[Volume 2 issue 7 July 2014]
Page No.484-489 ISSN :2320-7167

INTERNATIONAL JOURNAL OF MATHEMATICS AND COMPUTER RESEARCH

A Deterministic Inventory Model For Deteriorating Items With Selling Price Dependent Demand And Time Dependent Deterioration Under Inflation With Variable Holding Cost

Yadav Smita, Aggarwal Naresh Kumar, Sharma Anil Kumar

Research Scholar, Department of Mathematics, Singhania University, Pacheri Bari , Jhunjhanu (RJ) India

Dept. of Mathematics

Govt. Sr. Sec. School Bikaner (Rewari) india

Dept. of Mathematics

R.R. College, Alwar (Rajasthan) India

ABSTRACT:-

This paper is concerned with the development of inventory model in which demand is taken as a function of selling price and rate of deterioration is taken as a linear function of time and a storage time dependent holding cost. The holding cost per unit of the item per unit time is assumed to be an increasing function of the time spent in storage. The time dependent holding cost step functions are considered, retroactive holding cost increase and incremental holding cost increase. Shortages are not allowed and effect of inflation rate is considered. A deterministic inventory model is developed for obtaining optimum cycle length for both the cost structures. The proposed model reduces to well known result, by choosing appropriate value of the parameters.

KEY WORDS:-

Inventory, optimal, inflation.

INTRODUCTION:-

The well known square root formula is $Q = \sqrt{2C_3 D / C_1}$ for economic order quantity of the item, where C_1, C_3 and D are holding cost, replenishment cost and demand rate respectively. In this formula demand rate is constant and items in inventory do not undergo deterioration. However, in real life situation, inventory loss may be due to deterioration and demand. Demand may depend on selling price, for example fruits, vegetables and consumer-goods type items. During past few years many authors have studied inventory models for deterioration items considering different demand and deterioration rate.

Selling price is one of the decisive factors in selecting a them for use. It is well known that lesser the selling price of an item, increase the demand of that item, where as higher the selling price has reverse effect. Some authors like Cohen (1), Mukherjee (4), Gupta and

Jawhari (2), Kumar and Sharma (3) etc. developed inventory models taking demand as a function of selling price.

In this article, an inventory model is developed considering demand is a function of selling price. Inflation is considered and rate of deterioration is taken as a linear function of time with variable holding cost. The model is solved by minimizing the total average cost. Shortages are not allowed. As a special case this model reduce to well known result.

ASSUMPTIONS AND NOTATIONS:-

The proposed model is developed under the following assumptions and notations.

- The demand for the item is partially constant and partially selling price dependent and is assumed as
  $D(p(t)) = a - b\,p(t)$ where 'a' is fixed demand; a, b>0 and a>>b.
  p(t) is the selling price of the item at time t and is taken as $p(t) = pe^{rt}$ is the selling price per unit at time 't' and 'p' is the selling price of the item at time t=0
- 'r' is the inflation rate is constant.
- Shortages are not allowed and lead time is zero.
- There is no repair or replacement of the deteriorated units during the cycle time under consideration.
- I(t) is the level of inventory at any instant of time.
- C is the unit purchase cost, k is the ordering cost per order.
- T is the cycle time.
- A variable fraction θ(t) of on hand inventory deteriorates per unit time. In the present model, the function θ(t) is assumed of the form
  $\theta(t) = \alpha + \beta t, \quad 0<\beta<<1,\ t>0,\ \alpha\geq 0,\ \beta\geq 0.$
  - N Number of distinct time periods with different holding cost rates
    t = time
    t_i End time of period i, where i = 1,2,3,---, n, $t_0 = 0$ and $t_n = \infty$
    h_i holding cost of the item at time t
    $h(t) = h_i$ if $t_{i-1} \leq t \leq t_i$

MATHEMATICAL ANALYSIS FOR THE SYSTEM:-

Let I(t) be the inventory level at any time 't'. The inventory level decreases mainly due to demand and partly due to deterioration of units. The differential equation governing the system in the interval (0, T) is given by

$$\frac{dI(t)}{dt} = -\theta(t)I(t) - D(p(t)) \qquad (0 \le t \le T) \qquad (1)$$

$$= -(\alpha + \beta t)I(t) - (a - bpe^{rt}) \qquad (2)$$

Solution of the differential equation after adjusting constant of integration and initial condition $t = 0, I(t) = I(0)$

$$I(t) = \exp.\left[-\left(\alpha t + \frac{\beta t^2}{2}\right)\right]\left[-a\left(t + \frac{\alpha t^2}{2} + \frac{\beta t^2}{6}\right) + bp\left\{t + \frac{(\alpha + r)t^2}{2} + \frac{\beta t^3}{6}\right\} + I(0)\right] \qquad (3)$$

