[Volume 3 issue 10 October 2015]
PageNo.1191-1195 ISSN : 2320-7167

INTERNATIONAL JOURNAL OF MATHEMATICS AND COMPUTER RESEARCH

T_s* - Sg - Continuous Maps in Topological Spaces

Dr. T. Indira[1], S. Geetha[2]

[1,2]PG and Research Department of Mathematics, Seethalakshmi Ramaswami College, Trichy-2.

[1]E-mail:-drtindira.chandru@gmail.com

[2]E-mail:-gsvel@rediffmail.com

ABSTRACT

The aim of this paper is to introduec a new type of function called τ_s*-semi generalized continuous maps and study about some of their properties.

KEY WORDS

scl**, τ_s^* -topology, τ_s^*-sg-open set, τ_s^*-sg-closed set, τ_s^*- sg-continuous maps, 2010 Mathematics subject classification: Primary 57N05, Secondary 57N05

INTRODUCTION

The concept of generalized closed sets was introduced by Levine[ ]. Dunham[4] introduced the concept of closure operator cl* and a topology τ* and studied some of its properties. Pushpalatha, Easwaran and Rajarubi[11] introduced and studied τ*-generalized closed sets, and τ*-generalized open sets. Using τ*generalized closed sets, Eswaran and Pushpalatha[5] introduced and studied τ*-generalized continuous maps.

The purpose of this paper is to introduce and study the concept of a new class of maps, namely τ_s*-sg-continuous maps. Throughout this paper X and Y are topological spaces on which no separation axioms are assumed unless otherwise explicitly stated. For a subset A of a topological space X, cl(A), scl**(A) and A^C denote the closure, semi generalized closure and complement of A respectively.

PRELIMINARIES

Definition :2.1

For the subset A of a topological space X, the semi generalized closure of A (i.e.,scl**(A)) is defined as the intersection of all sg-closed sets containing A.

Definition:2.2

For the subset A of a topological space X, the topology

$$\tau_s^* = \{G : scl^{**}(G^C) = G^C \}.$$

Definition:2.3

A subset A of a topological space X is called τ_s*- semi generalized closed set[ ] (briefly τ_s*-sg-closed) if scl**(A) ⊆ G whenever A ⊆ G and G is τ_s*-semi open.

The complement of τ_s*- semi generalized closed set is called the τ_s*- semi generalized open set(briefly τ_s*-sg-open).

Definition:2.4

The τ_s*- semi generalized closure operator $cl_{\tau s*}$ for a subset A of a topological space (X, τ_s*) is defined by the intersection of all τ_s*- semi generalized closed sets containing A

$$\text{(i.e.,) } cl_{\tau s*}(A) = \cap\{G : A \subseteq G \text{ and } G \text{ is } \tau_s^* \text{-sg-closed}\}$$

Definition:2.5

A map $f : X \rightarrow Y$ from a topological space X into a topological space Y is called: continuous if the inverse image of every closed set (or open set) in Y is closed(or open) in X.
generalized continuous[2] (g-continuous) if the inverse image of every closed set in Y is g-closed in X.
(3) strongly sg-continuous if the inverse image of each gs-open set of Y is open in X.
(5) semi continuous[13] if the inverse image of each closed set of Y is semi-closed in X.
(6) sg-continuous[12] if the inverse image of each closed set of Y is sg-closed in X.
(7) gs-continuous[14] if the inverse image of each closed set of Y is gs-closed in X.
(8) gsp-continuous[3] if the inverse image of each closed set of Y is gsp-closed in x.
(9) αg-continuous[6] if the inverse image of each closed set of Y is g –closed in X.
(10)pre-continuous[9] if the inverse image of each open set of Y is pre-open in X.
(11)α-continuous[10] if the inverse image of each open set of Y is α-open in X.
(12)sp-continuous[1] if the inverse image of each open set of Y is semi-preopen in X.

Remark:2.6

In[7] it has been proved that every closed set is τ_s* - sg closed.
In[7] it has been proved that every sg-closed set in X is τ_s* - sg closed.

τ_s* - sg- CONTINUOUS MAPS IN TOPOLOGICAL SPACES

In this section, we introduce a new class of map namely τ_s*-semi generalized continuous map in topological spaces and study some of its properties and relationship with some existing mappings.

Definition: 3.1

A map $f:X \rightarrow Y$ from a topological space X into a topological space Y is called τ_s* -semi generalized continuous map(briefly τ_s* - sg-continuous) if the inverse image of every closed set in Y is τ_s* -sg-closed in X.

Theorem: 3.2

Let $f : X \rightarrow Y$ be a map from a topological space (X, τ_s^*) into a topological space (Y,σ_s^*).

