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# Early Detection Of Autism - An Application Of Fuzzy Graphs

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**Abstract**

Mental processing is the product of the huge number of synaptic interactions that occur in the brain. It is easier to understand how brain functions deteriorate than how they might be boosted. Lying at the border between the humanities, cognitive science and neurophysiology, some mental diseases offer new angles on this problematic issue.In this article we try to understand and quantify brain behaviour using fuzzy graphs. We illustrate a new technique for early detection of Autism in individuals using fuzzy graphs .

**Keyword:** Fuzzy Graphs, Distance in Fuzzy Graphs, Connectivity, ASD.

## Introduction

Autistic spectrum disorders(ASD) comprise the fastest growing class of developmental disabilities, with an incidence of around 6 per 10000 when higher functioning individuals are included. ASD's are characterised by impaired communication and social interaction, repetitive behaviours and restricted interests.

The exact neurological basis of ASD is still unknown. Recent studies using behavioural and neuroimaging techniques have shown anatomical and functional impairments in several brain regions, including the frontal lobe, medial temporal structures, and the cerebellum. The functional magnetic resonance imaging (***FMRI***) studies have shown differences in functional connectivity between ASD and control subjects for social cognition, working memory, visuomotor coordination and performance executive and cognitive tasks such as sentence comprehension[6]. Reduced functional connectivity was found between frontal and parietal brain areas. The motivation behind this chapter is the fact that there is some relation that connect the networks in the brain and ASD which can be analysed and studied using the theory of fuzzy graphs.

As mentioned earlier Autism is a highly popular neurodevelopmental disorder, which occurs in about 5 in 10,000 live births[3]. The neural connections in the brain can be used to explain the abnormalities of one's behaviour and it can also be used to help explain some of the causes of autism[8]. The main cause of autism is still unclear, but it is believed to occur as a result of disturbances in a distributed circuit of cerebellar, limbic, and cortical systems.

There are three different types of techniques that can be used to identify the abnormalities in the brain when looking for autism: circumference measurement, postmortem anatomy, and neuroimaging. Out of the three techniques, neuroimaging has been the best when it comes to unraveling the structural and functional changes of the autistic's brain[2]. Functional magnetic resonance imaging ***(FMRI)*** can be used to reveal the neural underpinnings of autism. The activation of long-distance extended networks is important in establishing long distance connectivity, organizing long-distance synchronised interactivity, and stabilizing specialized and lateralized functional networks. It is this area where techniques of graph theory can be applied[1]. As fuzzy graphs are more universal and flexible form of graphs, it is natural to think about the extension of methods of graph theory used in detection of Autism to fuzzy graphs.

Electroencephalography***(EEG)*** and ***FMRI*** gives information about the neural connections in the brain. It has been proved that there are less connections in the brain that occur for people who have autism and when we look at the different graphs, it is clear that there are less neural connections for any group of patients who had ASD. We make use of Neural networking which can be transferred to simple connectivity matrices in fuzzy graphs for easier analysis and comparison.

**What is FMRI ?**

FMRI stands for "functional magnetic resonance imaging" and is a technique used by scientists and medical professionals to map activity in the brain[5].

When our brains are active, such as when we talk or complete a math problem, the parts of the brain that are being used require more energy in the form of glucose.

In order to meet this increased energy demand, active brain tissue receives an increase in blood flow, which carries with it oxygen and glucose rich blood.

Using a magnetic resonance imaging scanner tuned to be sensitive to small differences in signal quality due to brain activity, scientists can measure the change in blood flow, specifically the resulting increase in oxygen content, and make inferences about the location of brain activity.

Through a process called the hemodynamic response, blood vessels selectively deliver glucose and oxygen to brain regions with high demands. This results in a localized increase in the ratio of oxygenated (versus deoxygenated) hemoglobin, which carries oxygen in the blood.

Oxygenated and deoxygenated hemoglobin have different magnetic properties due to the presence of iron in hemoglobin which is less exposed (and less magnetic) when oxygenated. Thus, oxygen rich blood and oxygen poor blood have different magnetic resonances and changes in the ratio of oxyhemoglobin and deoxyhemoglobin can be measured by tuning the imaging sequence so that it is sensitive to small magnetic inconsistencies in a region.

