

Static Spherical Fluid Models in Einstein–Cartan–Sciama–Kibble Theory A Study of Torsion, Spin–Gravity Coupling, And Self-Gravitating Configurations

Dr. Kanchan Singh

Department of Mathematics, Patliputra University Patna, Bihar, India

ARTICLE INFO	ABSTRACT
Published Online: 03 October 2026 Corresponding Author: Dr. Kanchan Singh KEYWORDS: Einstein–Cartan–Sciama–Kibble theory, torsion, intrinsic spin, perfect fluid sphere, static spherical symmetry, energy conditions, compact stars, spin–torsion coupling.	This paper develops a systematic framework for studying static, spherically symmetric perfect fluid configurations within the Einstein–Cartan–Sciama–Kibble (ECSK) theory of gravitation. The ECSK formalism extends classical general relativity by incorporating spacetime torsion as a geometrical field sourced by the intrinsic spin density of matter. We write down the governing field equations for the interior of a self-gravitating fluid sphere, exploit Hehl’s redefinition of effective thermodynamic variables to decouple the spin–torsion corrections, and derive a class of interior solutions by prescribing a barotropic equation of state together with a specific choice of one metric potential. The solutions are matched at the stellar surface to the Schwarzschild exterior via the Darmois–Israel junction conditions, yielding unique values for all integration constants. Explicit analytic expressions for the energy density, isotropic pressure, and spin density are presented and analysed. Physical viability is assessed through the standard energy conditions, regularity requirements, and monotonicity criteria. The investigation demonstrates that spin–torsion coupling introduces a repulsive pressure correction that can, under appropriate conditions, prevent the formation of central singularities and modify the mass–radius relationship of compact stellar objects. The astrophysical implications for neutron stars, quark stars, and other dense matter environments are discussed in detail.

1 INTRODUCTION

The geometric description of gravitation reached its zenith with Einstein’s field equations, which relate the curvature of a pseudo-Riemannian manifold to the energy–momentum distribution of matter. However, this picture leaves one fundamental property of matter—the intrinsic angular momentum, or spin, of elementary particles—entirely disconnected from the gravitational geometry. In a strictly Riemannian setting the connection is symmetric, and there is no room for a coupling between the spin of quantum-mechanical particles and the fabric of spacetime.

A geometrically natural resolution was independently proposed by Cartan [1], Sciama [2], and Kibble [3]. Their programme—now collectively referred to as the Einstein–Cartan–Sciama–Kibble (ECSK) theory—admits a non-symmetric affine connection whose antisymmetric part, the torsion tensor, acts

as an additional geometric degree of freedom sourced directly by the spin density of matter. The resulting framework preserves all well-tested predictions of classical general relativity in the low-density limit, while introducing qualitatively new physics at the extreme densities found in the interiors of compact stars or in the very early universe.

Unlike metric perturbation theories or scalar-tensor extensions, ECSK theory modifies the local structure of spacetime at the level of the connection rather than the metric. Torsion is algebraically related to the spin density—there are no torsion waves propagating in vacuo—so the spin–torsion interaction is inherently local and becomes important only when the spin density is non-negligible, that is, in matter of nuclear or supra-nuclear density.

The question of whether ECSK theory admits physically acceptable self-gravitating fluid configurations has attracted sustained interest since the 1970s. Pioneering work by Hehl

and collaborators [4] established the theoretical foundations and derived the general form of the field equations for matter with spin. Subsequent investigations by Kerlick [7], Kuchowicz [8], Prasanna [9], Skinner and Webb [11], and Sah and Chandra [12] explored specific fluid sphere solutions in the ECSK framework and examined the stability and energy properties of the resulting models.

The importance of studying spherically symmetric configurations lies in the fact that they represent the simplest non-trivial geometry for a self-gravitating object: they are the standard setting for stellar structure calculations in general relativity, and the classical Tolman–Oppenheimer–Volkoff (TOV) equation is their central result. In ECSK theory the TOV equation receives spin–torsion corrections that are potentially significant in the dense cores of neutron stars and quark stars, where nuclear matter may carry substantial net spin polarization.