(i) The following statements are equivalent:

(a) f is τ_s* - gs-continuous.

(b) the inverse image of each open set in Y is τ_s* - sg-open in X.

(ii) If $f : X \rightarrow Y$ is τ_s* - sg-continuous, then $f(cl_{\tau_s*}(A)) \subseteq cl(f(A))$ for every subset A of X.

Proof:

(i) Assume that $f : X \rightarrow Y$ is τ_s* - sg-continuous. Let F be open in Y. Then F^C is closed in Y. Since f is τ_s* - sg-continuous, $f^{-1}(F^C)$ is τ_s* - sg-closed in X.
But $f^{-1}(F^C) = X - f^{-1}(F)$. Thus $X - f^{-1}(F)$ is τ_s* -sg-closed in X.
Therefore (a) $\Longrightarrow$ (b).

Conversely, assume that the inverse image of each open set in Y is τ_s* - sg-open in X.
Let F be any closed set in Y. Then F^C is open in Y. By assumption, $f^{-1}(F^C)$ is τ_s* - sg-open in X. But $f^{-1}(F^C) = X - f^{-1}(F)$. Therefore, $X - f^{-1}(F)$ is τ_s* - sg-open in X and so $f^{-1}(F)$ is τ_s* -sg-closed in X. Therefore, f is τ_s* -sg-continuous.

Hence (b) $\Longrightarrow$ (a). Thus (a) and (b) are equivalent.

(ii) Assume that f is τ_s* -sg-continuous. Let A be any subset of X, f(A) is a subset of Y. Then cl(f(A))) is a closed subset of Y. Since f is τ_s* -sg-continuous, f^{-1}(cl(f(A))) is τ_s*-sg-closed in X and it containing A. But cl $_{\tau s*}$ (A) is the intersection of all τ_s* -sg-closed sets containing A.

$$cl_{\tau s*}(A) \subseteq f^{-1}(cl(f(A))).$$

$$\text{(i.e.,)}\ f(cl_{\tau s*}(A)) \subseteq cl(f(A)).$$

Theorem:3.3

If a map f : X →Y from a topological space X into a topological space Y is continuous then it is τ_s* -sg-continuous but not conversely.

Proof:

Let f : X →Y be continuous. Let V be a closed set in Y. Since f is continuous, f^{-1}(V) is closed in X. By Remark:2.6(2), f^{-1}(V) is τ_s* -sg-closed. Thus, f is τ_s* -sg-continuous.

Remark:

The converse of the theorem need not be true as seen from the following example.

Example:

Let X = Y = {a,b,c}, τ = {X,Φ,{a,b}} and **σ** = {Y,Φ, {b},{c},{b,c},{a,b}}.

Let f : X →Y be a map defined by f(a)=a, f(b)=b.f(c)=c. Here f is τ_s* -sg-continuous. But f is not continuous. Since for the sets {a,c},{a,b},{a} are closed in Y, but {a,c},{a,b},{a} are not closed on X.

Theorem:3.5

If a map f : X →Y from a topological space X into a topological space Y is sg-continuous then it is τ_s* -sg-continuous but not conversely.

Proof:

Let f : X →Y be sg-continuous. Let F be any closed set in Y. Then the inverse image f^{-1}(F) is sg-closed in X. Also, by Remark: 2.6(2), f^{-1}(F) is τ_s* - sg-closed. Then, f is τ_s* -sg-continuous.

Remark:

The converse of the theorem need not be true as seen from the following example.

Example:

Let X = Y = {a,b,c} with topologies τ={X,Φ,{c}} and **σ**= {Y,Φ,{b},{c},{b,c}}.

Let f : X →Y be an identity map. Then f is τ_s* -sg-continuous. But it is not sg-continuous. Since for the closed set V = {a,c} in Y, f^{-1}(V) = {a,c} is not sg-closed in X.

Theorem:3.7

If a map f : X→Y from a topological space X into a topological space Y is strongly sg-continuous then it is τ_s* - sg-continuous but not conversely.

Proof:

Let f : X → Y be strongly sg-continuous. Let F be a closed set in Y, then F is sg-closed. Hence F^C is sg-open in Y. Since f is strongly sg-continuous $f^{-1}(F^C)$ is open in X.

But $f^{-1}(F^c)$ = X – f^{-1}(F). Therefore f^{-1}(F) is closed in X. By Remark:2.6(1), f^{-1}(F) is τ_s* - sg-closed in X. Therefore f is τ_s* -sg-continuous.

Remark:3.8

From the above discussion, we obtain the following implications.