The resulting blood-oxygenation level dependent, or ***BOLD***, signal is the central metric used in establishing an image of brain function using ***FMRI***.

**Preliminaries**

The following basic definitions are taken from[7].A fuzzy graph is a pair $\boldsymbol{G}: (\boldsymbol{\sigma}, \boldsymbol{\mu})$, where $\boldsymbol{\sigma}$ is a fuzzy subset of a set V and $\mu$ is a fuzzy relation on $\sigma$, i.e, $\mu(u,v) \leq Min[\sigma(u), \sigma(v)] \forall u, v \in V$. We assume that V is finite and non empty, $\mu$ is reflexive and symmetric. In all the examples, $\sigma$ is chosen suitably. Also we denote the underlying crisp graph by $G^*: (\sigma^*, \mu^*)$, where $\sigma^* = \{u \in V \,/ \sigma(u) > 0\}$ and $\mu^* = \{(u,v) \in VXV / \, \mu(u,v) > 0\}$.

$\boldsymbol{H}: (\boldsymbol{\tau}, \boldsymbol{\nu})$**,** is called a partial fuzzy subgraph of **G** if $\tau \leq \sigma$ and $\nu \leq \mu$. We call $\boldsymbol{H}: (\boldsymbol{\tau}, \boldsymbol{\nu})$, a spanning fuzzy subgraph of **G** if $\tau = \sigma$.

A path **P** of length ***n***is a sequence of distinct nodes $v_1, v_2 \ldots, v_n$such that$\mu\{v_i, v_{i-1}\} > 0$. The membership of a weakest arc is defined as its strength. If $v_0 = v_n$ and $n \geq 3$, then **P** is called a cycle and it is a fuzzy cycle if there is more than one weak arc.

The strength of connectedness between two nodes ***u,v*** is defined as the maximum of strengths of all paths between***u*** and ***v*** and is denoted by $\boldsymbol{CONN_G\,(u,v)}$.

The adjacency matrix of a fuzzy graph $\boldsymbol{G}: (\boldsymbol{\sigma}, \boldsymbol{\mu})$is $M_G = \left(m_{i,j}\right)$where $m_{i,j} = \mu\{v_i, v_j\} \; if \; i \neq j$ and $m_{i,j} = \sigma(u_i) \; if \; i = j$ .

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The matrix $(M_G)^2 = M_G.M_G$ is computed similar to matrix multiplication where addition is replaced by ***Max***and multilication by ***Min***. Using the same technique the matrices$(M_G)^3, (M_G)^4 \ldots etc$ can be calculated until $(M_G)^k = (M_G)^{k+1}$ for an integer ***k.*** The matrix $(M_G)^k$ is called the reachabilty of the fuzzy graph G. It is denoted by $R_G = (r_{i,j})$ and $CONN_G(v_i, v_j) = r_{i,j}$ .

**Fuzzy Graph, Connectivity Matrix and Computation Of $CONN_G(v_i, v_j)$**

Let $G: (\sigma, \mu)$, be a fuzzy graph as in Fig: 1. The connectivity matrix and computation of strength of connectedness between two nodes in fuzzy graph is illustrated below.

