualize the temperature distribution and fluid flow by the effects of Rayleigh number Ra , Hartmann number Ha, heat flux q for steady- state, laminar and MHD natural convection flow in a square open cavity with a heated circular cylinder. The flow with all Ra in this work have been affected by the buoyancy force. Temperature fields are illustrated in the flow region. The physical properties are discussed for different values of parameters and important findings of this investigation are given below.

The high temperature region remains at the lower half of the open side for Ha = 0-100 and at upper half for Ha = 125,150. The isothermal lines are nonlinear for all Ra used in this works and they occupied more than right half region of the cavity. The significant findings of this work are that for all cases of Ha and Ra the isothermal lines concentrated to the right lower corner of the cavity and there are recirculations around the cylinder and one small vortex has been created above the cylinder in the cavity. The total heat flux is increasing with increasing Ra. The recirculation region are increasing with increasing Ra. Which are good agreement with the existing Heat Transfer Theory. It is expected that the findings of this study may be useful in both gases and liquid.

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