Let X = Y = {a,b,c}. Let f : X →Y be an identity map.

Let τ = {X,Φ,{a},{a,c}} and **σ** = {Y,Φ,{a},{a,b},{a,c}}. Then f is semi-continuous .

But it is not τ_s* - sg-continuous. Since for the closed set V = {c} in Y,

f^{-1}(V) = {c} is not τ_s* - sg-closed in X.

(2) Let $\tau = \{X,\Phi,\{c\}\}$ and $\boldsymbol{\sigma} = \{Y,\Phi,\{b\},\{c\},\{b,c\}\}$. Then f is τ_s^* - sg-continuous. But it is not semi-continuous. Since for the closed set

$V = \{a,c\}$ in Y, $f^{-1}(V) = \{a,c\}$ is not semi-closed in X.

Let Let $\tau = \{X,\Phi,\{c\}\}$ and $\boldsymbol{\sigma} = \{Y,\Phi,\{b\},\{c\},\{b,c\}\}$. Then f is τ_s^* - sg-continuous. But it is not sg-continuous. Since for the closed set $V=\{a,c\}$ in Y,

$f^{-1}(V) = \{a,c\}$ is not sg-closed in X.

Let $\tau = \{X,\Phi,\{a\}\}$ and $\boldsymbol{\sigma} = \{Y,\Phi,\{b\},\{c\}\{b,c\},\{a,c\}\}$. Then f is τ_s^* - sg-continuous But it is not gs-continuous. Since for the closed set $V = \{a\}$ in Y, $f^{-1}(V) = \{a\}$ is not gs-closedin X.

Let $\tau = \{X,\Phi,\{a,c\}\}$ and $\boldsymbol{\sigma} = \{Y,\Phi,\{a\},\{b\},\{a,b\},\{b,c\}\}$. Then f is gsp-continuous. But it is not τ_s^* - sg-continuous. Since for closed set $V = \{c\}$ in Y, $f^{-1}(V) = \{c\}$ is not

τ_s^* - sg-closed in X.

Let $\tau = \{X,\Phi,\{c\}\}$ and $\boldsymbol{\sigma} = \{Y,\Phi,\{a\},\{b\},\{a,b\},\{b,c\}\}$. Then f is τ_s^* - sg-continuous. But it is not gsp-continuous. Since for closed set $V = \{c\}$ in Y, $f^{-1}(V) = \{c\}$ is not

gsp-closed in X.

Let $\tau = \{X,\Phi,\{b\}\}$ and $\boldsymbol{\sigma} = \{Y,\Phi,\{c\},\{a,c\}\}$. Then f is τ_s^* - sg-continuous. But it is not αg-continuous. Since for closed set $V = \{b\}$ in Y, $f^{-1}(V) = \{b\}$ is not

αg-closed in X.

Let $\tau = \{X,\Phi,\{a\}\}$ and $\boldsymbol{\sigma} = \{Y,\Phi,\{c\},\{a,c\},\{b,c\}\}$. Then f is τ_s^* - sg-continuous. But it is not pre-continuous. Since for open set $V = \{b,c\}$ in Y, $f^{-1}(V) = \{b,c\}$ is not

Pre-open in X.

Let $\tau = \{X,\Phi,\{c\},\{b,c\}\}$ and $\boldsymbol{\sigma} = \{Y,\Phi,\{c\},\{a,c\},\{b,c\}\}$. Then f is α-continuous. But it is not τ_s^* - sg-continuous. Since for closed set $V = \{b\}$ in Y, $f^{-1}(V) = \{b\}$ is not

τ_s^* - sg-closed in X.

Let $\tau = \{X,\Phi,\{a\},\{b,c\}\}$ and $\boldsymbol{\sigma} = \{Y,\Phi,\{a\},\{c\},\{a, c\},\{b,c\}\}$. Then f is τ_s^* - sg-continuous. But it is not α-continuous. Since for the open set $V=\{a,c\}$ in Y,

$f^{-1}(V) = \{a,c\}$ is not α-open in X.

Let $\tau = \{X,\Phi,\{a\},\{a,c\}\}$ and $\boldsymbol{\sigma} = \{Y,\Phi,\{a\},\{a,b\},\{a,c\}\}$. Then f is sp-continuous. But it is not τ_s^* - sg-continuous. Since for closed set $V = \{c\}$ in Y, $f^{-1}(V) = \{c\}$ is not

τ_s^* - sg-closed in X.

Let $\tau = \{X,\Phi,\{c\}\}$ and $\boldsymbol{\sigma} = \{Y,\Phi,\{b\},\{c\},\{a, c\},\{b,c\}\}$. Then f is τ_s^* - sg-continuous. But it is not sp-continuous. Since for the open set $V=\{b\}$ in Y,

$f^{-1}(V) = \{b\}$ is not sp-open in X.