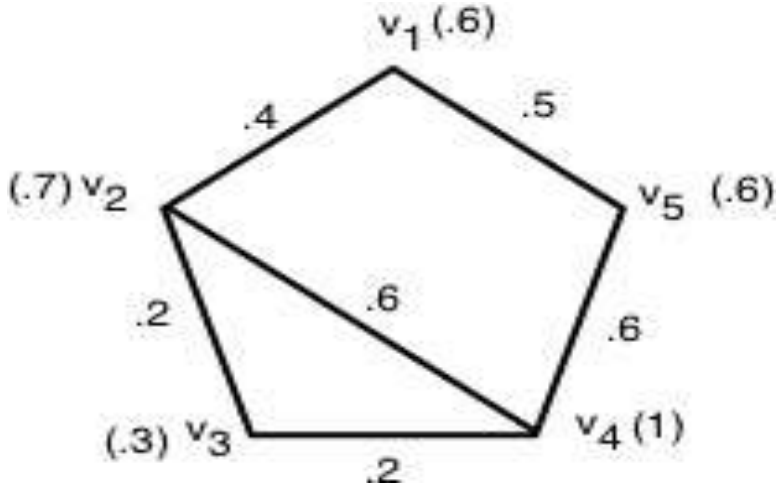


**Figure: 1**

$$M_G = \begin{bmatrix} .6 & .4 & 0 & 0 & .5 \\ .4 & .7 & .2 & .6 & 0 \\ 0 & .2 & .3 & .2 & 0 \\ 0 & .6 & .2 & 1 & .6 \\ .5 & 0 & 0 & .6 & .6 \end{bmatrix} M_G^2 = \begin{bmatrix} .6 & .4 & .2 & .5 & .5 \\ .4 & .7 & .2 & .6 & .6 \\ .2 & .2 & .3 & .2 & .2 \\ .5 & .6 & .2 & 1 & .6 \\ .5 & .6 & .2 & .6 & .6 \end{bmatrix}$$

$$M_G^3 = \begin{bmatrix} .6 & .4 & .2 & .5 & .5 \\ .5 & .7 & .2 & .6 & .6 \\ .2 & .2 & .3 & .2 & .2 \\ .5 & .6 & .2 & 1 & .6 \\ .5 & .6 & .2 & .6 & .6 \end{bmatrix} M_G^4 = \begin{bmatrix} .6 & .4 & .2 & .5 & .5 \\ .5 & .7 & .2 & .6 & .6 \\ .2 & .2 & .3 & .2 & .2 \\ .5 & .6 & .2 & 1 & .6 \\ .5 & .6 & .2 & .6 & .6 \end{bmatrix}$$

In above example $(M_G)^3 = (M_G)^4$, so the reachability matrix is $(M_G)^3$.
Thus $CONN_G(v_1, v_2) = 0.5, CONN_G(v_1, v_3) = 0.2, CONN_G(v_1, v_4) = 0.5, CONN_G(v_1, v_5) = 0.5$
$CONN_G(v_2, v_3) = 0.2, CONN_G(v_2, v_4) = 0.6, CONN_G(v_2, v_5) = 0.6, CONN_G(v_3, v_4) = 0.2,$
$CONN_G(v_3, v_5) = 0.2, CONN_G(v_4, v_5) = 0.6.$

**Graph Theoretical Analysis Of Brain**

The anatomical configuration of brain networks, ranging from inter-neuronal connectivity to inter-regional connectivity, has long been a focus of empirical neuro- science. network analysis, and in particular graph theory , offers new ways to quantitatively characterize anatomical patterns.

According to graph theory, structural brain networks can be described as graphs that are composed of nodes (vertices) denoting neural elements (neurons or brain regions) that are linked by edges representing physical connections (synapses or axonal projections). The first such study used a set of neuronographic measurements of the propagation of epileptiform activity following localised applications of strychnine to the macaque cortex63. This demonstrated a pattern of functional connections between cortical areas that was consistent with a small-world network. These findings have been extended by studies based on functional MRI (FMRI), electroencephalography (EEG), magnetoencephalography (MEG) or multielectrode array (MEA) data. Sructural and functional brain networks can be explored using graph theory through the following four steps (see Figure 2).

These informations are sourced from a review by Ed Bullmore and Olaf Sporns in [5].

This was further extended by Dae Jin Kim and Amanda[4].

The authors investigated whether the resting electroencephalogram (EEG) in patients with bipolar disorder(BD) showed altered synchronization or network properties. It was noticed that the normalized characteristic path length and small-worldness were significantly correlated with depression scores in BD patients. These results suggest that BD patients show impaired neural synchronization at rest and a disruption of resting state functional connectivity.

**Steps to analyse structural and functional brain networks using graph theory** [5] **[Fig: 2]**

**Step 1:**

Define the network nodes. These could be defined as electroencephalography or multielectrode-array electrodes, or as

anatomically defined regions of histological, MRI or diffusion tensor imaging data.