The present investigation revisits and significantly extends the existing literature in several directions. We formulate the ECSK field equations for a static, spherically symmetric perfect fluid from first principles, carefully accounting for the spin density tensor and the Hehl

redefinition of effective thermodynamic variables. We then construct exact analytic interior solutions by imposing a linear equation of state and a specific ansatz for one of the metric functions. The interior solution is matched to the exterior Schwarzschild geometry through the Darmois–Israel junction conditions. Physical viability is assessed systematically through the standard energy conditions, and the astrophysical significance of the spin–torsion corrections is discussed in the context of contemporary observations of compact stellar remnants.

The paper is organised as follows. Section 2 reviews the geometrical foundations of ECSK theory and states the field equations. Section 3 introduces the static spherically symmetric metric and the matter model. Section 4 reduces the ECSK equations to a manageable system using Hehl’s approach. Section 5 derives the exact interior solution and applies boundary conditions. Section 7 examines the energy conditions and physical behaviour. Section 6 presents the modified TOV equation with spin corrections. Section 8 discusses astrophysical implications, and Section 10 summarises the findings.

2 GEOMETRICAL FOUNDATIONS OF ECSK THEORY

2.1 Torsion and the Affine Connection

In ECSK theory the spacetime manifold M is equipped with a metric tensor $g_{\mu\nu}$ and an independent affine connection $\Gamma^\lambda_{\mu\nu}$. The torsion tensor is defined as the antisymmetric part of the connection:

$$T^\lambda_{\mu\nu} \equiv \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu}. \quad (1)$$

The connection may be decomposed as

$$\Gamma^\lambda_{\mu\nu} = \left\{ \begin{smallmatrix} \lambda \\ \mu\nu \end{smallmatrix} \right\} + K^\lambda_{\mu\nu}, \quad (2)$$

where $\left\{ \begin{smallmatrix} \lambda \\ \mu\nu \end{smallmatrix} \right\}$ denotes the Christoffel symbol of $g_{\mu\nu}$ and $K^\lambda_{\mu\nu}$ is the contortion tensor, which is algebraically expressible in terms of the torsion:

$$K^\lambda_{\mu\nu} = \frac{1}{2} (T^\lambda_{\mu\nu} + T^\lambda_{\nu\mu} + T^\lambda_{\mu\lambda}). \quad (3)$$

2.2 The ECSK Field Equations

The gravitational field equations of ECSK theory consist of two coupled sets. The first, relating curvature to the canonical energy–momentum, is:

$$G^\mu_{\nu} \equiv R^\mu_{\nu} - \frac{1}{2} \delta^\mu_{\nu} R = -\kappa t^\mu_{\nu}, \quad (4)$$

and the second, the Cartan equation relating torsion to spin density, reads:

$$T^\lambda_{\mu\nu} + \delta^\lambda_{\mu} T^\sigma_{\sigma\nu} - \delta^\lambda_{\nu} T^\sigma_{\sigma\mu} = -\kappa S^\lambda_{\mu\nu}, \quad (5)$$

where $\kappa = 8\pi G/c^4$ (with $G = c = 1$ hereafter), R^μ_{ν} is the Ricci tensor constructed from the full connection Γ , t^μ_{ν} is the canonical energy–momentum tensor (asymmetric in general), and $S^\lambda_{\mu\nu}$ is the spin density tensor of the matter field.

A crucial feature of equation (5) is its *algebraic* character: torsion at any spacetime point is determined instantaneously by the spin density at that point, with no propagation. Substituting (5) back into (4) one obtains an effective Einstein equation whose source includes quadratic spin contributions—the Hehl effective terms—in addition to the usual symmetric energy–momentum tensor.

2.3 Canonical Energy–Momentum and Spin Density

For a perfect fluid with intrinsic spin, the canonical energy–momentum tensor is related to the symmetric (metric) energy–momentum tensor T^μ_{ν} by

$$t^\mu_{\nu} = T^\mu_{\nu} + \frac{1}{2} \nabla_\lambda (S^{\mu\lambda}_{\nu} + S^{\lambda\mu}_{\nu} - S^{\lambda\mu}_{\nu}). \quad (6)$$

In the particular case of a *Weyssenhoff fluid*—the standard macroscopic spin fluid used in ECSK cosmology and astrophysics—the spin density tensor satisfies the Frenkel condition $S^{\mu\nu}{}_{;\lambda}u^\lambda = 0$, where u^λ is the fluid four-velocity. This constraint effectively reduces the independent components of $S^{\mu\nu}{}_{;\lambda}$ and leads to the simplified field equations discussed in the next section.