Let $\tau = \{X,\Phi,\{c\},\{b,c\}\}$ and $\boldsymbol{\sigma} = \{Y,\Phi,\{c\},\{a,c\},\{b,c\}\}$. Then f is weakly sg-continuous. But it is not τ_s^* - sg-continuous. Since for closed set $V = \{b\}$ in Y,

$f^{-1}(V) = \{b\}$ is not τ_s^* - sg-closed in X.

Let $\tau = \{X,\Phi,\{a\},\{b,c\}\}$ and $\boldsymbol{\sigma} = \{Y,\Phi,\{c\},\{b,c\}\}$. Then f is τ_s^* - sg-continuous. But it is not weakly-sg-continuous. Since for the open set $V=\{a,c\}$ in Y,

$f^{-1}(V) = \{a,c\}$ is not semi-open in X.

Let $\tau = \{X,\Phi,\{c\},\{a,c\}\}$ and $\boldsymbol{\sigma} = \{Y,\Phi,\{c\},\{a,c\},\{b,c\}\}$. Then f is weakly gs-continuous. But it is not τ_s^* - sg-continuous. Since for closed set $V = \{a\}$ in Y,

$f^{-1}(V) = \{a\}$ is not τ_s^* - sg-closed in X.

Let $\tau = \{X,\Phi,\{a\},\{b,c\}\}$ and $\boldsymbol{\sigma} = \{Y,\Phi,\{c\},\{a,c\},\{b,c\}\}$. Then f is τ_s^* - sg-continuous. But it is not weakly-gs-continuous. Since for the open set $V=\{c\}$ in Y,

$f^{-1}(V) = \{c\}$ is not semi-open in X.

CONCLUSION

The class of τ_s* -sg-continuous maps defined using τ_s* -sg-closed sets. The τ_s* -sg-closed sets can be used to derive a new homeomorphism, connectedness, compactness and new separation axioms.This concept can be extended to bitopological and fuzzy topological spaces.

REFERENCES

1. M.E. Abd El-Monsef, S.N.El.DEEb and R.A.Mahmoud, β-open sets and and β-continuous mappings, Bull. Fac. Sci. Assiut Univ.12(1)(1983),77-90.
2. K. Balachandran, P. Sundaram and J.Maki, On generalized continuous maps in topological spaces. Em. Fac. Sci. K.ochi Univ.(Math.) 12 (1991), 5-13. Monthly,70(1963),36-41.
3. J.Dontchev, On generalizing sempreopen sets, Mem. Fac. Sci. Kochi Uni. Ser A, Math.,16(1995), 35-48.
4. J.Dontchev, On generalizing semipreopen sets, Mem.Fac.Sci.Kochi Uni.Ser A,Math.,16(1995),35-48.
5. S. Eswaran and A Pushpalatha, τ* - generalized continuous maps in topological spaces, International J. of Math Sci & Engg. Apppls.(IJMSEA) ISSN 0973-9424 Vol.3, No.IV,(2009),pp.67-76.
6. Y. Gnanambal, On generalized preregular sets in topological space, Indian J. Pure Appl. Math.(28)3(1997),351-360.
7. T. Indira and S.Geetha ,τ_s*-sg -closed sets in topological spaces, International Journal ofMathematics Trends and Technology-Volume 21 No.1-May 2015.
8. M. Levine, Generalized closed sets in topology, Rend. Circ. Mat..Palermo,19,(2)(1970),89- 96.
9. A.S.Mashhour, I.A.Hasanein and S.N.El-Deeb, On precontinuous and weak precontinuous functions, Proc. Math. Phys. Soc.Egypt 53(1982),47-53.
10. A.S. Mashhour, I.A.Hasanein and S.N.El-Deeb, On α-continuous and α-open mappings, Acta. Math.Hunga.41(1983),213-218.
11. A.Pushpalatha,S.Eswaran and P.Rajarubi,τ*-generalized closed sets in topological spaces, Prodeedings of World Congress on Engineering 2009 Vol II WCE 2009, July 1-3,2009, London, U.K., 1115-1117.
12. P.Sundarm, H.Maki and K.Balachandran, sg-closed sets and semi-$T_{1/2}$ spaces. Bull.Fukuoka Univ. Ed..Part III,40(1991),33-40.
13. Semi open sets and semicontinuity in topological spaces, Amer. Math.,
14. Semigeneralized closed and generalized closed maps, Mem.Fac.Sci.Kochi.