**Step 2:**

Estimate a continuous measure of association between nodes. This could be the spectral coherence or Granger causality measures between two magnetoencephalography sensors, or the connection probability between two regions of an individual diffusion tensor imaging data set, or the inter-regional correlations in cortical thickness or volume MRI measurements estimated in groups of subjects.

**Step 3:**

Generate an association matrix by compiling all pairwise associations between nodes and (usually) apply a threshold to each element of this matrix to produce a binary adjacency matrix or undirected graph.

**Step 4:**

Calculate the network parameters of interest in this graphical model of a brain network and compare them to the equivalent parameters of a population of random networks.

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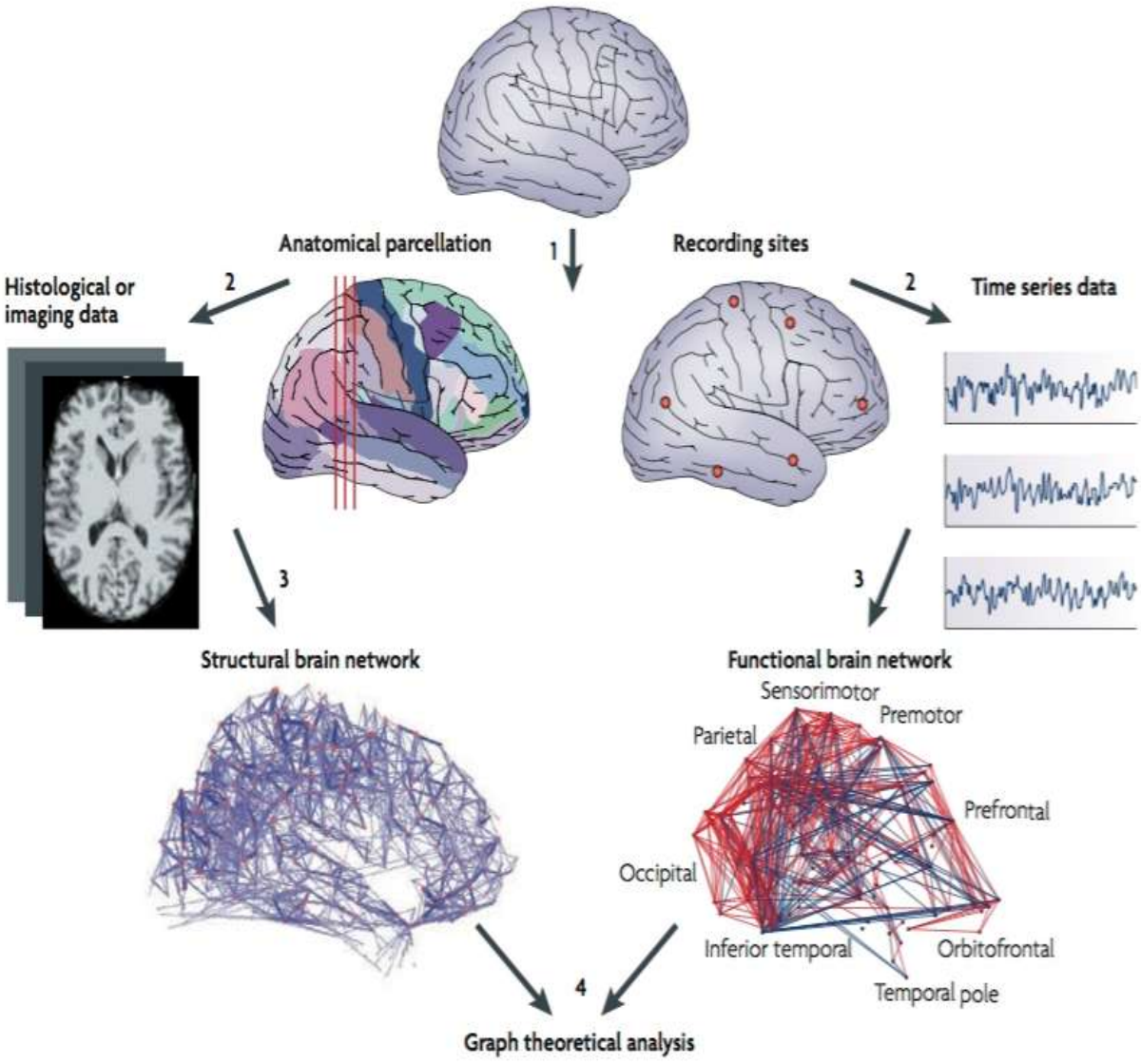