3 STATIC SPHERICALLY SYMMETRIC SPACETIME AND MATTER CONTENT

3.1 Interior Line Element

We consider the most general static, spherically symmetric line element in curvature coordinates:

$$ds^2 = -e^{2\nu(r)} c^2 dt^2 + e^{2\lambda(r)} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2), \quad (7)$$

where $\nu(r)$ and $\lambda(r)$ are metric potentials that depend solely on the radial coordinate r . The exterior region $r > r_*$ (where r_* is the stellar radius) is described by the Schwarzschild vacuum metric with mass parameter M .

3.2 Matter Tensor

For an isotropic perfect fluid the symmetric energy–momentum tensor takes the diagonal form

$$T^\mu{}_\nu = \text{diag} \left(-\rho c^2, P, P, P \right), \quad (8)$$

where ρ is the total energy density (including rest mass) and P is the isotropic pressure, both functions of r alone.

3.3 Spin Density in Spherical Symmetry

In a spherically symmetric geometry, a consistent non-zero spin density tensor must be polarized along the radial direction. Adopting the Weyssenhoff prescription, the only non-vanishing independent component of the spin density is $S_{t\theta}$, which we parameterize through a scalar spin density function $\sigma(r)$. Following the conventions of Hehl et al. [4], one defines a scalar spin density $k(r)$ such that

$$S_{\mu\nu\lambda} = k(r)\epsilon_{\mu\nu\lambda\sigma} u^\sigma, \quad (9)$$

where $\epsilon^{\mu\nu\lambda\sigma}$ is the Levi–Civita tensor and u^σ is the static fluid four-velocity $u^\sigma = (e^{-\nu}, 0, 0, 0)$.

From the Cartan equation (5) applied to the metric (7), the torsion components are expressible in terms of $k(r)$. Compatibility of the torsion equations with the metric then yields the first integral

$$k(r) = H e^{-\nu(r)}, \quad (10)$$

where H is an integration constant with dimensions of spin density, to be fixed by the boundary conditions. This result is a direct analogue of the cylindrically symmetric result obtained by Prasanna [10].

4 REDUCTION OF THE FIELD EQUATIONS

4.1 Hehl Effective Variables

Substituting the Cartan equation (5) into the curvature equation (4) and using the Weyssenhoff spin tensor, the effective field equations involve quadratic spin contributions. Following Hehl’s approach [5,6], one introduces the effective energy density and pressure:

$$\bar{\rho} = \rho - 2\pi k^2, \quad (11)$$

$$\bar{P} = P - 2\pi k^2. \quad (12)$$

In terms of these effective variables the ECSK field equations for the metric (7) reduce to a form structurally identical to the standard Einstein equations:

$$\frac{d}{dr} (r e^{-2\lambda}) = 1 - 8\pi r^2 \bar{\rho}, \quad (13)$$

$$\frac{2\nu'}{r} e^{-2\lambda} - \frac{1 - e^{-2\lambda}}{r^2} = 8\pi \bar{P}, \quad (14)$$

$$e^{-2\lambda} \left(\nu'' + \nu'^2 - \nu'\lambda' + \frac{\nu' - \lambda'}{r} \right) = 8\pi \bar{P}, \quad (15)$$

where primes denote d/dr . The spin density $k(r)$ enters only through the effective variables $\bar{\rho}$ and $\bar{P}$ via equations (11)–(12), greatly simplifying the analysis.

4.2 Mass Function

Defining the interior mass function $m(r)$ through $e^{-2\lambda} = 1 - 2m(r)/r$, equation (13) yields

$$\frac{dm}{dr} = 4\pi r^2 \bar{\rho}(r) = 4\pi r^2 [\rho(r) - 2\pi k^2(r)] \quad (16)$$

The total gravitational mass of the star is $M = m(r_*)$. Equation (16) shows that the effective contribution of spinning matter to the gravitational mass is *reduced* relative to pure matter by the spin term $8\pi^2 k^2$, which is always non-negative.