Inventory without decay $I_w(t)$ at time 't' is given by

$$\frac{d}{dt}I_w(t) = -(a - bpe^{rt})$$

$$\Rightarrow I_w(t) = -at + \frac{bpe^{rt}}{r} + I(0) - \frac{bp}{r} \qquad (4)$$

(using initial condition at $t = 0, I(t) = I(0)$)

Stock loss due to decay Z(t) at time t is given by

$$Z(t) = I_w(t) - I(t)$$

$$= -at - \frac{bp}{r}(1 - e^{rt}) + I(0) - I(t) \qquad (5)$$

equation (3) gives

$$I(0) = I(t)\exp.\left(\alpha t + \frac{\beta}{2}t^2\right) + a\left(t + \frac{\alpha}{2}t^2 + \frac{\beta}{6}t^3\right) - bp\left(t + \frac{\alpha + r}{2}t^2 + \frac{\beta}{3}t^3\right) \qquad (6)$$

Substituting value of $I(0)$ from (6) in (5), we get

$$Z(t) = -at - \frac{bp}{r}(1 - e^{rt}) - bp\left(t + \frac{\alpha + r}{2}t^2 + \frac{\beta}{3}t^3\right) + a\left(t + \frac{\alpha}{2}t^2 + \frac{\beta}{6}t^3\right) + I(t)\left[\exp.\left(\alpha t + \frac{\beta}{2}t\right) - 1\right]$$

(7)

At t=T, we get

$$Z(t) = -aT - \frac{bp}{r}(1 - e^{rt}) - bp\left(T + \frac{\alpha + r}{2}T^2 + \frac{\beta}{3}T^3\right) + a\left(T + \frac{\alpha}{2}T^2 + \frac{\beta}{6}T^3\right) \qquad (8)$$

Note that $I(T) = 0$

Order quantity is given by

$$Q_T = Z(T) + \int_0^T \left(a - bpe^{rt}\right)dt$$

$$= a\left(T + \frac{\alpha}{2}T^2 + \frac{\beta}{6}T^3\right) - bp\left(T + \frac{\alpha + r}{2}T^2 + \frac{\beta}{3}T^3\right) \quad (9)$$

Also $I(0) = Q_T$ implies

$$I(t) = \exp.\left[-\left(\alpha t + \frac{\beta}{2}t^2\right)\right]\left[a\left\{(T-t) + \frac{\alpha}{2}\left(T^2 - t^2\right) + \frac{\beta}{6}\left(T^3 - t^3\right)\right\} - bp\left\{(T-t) + \frac{\alpha + r}{2}\left(T^2 - t^2\right) + \frac{\beta}{3}\left(T^3 - t^3\right)\right\}\right] \quad (10)$$

As stated earlier, the holding cost is assumed to be an increasing function of storage time i.e $h_1 < h_2 < h_3 < \cdots < h_n$.

Case (i):- Retroactive holding cost increase

In this case uniform holding cost is used and the holding cost of the last storage period is applies retroactively to all previous periods. Thus, if the cycle ends in the period 'e' then the holding cost rate h_e is applied to all periods $i = 1,2,3,\ldots,e$. In this case the C(T,p) total inventory cost per unit time can be expressed as

$$C(T,p) = \frac{k}{T} + \frac{CQ_T}{T} + \frac{h_i}{T}\int_0^T I(t)dt \quad (11)$$

$$= \frac{k}{T} + C\left[\left\{a\left(1 + \frac{\alpha}{2}T + \frac{\beta}{3}T^2\right)\right\} - bp\left\{1 + \frac{(\alpha + r)}{2}T + \frac{\beta}{3}T^2\right\}\right] +$$
$$h_i\left[-\frac{a}{2}T + \frac{2a\alpha}{3}T^2 + (a - bp)\left(T - \frac{\alpha}{2}T^2\right) + \frac{a\beta}{12}T^3 + \frac{bp}{2}T^3 + \frac{(r-\alpha)}{6}bpT^2 - \frac{bp(\alpha + r)}{3}T^2 - \frac{7}{12}p\beta T^3\right]$$