**Figure 2**

Each step entails choices that can influence the final results and must be carefully informed by the experimental question. At step 1, parcellation schemes can use prior anatomical criteria or be informed by the functional connectivity profiles of different regions. Several such parcellation schemes may be available and can affect network measures. In most magnetoencephalography and electroencephalography studies, network nodes are equivalent to individual electrodes or sensors, but networks could also be based on reconstructed anatomical sources. However, some reconstruction algorithms will determine the brain location of each source by minimizing the covariance between sensors, which has major effectson the configuration of functional networks. At step 2, a range of different coupling metrics can be estimated, including measures of both functional and effective connectivity.

A crucial issue at step 3 is the choice of threshold used to generate an adjacency matrix from the association matrix: different thresholds willgenerate graphs of different sparsity or connection density, and sonetwork properties are often explored over a range of plausible thresholds. Finally, at step 4, a largenumber of network parameters can bequantified. These parameters can be compared with equivalent parameters estimated in random networks containing the same number of nodes and connections.

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**Mathematical Modelling Of brain using Fuzzy Graphs**

As we all know that human brain consists of a large quantity of neurones and the connections between them which results in a complicated network.

It is through this network the communication and data processing in the brain occurs.

It is quite cumbersome to model brain using a fuzzy graph by considering each neurone individually. So we resort to the method of grouping the neurones in the form of some simple nodes and giving a membership value for each node depending on the number of neurones forming the node.

The exceptional skills found in autistic subjects may be explained by their special mental functioning, in particular by the weak central coherence and strong local connectivity.

A consequence of this local overactivation, is the generation of patterns of weak long-range connectivity. Particularly implicated in deficits of long-range connectivity is the cerebellum, which is strongly involved not only in sensorimotor processing, but also in emotion and cognition. In autism, the cerebellum has been shown to present hypoplasia of the vermis and hemispheres and reduced numbers of Purkinje cells .

Notably, the reduction in Purkinje cells may have the effect of disinhibiting the deep cerebellar nuclei, producing abnormally strong local connectivity associated with weak connectivity along the cerebello-thalamo- cortical circuit.

This altered connectivity may be related to the abnormal overgrowth observed in prefrontal lobes (PFLs), to which the cerebellar hemispheres are closely connected.

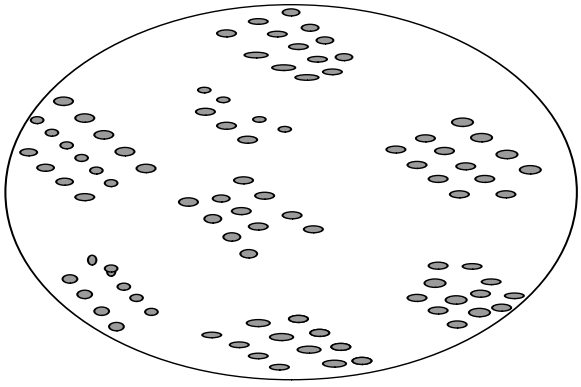
Hence when we deal with autism we give importance to long range connectivity in the brain. This is the reason behind the method of grouping the neurones from selected regions of brain in the form of some simple nodes(Cluster Nodes or C Nodes) and giving a membership value for each node depending on the number of neurones forming the node.

We propose a method in which the brain is divided into a number of clusters. These clusters can be found using any of the clustering algorithms that have been developed recently.

Each of these clusters are considered as node(C- Nodes) of the fuzzy graph that is to be constructed.

Without loss of generality we take the membership of each C- node as 1.