4.3 Conservation Equation

The contracted Bianchi identity $\nabla_\mu t^\mu_\nu = 0$, once the torsion is eliminated via equation (5), yields the modified hydrostatic equilibrium equation:

$$\frac{d\bar{P}}{dr} = -(\bar{\rho} + \bar{P})\nu' + \mathcal{F}_{\text{spin}}(r), \quad (17)$$

where $\mathcal{F}_{\text{spin}}$ is a spin-correction term that depends on $k(r)$ and $k(r)\nu'(r)$. In the limit of vanishing spin density ($k \rightarrow 0$) this reduces to the standard generalrelativistic Tolman–Oppenheimer–Volkoff (TOV) equation.

5 EXACT INTERIOR SOLUTION

5.1 Closure Conditions

The system (13)–(15) contains four unknown functions (ν , λ , $\bar{\rho}$, P^-) and three independent equations, rendering it under-determined. Two supplementary conditions are required.

5.1.1 Equation of State

We impose a barotropic linear equation of state relating the effective variables:

$$\bar{\rho}^- = nP^-, \quad n = \text{const} > 0. \quad (18)$$

This generalizes the polytropic family and encompasses physically relevant cases such as radiation-dominated matter ($n = 3$ in the non-relativistic limit), stiff matter ($n = 1$), and soft equations of state ($n \gg 1$).

5.1.2 Metric Ansatz

As the second supplementary condition, we prescribe a specific algebraic form for the metric potential $e^{2\lambda}$:

$$e^{2\lambda} = \frac{1}{1 - Ar^2}, \quad (19)$$

which corresponds to

$$m(r) = \frac{Ar^3}{2}, \quad (20) \text{ i.e., uniform effective energy density } \bar{\rho} = 3A/(8\pi) = \text{const.}$$

The constant $A > 0$ has dimensions of $(\text{length})^{-2}$ and sets the density scale of the configuration. This choice is the natural generalisation of the Schwarzschild interior (constant-density) solution within ECSK theory.

5.2 Solution for the Metric Potential ν

Substituting (18) and (19) into (14), one obtains a first-order linear ODE for e^ν :

$$\nu' = \frac{r[4\pi\bar{P}(1 - Ar^2) + A]}{1 - Ar^2}. \quad (21)$$

Since $\bar{\rho} = \text{const}$ and the equation of state (18) forces P^- to vary with position (it cannot be simultaneously constant unless $n \rightarrow \infty$), we solve for $P^-(r)$ first. From (13) and (14) combined with (18):

$$\bar{P}(r) = \bar{\rho} \frac{\sqrt{1 - Ar^2} - \sqrt{1 - Ar_*^2}}{n\sqrt{1 - Ar_*^2} - (n - 1)\sqrt{1 - Ar^2}} \cdot \frac{n}{1}, \quad (22)$$

and the metric potential is then:

$$e\nu(r) = D \ln \left[\frac{1 - Ar_*^2 - (n - 1)\sqrt{1 - Ar^2}}{1 - Ar_*^2} \right], \quad (23)$$

where D is a constant of integration. For $n = 1$ (stiff matter) the exponent $n/(n - 1)$ diverges, and the solution takes a different logarithmic form; we henceforth assume $n \neq 1$.

The actual energy density and pressure are recovered from the effective quantities via the spin density:

$$\rho(r) = \bar{\rho} + 2\pi k^2(r) = \bar{\rho} + 2\pi H^2 e^{-2\nu(r)}, \quad (24)$$

$$P(r) = \bar{P}(r) + 2\pi k^2(r) = \bar{P}(r) + 2\pi H^2 e^{-2\nu(r)}. \quad (25)$$

5.3 Boundary Conditions and Junction

The interior solution (19)–(23) contains three free parameters: A , D , and H . These are fixed by the following conditions at the stellar surface $r = r_*$:

1. **Vanishing pressure:** Physical requirement that $\bar{P}(r_*) = 0$, which determines $\bar{P}(r_*) = 0 \Rightarrow$ the effective pressure vanishes at the stellar surface. (26)

2. **Continuity of the metric:** The interior metric must match the exterior Schwarzschild metric

$$ds_{\text{ext}}^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad (27)$$

at $r = r_*$. Continuity of g_{tt} gives

$$e^{2\nu(r_*)} = 1 - \frac{2M}{r_*} = 1 - Ar_*^2, \quad (28)$$

which fixes D in terms of A and r_* .

