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Effect Of Heat And Mass Transfer On An Oscillatory Mhd Mixed Convection Past An Infinite Vertical Porous Plate With Variable Suction And Radiation

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ABSTRACT

The effect of heat and mass transfer on an oscillatory MHD mixed convection past an infinite vertical porous plate with variable suction and radiation has been investigated. A uniform magnetic field is applied normal to the plate. A perturbation method was employed to solve the momentum equation, energy equation, species concentration equation, the skin friction, the Nusselt number and the Sherwood number. The effects of various parameters on velocity, temperature and concentration distribution profiles are considered and discussed in details through graphs and tables. However, our investigation of the problem setup leads to the following conclusion.The fluid motion is decelerated under the action of transverse magnetic field and radiation conduction parameter. Temperature falls due to effect of radiation and Prandtl numbers. The concentration distribution reduced with increase in Schmidt number and chemical parameter.

Keywords: Heat and Mass Transfer, Convection, Variable Suction, Oscillatory Flow, Radiation

1. INTRODUCTION

The problem of interaction of free and forced convection with thermal radiation of viscous incompressible MHD mixed convective flow past an infinite vertical porous plates with mass transfer are being studied nowadays due to the many applications of such problems in Astrophysics, Geophysical and different engineering fields. The heating of rooms and buildings by use of radiators is an example of heat transfer by free convection. The study of MHD is quite important in the field of aeronautics, especially in missile aerodymics, since the temperature that occurs in such flight speed are sufficient to dissociate or even ionize the air appreciably. For example, when a high speed missile re-enters the earth's atmosphere, a very large amount of heat is generated due to the friction of air molecules and this viscous heating may sometimes be so considerable as to ionize the air near the stagnation point. This ionized gas or plasma interacts with the magnetic field and alter the heat transfer and friction characteristics. In addition to this, some fluids can also emit and absorb thermal radiation. In such a situation, the study of the effect of the magnetic field on the flow is not only electrically conducting but also capable of emitting and absorbing thermal radiation. This type of investigation is carried out because of its importance in space and temperature related problems.

MHD is the science of motion of electrically conducting fluid in presence of magnetic field. There are numerous examples of application of MHD principles, these include MHD generators, MHD pumps and MHD flow meters etc. The dynamo and motor is a classical example of MHD principle. MHD principles also find its application in medicine and biology. The principle of MHD is also used in stabilizing a flow the transition from laminar to turbulent flow. Convection problems of electrically conducting fluid in the presence of transverse magnetic field have got much importance because of its wide application in geophysics, Astrophysics, Plasma physics and missile technology etc. The present form of MHD is due to the pioneer contribution of several authors like Alfven (1942), Cowling (1957), Crammer and Pai (1978), Shercliff (1968) and Ferraro and Pulmption (1966). Model studies on MHD free and forced convection with heat and mass transfer problems have been carried out by many of the authors due to their application in many branches of science and technology. Some of them are Ahmed (2010) , Elbashbeshy (2003) and Singh and Singh (2000).

Radiation is a process of heat transfer through electromagnetic waves. Radiative conductive flows are encountered in industrial environment. For example heating and cooling chambers, fossil fuel combustion energy process, evaporation from large open water reservoirs, astrophysical flow etc. Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. If the temperature of the surrounding field is rather high, radiation effect play an important role in space related technology. The effect of radiation on various convective flows under different conditions has been studied by many researchers including Hussain and Takhar (1996), Ahmed and Sarmah (2009), Rajesh and Varma (2010) and Ahmed et al (2013). Radiation and free convection on the oscillatory flow past a vertical plate was studied by Mansour (1990). Ahmed and Sinha (2013) studied the soret effect on an oscillatory flow past an infinite vertical porous plate with variable suction.

2. PROBLEM FORMULATION

We now consider an unsteady MHD conductive flow of a viscous incompressible electrically conducting fluid past an infinite vertical porous plate with variable suction under the influence of a uniform transverse magnetic field. We introduced a coordinate system (x', y', z') with $X - axis$ vertically upwards along the plate, $Y - axis$ perpendicular to the plate and directed into the fluid region and $Z - axis$ along the width of the plate. Let the components of velocity along X and Y axes be $u' and\ v'$. Let these velocity components chosen be along the upward direction of the plate and normal to the plate respectively.

Under these assumptions, the equations that describe the physical situation are given by

$$\frac{\partial v'}{\partial y'} = 0 \qquad (2.1)$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \nu\frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) - \nu\frac{u'}{K'} - \frac{\sigma\beta_0^2 u'}{\rho} \qquad (2.2)$$

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{K_T}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q}{\partial y'} \qquad (2.3)$$

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2}{\partial y'^2} - K'_c(C' - C'_\infty) \qquad (2.4)$$

All the physical quantities involved in the above equations are defined in the nomenclature.

The boundary conditions are:

$$u' = 0, T' = T_\infty + \left(1 + \varepsilon e^{i\omega t}\right)(T'_w - T'_\infty), C' = C'_\infty + \left(1 + \varepsilon e^{i\omega t}\right)(C'_w - C'_\infty)\ at\ y = 0$$

$$u' \longrightarrow U' = U_0\left(1 + \varepsilon e^{i\omega t}\right), T' \longrightarrow T'_\infty, C' \longrightarrow C'_\infty\ as\ y = 1 \qquad (2.5)$$

The equation (3.1) yield that the suction velocity at the plate is either a constant or a function of time and we take the suction velocity normal to the plate in the form

$$V = -V_0(1 + \varepsilon A e^{i\omega t}) \qquad (2.6)$$

Where A is a real positive constant, ϵ is a small value less than unity, V_o is a scale of function of suction velocity which is non-zero positive. The negative sign indicates that the suction is towards plate.