The arcs between these C-nodes are formed depending on the number of arcs between the neurones in the corresponding cluster and a membership value(normalised) is given according to the number of connections between the included neurons. The figures 3, 4 and 5illustrates the method to fuzzify the brain networks. In figure 4 there are 3 arcs between $\mathbf{C_1}$ and $\mathbf{C_4}$. In general the membership value of the arc between $\mathbf{C_i}$ and $C_j$ will be proportional to the number connections between the neurones in $\mathbf{C_i}$and $\mathbf{C_j}$ **.**



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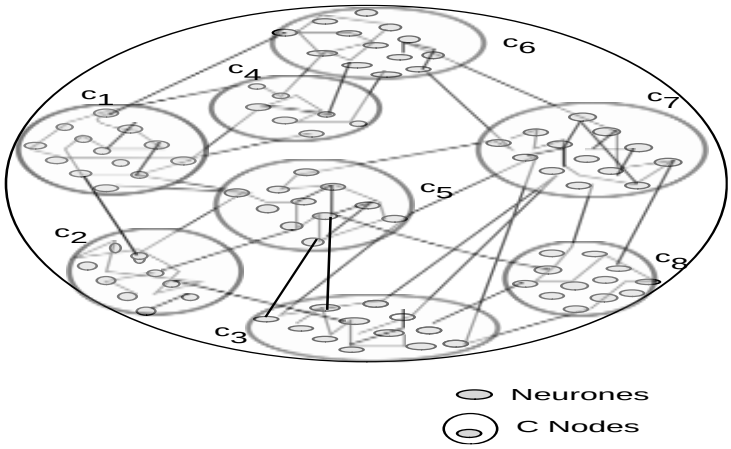
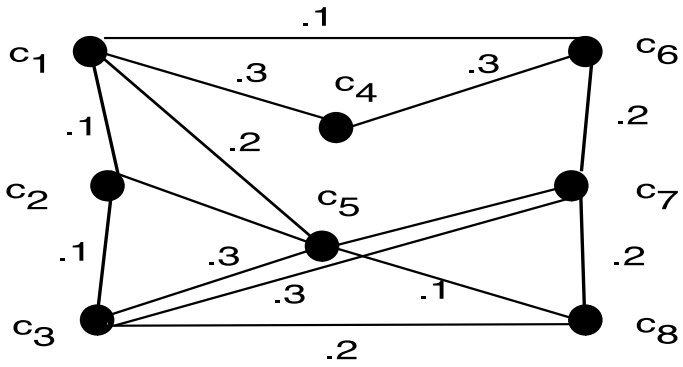
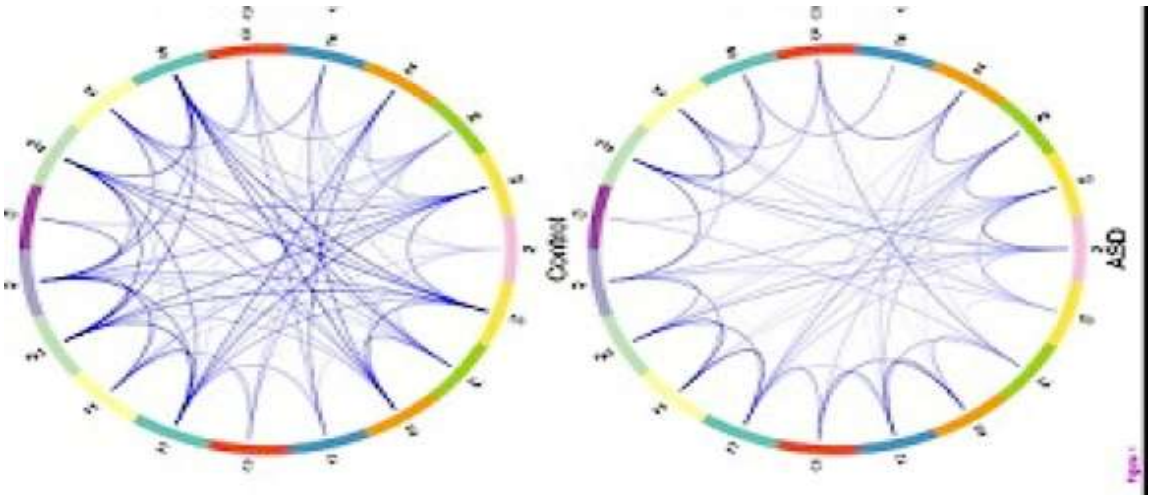
**Brain & Neurones: Figure 3**