3. **Continuity of g_{rr} :** Requires $e^{-2\lambda(r_*)} = 1 - 2M/r_*$, which is automatically satisfied with $M = Ar_*^3/2$.

4. **Vanishing spin density at surface:** We impose $k(r_*) = 0$, which from equation (10) gives $H = 0$ or, more generally, provides a relation between H and the surface value of $e^{-\nu}$.

After applying these conditions, the complete interior solution is uniquely determined in terms of the two observable parameters M and r_* :

$$A = \frac{2M}{r_*^3}, \quad (29)$$

(30)

These reduce exactly when $H \rightarrow 0$ and $n \rightarrow \infty$ (dust limit), confirming consistency.

$$e^{2\nu(r)} = \left[\frac{3}{2} \sqrt{1 - \frac{2M}{r_*}} - \frac{1}{2} \sqrt{1 - \frac{2Mr^2}{r_*^3}} \right]^2, \quad (31)$$

$$\bar{P}(r) = \bar{\rho} \frac{\sqrt{1 - 2Mr^2/r_*^3} - \sqrt{1 - 2M/r_*}}{3\sqrt{1 - 2M/r_*} - \sqrt{1 - 2Mr^2/r_*^3}}$$

6 MODIFIED TOLMAN–OPPENHEIMER–VOLKOFF EQUATION

The full hydrostatic equilibrium condition in ECSK theory with spherical symmetry, obtained from the contracted Bianchi identity, takes the form:

$$\frac{dP}{dr} = -\frac{(\rho + P)(m + 4\pi r^3 P)}{r(r - 2m)} + \frac{d(2\pi k^2)}{dr} + \frac{2\pi k^2}{r(r - 2m)} \left[2m - r + r(r - 2m) \frac{d \ln k}{dr} \right]. \quad (32)$$

The first term on the right-hand side is the standard TOV expression. The remaining terms represent spin–torsion corrections. Since $k^2 \geq 0$, the sign of these corrections depends on the gradient dk/dr and the profile of the mass function $m(r)$.

For the solution (10), where $k = He^{-\nu}$, the spin density decreases outward

(since ν increases outward in a physically reasonable star), so $dk/dr < 0$. Numerical evaluation shows that the spin–torsion correction terms in (32) collectively act as a *repulsive pressure* contribution, reducing the net inward gravitational force and therefore supporting a slightly more massive star for a given central density compared to the pure general relativistic case.

7 ENERGY CONDITIONS AND PHYSICAL ANALYSIS

7.1 Standard Energy Conditions

The physical viability of a stellar model requires the satisfaction of several energy conditions throughout the interior $0 \leq r \leq r_*$:

- **Null Energy Condition (NEC):** $\rho + P \geq 0$.
- **Weak Energy Condition (WEC):** $\rho \geq 0$ and $\rho + P \geq 0$.
- **Strong Energy Condition (SEC):** $\rho + 3P \geq 0$ (in the isotropic case).
- **Dominant Energy Condition (DEC):** $\rho \geq |P|$.

For the present model, using (24)–(25) and the positivity of k^2 :

$$\rho + P = \bar{\rho} + P^- + 4\pi k^2 \geq \bar{\rho} + P^-, \quad (33)$$

so the NEC for the ECSK model is *easier to satisfy* than for the corresponding pure-GR model, because the spin density provides a positive additive contribution to $\rho + P$.

7.2 Regularity at the Centre

Physically acceptable solutions must satisfy:

1. $\rho(0)$, $P(0)$ finite and positive (no central singularity).

2. $v'(0) = 0, \lambda(0) = 0$ (smooth geometry at the centre).
3. $dP/dr \leq 0$ for $0 \leq r \leq r_*$ (monotonically decreasing pressure).
4. $d\rho/dr \leq 0$ for $0 \leq r \leq r_*$ (monotonically decreasing density).

From (19), $e^{2\lambda(0)} = 1$ (regular) and $m(0) = 0$ (no point mass). The metric potential $e^{2\nu}$ given by (30) is positive and finite at $r = 0$, and $v'(0) = 0$ by inspection. Therefore, the solution is regular at the centre.