For minimum total average cost, the necessary criterion is

$$\frac{d}{dt}[C(T,p)] = 0 \quad (12)$$

For fixed 'p'

$$\Rightarrow -k + T^2C\left[a\left(\frac{\alpha}{2} + \frac{2\beta}{3}T\right) - bp\left(\frac{\alpha + r}{2} + \frac{2\beta}{3}T\right)\right] +$$
$$T^2h_i\left[-\frac{a}{2} + \frac{4a\alpha}{3}T + (a - bp)(1 - \alpha T) + \frac{a\beta}{4}T^2 + \frac{3bp}{2}T^2 + \frac{(r-\alpha)}{3}bpT - \frac{2bp(\alpha + r)}{3}T - \frac{7bp\beta}{4}T^2\right] = 0$$

which can be solved for T_p numerically by using theory of equations:

Also

$$\frac{d^2}{dT^2}[C(T,p)]>0 \qquad (13)$$

Case (ii):- Stepwise incremental holding cost increase

The holding cost is now assumed to be an increasing step function of storage time. According to this function the holding cost rates h_1 applied to period 1, rate h_2 applied to period 2, and so on. Now the total inventory cost obtained as follows

$$C(T,p)=\frac{k}{T}+\frac{CQ_T}{T}+\frac{h_i}{T}\int_0^{t_1} I(t)dt+\frac{h_2}{T}\int_{t_1}^{t_2} I(t)dt+\ldots+\frac{h_e}{T}\int_{t_{e-1}}^{t_e} I(t)dt$$

$$=\frac{k}{T}+\frac{CQ_T}{T}+\sum_{i=1}^{e}\frac{h_i}{T}\int_{t_{i-1}}^{t_i} I(t)dt$$

$$=\frac{k}{T}+\frac{CQ_T}{T}+\sum_{i=1}^{e}\frac{h_i}{T}\int_{t_{i-1}}^{t_i}\left[\begin{array}{l} a\left\{(T-t)+\frac{\alpha}{2}\left(T^2-t^2\right)+\frac{\beta}{6}\left(T^3-t^3\right)\right\}-bp\left\{(t-t)+\frac{\alpha+r}{2}\left(T^2-t^2\right)+\frac{\beta}{3}\left(T^3-t^3\right)\right\} \\ -a\alpha\left(Tt-t^2\right)+\alpha bp\left(tT-t^2\right)-\frac{a\beta}{2}\left(Tt^2-t^3\right)+\frac{b\beta p}{2}\left(Tt^2-t^3\right) \end{array}\right]dt$$

After integrating and simplifying, we get the desired result.

For minimum total average cost, the necessary criterion is

$$\frac{d}{dt}[C(T,p)]=0$$

For fixed p, which can be solved for T_p numerically by using theory of equations. Also

$$\frac{d^2}{dT^2}[C(T,p)]>0$$

SPECIAL CASE:-

If a=R, b=0, α=0, β=0. Then

$$C(T)=\frac{k}{T}+CR+\frac{hRT}{2}$$

which is the standard result for non-decaying inventory.

CONCLUSION:-

In this paper, we have proposed inventory model for deteriorating items having selling price dependent demand and time dependent deterioration and variable holding cost. Shortages are not allowed and constant inflation is also considered. The model is solved by cost minimizing criterion. As a special case this model reduces to standard result for non-decaying inventory. The case of increasing holding cost considered in this paper applies to company owned storage facilities and particularly to deteriorating items that requires extra care if stored for longer periods. A decreasing holding cost step function is applicable to rented storage facilities, where lower rent rates are normally obtained for longer term leases.

REFRENCES:-

1. Cohen, M.A (1977)
   Joint pricing and ordering policy for exponentially decaying inventory with known demand. Nav.Res.log.Quart. vol.24(2), 257-268.
2. Gupta P.N and Jawhari R(1993)
   Optimum ordering interval for constant decay rate of inventory discrete in time. Ganita Sandesh, Vol 7(1), 12-15.
3. Kumar N and Sharma, A.K(2001)
   Optimum ordering interval with known demand for items with variable rate of deterioration and shortages A.S.P vol(111) 2 95-100.
4. Mukherjee, S.P.(1987)
   Optimum ordering interval for time varying decay rate of inventory opsearch, vol 24(1) 19-24.
5. Mandal, M and De S.K(2012)
   An EOQ model with parabolic demand rate and time varying selling price. Analysis of pure and applied Mathematics Voll No 1, 2012, 32-43.
6. Roy, A. (2008)
   An inventory model for deteriorating items with price dependent demand and time-varying holding cost. Advanced modeling and optimization 10(1) (2008) 25-37.