**Construction Of C- nodes: Figure 4**




**Proposed Fuzzy Graph Model: Figure 5**




**Connections in person with normal brain & brain affected with ASD: Figure 6**

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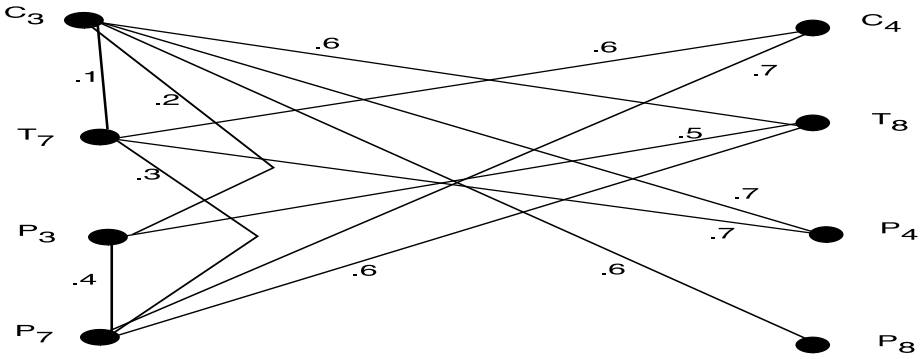
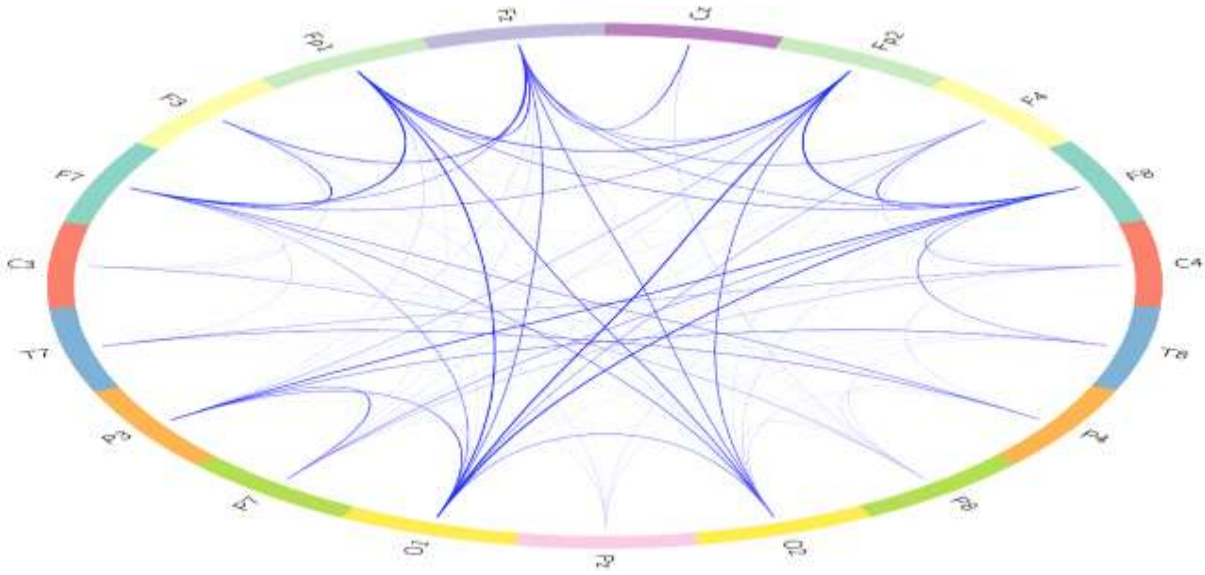
Choose a small portion of the above brain diagrams.
Consider the fuzzy graph formed by the nodes ***$T_3$, $C_4$, $T_7$, $T_8$, $P_3$, $P_4$, $P_7$, $P_8$***. The fuzzy model can be drawn as below. The figure 7 gives the model for normal brain and figure 8 and 9 is that of autistic brain.