7.3 The Buchdahl–Bondi Compactness Bound

In classical general relativity, Buchdahl’s theorem establishes that for any isotropic perfect fluid sphere satisfying $d\rho/dr \leq 0$, the compactness parameter $C = 2M/r_*$ must satisfy $C \leq 8/9$. In ECSK theory the spin–torsion correction modifies this bound. Since the effective energy density $\bar{\rho} < \rho$ (spin density reduces effective mass), the ECSK bound is:

$$\frac{2M}{r_*} \leq \frac{8}{9} [1 + \mathcal{O}(\mathcal{H}^2)], \quad (34)$$

where the correction term $\mathcal{O}(\mathcal{H}^2)$ is positive, implying that ECSK stars can in principle be *slightly more compact* than their GR counterparts before pressure divergence at the centre.

7.4 Sound Speed and Causality

The causality condition requires the speed of sound v_s not to exceed the speed of light:

$$v_s^2 \equiv \frac{d\bar{P}}{d\bar{\rho}} = \frac{1}{n} \leq 1 \implies n \geq 1 \quad (35)$$

For stiff matter ($n = 1$), $v_s = c$ (marginal causality); for softer equations of state ($n > 1$), the sound speed is sub-luminal. The spin density itself does not directly enter the sound speed, since its contribution to both ρ and P through the term $2\pi k^2$ is the same, and therefore cancels in the derivative $dP/d\rho$.

8 ASTROPHYSICAL IMPLICATIONS

8.1 Neutron Stars and Spin Polarization

At the extreme densities encountered in neutron star interiors ($\rho \sim 10^{14} - 10^{15} \text{ gcm}^{-3}$), several theoretical models predict that the nuclear matter may develop a non-zero net spin polarization due to ferromagnetic-like instabilities in the neutron gas [4,7]. If such polarization exists, the spin–torsion coupling of ECSK theory would contribute an effective repulsive pressure through the $2\pi k^2$ terms in equations (11)–(12).

Quantitatively, the ratio of the spin correction to the ordinary energy density scales as

$$\frac{2\pi k^2}{\rho} \sim \frac{2\pi \mathcal{H}^2}{\rho e^{2\nu}}. \quad (36)$$

For $\mathcal{H} \sim \hbar \cdot n_{\text{spin}}$ (where n_{spin} is the number density of polarized neutrons), at nuclear saturation density this ratio can reach values of order $10^{-3} - 10^{-2}$ —small but potentially observable through precise measurements of neutron star radii by X-ray observatories such as NICER.

8.2 Quark Stars

Quark stars, composed entirely of deconfined quark matter, represent an even more extreme matter state than neutron stars. The equation of state for quark matter is well approximated by the MIT Bag model, which in its simplest form gives $P = (\rho - 4B)/3$, where B is the bag constant. Within the ECSK framework, the effective quantities $\bar{\rho}$ and $\bar{P}$ satisfy the same relation with B corrected by spin–torsion terms. The present model with the linear equation of state (18) can mimic this behaviour for $n = 3$, providing a qualitative analogue of quark star structure with spin corrections.

8.3 Singularity Avoidance in Gravitational Collapse

One of the most physically important consequences of ECSK theory is the potential avoidance of spacetime singularities in gravitational collapse. In classical general relativity, the Penrose–Hawking singularity theorems guarantee the formation of a singularity for collapsing matter satisfying the strong energy condition. In ECSK theory, however, the effective energy density $\bar{\rho} = \rho - 2\pi k^2$ can, in principle, violate the SEC even when the physical energy density ρ does not, provided k^2 is sufficiently large. This effective violation generates a repulsive contribution at extreme densities that may halt the collapse, converting what would be a singularity in GR into a non-singular bounce.

The condition for this singularity avoidance is:

$$2\pi k^2 > \frac{\rho + 3P}{4}, \quad (37)$$

which requires the spin density to be sufficiently large. Using (10), this translates to a condition on the constant $\mathcal{H}$:

$$\mathcal{H}^2 > \frac{(\rho + 3P) e^{2\nu}}{8\pi}. \quad (38)$$

Although this condition is not satisfied under ordinary astrophysical conditions, it may be reached during the final stages of gravitational collapse when densities approach the Planck scale.

8.4 Early Universe Cosmology

The ECSK framework has significant implications for early-universe cosmology. In a homogeneous, isotropic Friedmann–Lemaître–Robertson–Walker (FLRW) background, the spin–torsion correction introduces an effective equation of state with $w_{\text{eff}} < -1$ at extremely high densities, which can drive a pre-Big-Bang bounce and resolve the initial singularity of standard cosmology. The spherically symmetric models studied here, while not directly applicable to cosmology, provide a test bed for understanding the competition between spin-induced repulsion and gravitational attraction that underlies the cosmological singularity avoidance mechanism.

9 DISCUSSION

The solutions derived in the present work share several structural features with their pure-GR counterparts while exhibiting important qualitative differences attributable to the spin–torsion coupling. We highlight the following points.

(i) Modified mass–radius relation: For fixed central density, an ECSK star is predicted to have a slightly smaller gravitational mass than the corresponding GR star, because the effective mass-generating density $\bar{\rho} = \rho - 2\pi k^2$ is reduced by the spin correction. This prediction, if confirmed observationally, could distinguish ECSK theory from GR for dense compact objects.

(ii) Stiffening of the effective equation of state: Although the bare equation of state parameter n is fixed, the presence of the spin term increases the actual pressure P relative to the effective pressure P^- at the same density. This effective stiffening increases the maximum mass of a stable neutron star, potentially resolving tensions between the soft nuclear equation of state inferred from terrestrial experiments and the high masses ($M \sim 2M_\odot$) observed for some neutron stars.

(iii) Spin density profile: The spin density $k(r) = He^{-\nu(r)}$ increases inward as ν decreases toward the centre, consistent with the expectation that spin polarization is strongest at the highest densities. The parameter H sets the overall magnitude of the spin effect and must ultimately be constrained by microphysical nuclear or quark matter calculations.

(iv) Comparison with spherically symmetric ECSK literature: Our solutions extend the work of Prasanna [9] and Kerlick [7] by providing explicit analytic expressions for the metric potentials and thermodynamic variables under a specific but physically motivated equation of state. The matching to the Schwarzschild exterior is more carefully

handled here, and the Buchdahl-type compactness constraint is derived explicitly for the ECSK context.

10 CONCLUSION

We have investigated static, spherically symmetric perfect fluid configurations within the Einstein–Cartan–Sciama–Kibble (ECSK) theory of gravitation, wherein spacetime torsion is sourced by the intrinsic spin density of the fluid constituents. The principal results of this investigation are summarised below.

1. The ECSK field equations for the static, spherically symmetric interior reduce to a system formally identical to the standard Einstein equations when written in terms of the Hehl effective energy density and pressure, $\bar{\rho} = \rho - 2\pi k^2$ and $\bar{P} = P - 2\pi k^2$, where $k(r) = He^{-\nu(r)}$ is the scalar spin density function. This decoupling is a key structural feature that greatly facilitates the derivation of exact solutions.
2. By imposing the linear barotropic equation of state $\bar{\rho} = n\bar{P}$ together with the constant-effective-density ansatz for the metric potential, a closed-form exact interior solution has been obtained. The integration constants are uniquely fixed by the Darmois–Israel junction conditions at the stellar surface and by the requirement of central regularity.
3. The actual physical energy density and pressure exceed their effective counterparts by the spin density term $2\pi k^2$, which is positive definite. Consequently, ECSK stars with the same gravitational mass and radius as GR stars have a larger actual central pressure and energy density—a signature that could, in principle, be tested through precise neutron star observations.
4. A modified TOV equation incorporating spin–torsion corrections has been derived. The correction terms act as a net repulsive force contribution, supporting the star against gravitational collapse and potentially allowing configurations with compactness parameters slightly exceeding the classical Buchdahl bound.
5. The energy conditions are analysed and shown to be more easily satisfied in the ECSK framework than in GR, owing to the positive contribution of $4\pi k^2$ to $\rho + P$.
6. Astrophysical implications include: a predicted softening of the effective equation of state (reduced gravitational mass), a mechanism for singularity avoidance in gravitational collapse, and potential observational signatures in neutron star mass–radius measurements by X-ray timing observatories.

Future extensions of this work will include the study of charged (Reissner–Nordström exterior) configurations, the inclusion of a cosmological constant, generalisation to non-isotropic (anisotropic pressure) fluid models, and investigation of the dynamical stability of the present solutions against radial perturbations.

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