**Fuzzy Graph Model - Normal Brain: Figure 7**



**An Autistic brain: Figure 8**

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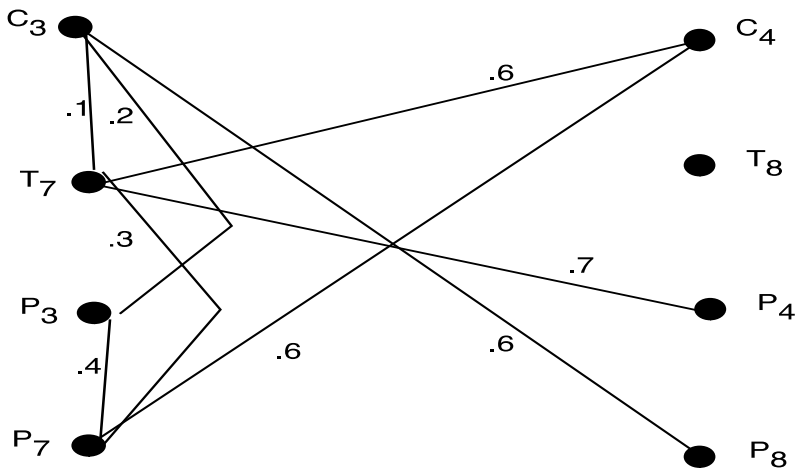


**Autistic Brain Fuzzy Graph Model: Figure 9**

Note that in the autistic brain some of the arcs namely ***[C3, T8], [C3, P4], (P3,T8], [P7, T8]*** are not present. Next we write the connectivity matrix of both the fuzzy graphs and we can find the connectivity between the nodes using max-min composition illustrated in example section 4. The connectivity matrix of normal brain is

$$M_1 = \begin{array}{c} Nodes \\ C_3 \\ C_4 \\ T_7 \\ T_8 \\ P_3 \\ P_4 \\ P_7 \\ P_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} C_3 & C_4 & T_7 & T_8 & P_3 & P_4 & P_7 & P_8 \end{array} \\ \begin{bmatrix} 1 & 0 & .1 & .6 & .2 & .7 & 0 & .6 \\ 0 & 1 & .6 & 0 & 0 & 0 & .7 & 0 \\ .1 & .6 & 1 & 0 & 0 & .7 & .3 & 0 \\ .6 & 0 & 0 & 1 & .5 & 0 & .6 & 0 \\ .2 & 0 & 0 & .5 & 1 & 0 & .4 & 0 \\ .7 & 0 & .7 & 0 & 0 & 1 & 0 & 0 \\ 0 & .7 & .3 & .6 & .4 & 0 & 1 & 0 \\ .6 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

The connectivity matrix of Autistic brain is as below

$$M_2 = \begin{array}{c} Nodes \\ C_3 \\ C_4 \\ T_7 \\ T_8 \\ P_3 \\ P_4 \\ P_7 \\ P_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} C_3 & C_4 & T_7 & T_8 & P_3 & P_4 & P_7 & P_8 \end{array} \\ \begin{bmatrix} 1 & 0 & .1 & 0 & .2 & 0 & 0 & .6 \\ 0 & 1 & .6 & 0 & 0 & 0 & .7 & 0 \\ .1 & .6 & 1 & 0 & 0 & .7 & .3 & 0 \\ .6 & 0 & 0 & 1 & .5 & 0 & .6 & 0 \\ .2 & 0 & 0 & 0 & 1 & 0 & .4 & 0 \\ .7 & 0 & .7 & 0 & 0 & 1 & 0 & 0 \\ 0 & .7 & .3 & 0 & .4 & 0 & 1 & 0 \\ .6 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

From the matrix **M2** and also from the figure 9, it is clear that the C- node **T8** has been disconnected. Obviously this will affect the connectivity of the whole network. Connectivity between **C3** and **P4** has been reduced to **.2** from **.4** and **P7** and **T8** are disconnected.

**Conclusion**

We have put forward a technique to calculate the strength of connectivity between neurones in a human brain using the fuzzy graphs. There is a significant difference in the connectivity between nodes of normal brain and that of an autistic brain. The discussed method will be an opening window to the development of

more powerful tools that may give strong inference about the complex neural structure of human brain. The tool can be used as a method to analyse the early defects in the brain that may lead to disorders like Autism.

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