The radiative heat flux is given by

$$\frac{\partial q}{\partial y'} = 4\alpha^2(T'_\infty - T') \qquad (2.7)$$

In order to write the governing equations and the boundary conditions in the dimensional form, the following non-dimensional quantities are introduced:

$$u = \frac{u\prime}{V_o}, V = \frac{V\prime}{V_o}, y = \frac{V_o y\prime}{\upsilon}, U = \frac{U\prime}{U_o}, t = \frac{tV_o^2}{4\upsilon}, \boldsymbol{\theta} = \frac{\boldsymbol{T'-T\prime_\infty}}{\boldsymbol{T\prime_w - T_\infty}}, \boldsymbol{C} = \frac{\boldsymbol{C'-C\prime_\infty}}{\boldsymbol{C\prime_w - C\prime_\infty}}, \boldsymbol{S_c} = \frac{\boldsymbol{\upsilon}}{\boldsymbol{D}}, \boldsymbol{M} = \frac{\boldsymbol{\sigma B_0^2 \upsilon}}{\boldsymbol{\rho V_o}}, \boldsymbol{G_r} = \boldsymbol{\upsilon\beta g}\frac{\boldsymbol{(T'-T\prime_\infty)}}{\boldsymbol{V_o^3}}, \boldsymbol{\omega} = \frac{\boldsymbol{4\upsilon\omega\prime}}{\boldsymbol{V_o^2}}, \boldsymbol{K} = \frac{\boldsymbol{K\prime V_o^2}}{\boldsymbol{\upsilon}}, \boldsymbol{P_r} = \frac{\boldsymbol{\upsilon}}{\propto}, \boldsymbol{G_m} = \frac{\boldsymbol{\upsilon\beta^* g(C'_w - C'_\infty)}}{\boldsymbol{V_o^3}}, \boldsymbol{S_r} = \frac{\boldsymbol{D_1(T'_w - T'_\infty)}}{\boldsymbol{C\prime_w - C\prime_\infty}}, \boldsymbol{K_c} = \frac{\boldsymbol{K_c \upsilon}}{\boldsymbol{V_o^2}}$$

(2.8)

In view of the equations (2.6)-(2.8), the equations (2.2)-(2.4) reduce to the following dimensional form:

$$\frac{1}{4}\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G_r\theta + G_m C - uB \ \ where\ B = u(1+M) \qquad (2.9)$$

$$\frac{1}{4\upsilon}\frac{\partial \theta}{\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} + N^2\theta \qquad (2.10)$$

$$\frac{1}{4}\frac{\partial C}{\partial t} - \left(1 + \varepsilon A e^{i\omega t}\right)\frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} - K_c C \qquad (2.11)$$

The corresponding boundary conditions are:

$$u = 0, \theta = 1 + \varepsilon A e^{i\omega t}, C = 1 \ \ at\ y = 0$$

$$u \to U = 1 + \varepsilon e^{i\omega t}, \theta =\to 0, C \to 0\ as\ y \to 1 \qquad (2.12)$$

3. METHOD OF SOLUTION/SOLUTION OF THE PROBLEM

The momentum equation, energy equation and concentration equation can be reduced to the set of ordinary differential equations, which are solved analytically. This can be done by representing the velocity, temperature and concentration of the fluid in the perturbation series as follows

$$u(y,t) = u_0(y) + \varepsilon u_1(y)e^{i\omega t} + o(\varepsilon^2) \qquad (3.1)$$

$$\theta(y,t) = \theta_0(y) + \varepsilon \theta_1(y)e^{i\omega t} + o(\varepsilon^2) \qquad (3.2)$$

$$C(y,t) = C_0(y) + \varepsilon C_1(y)e^{i\omega t} + o(\varepsilon^2) \qquad (3.3)$$

Substituting equations (3.1) – (3.3) into equations (2.9)-(2.12), equating the coefficients of harmonic and non-harmonic term and neglecting the coefficients of higher order of ε^2, we get:

$$u_0''(y) + u_0'(y) - Bu_0(y) = G_r\theta_0(y) + G_mC_0(y) \qquad (3.4)$$

$$u_1''(y) + u_1'(y) - \left(\frac{i\omega}{4} + B\right)u_1(y) = -Au_0'(y) - G_r\theta_1(y) - G_mC_1(y) \qquad (3.5)$$

$$\theta_0''(y) + P_r\theta_0'(y) + N^2P_r\theta_0 y = 0 \qquad (3.6)$$

$$\theta_1''(y) + P_r\theta_1'(y) - \left(\frac{i\omega}{4} - N^2\right)P_r\theta_1(y) = -AP_r\theta_0(y) \qquad (3.7)$$

$$C_0''(y) - (K_cS_c - S_c)C_0(y) = 0 \qquad (3.8)$$

$$C_1''(y) + S_cC_1'(y) - \left(\frac{i\omega}{4} + K_c\right)S_cC_1(y) = -S_cAC_0'(y) \qquad (3.9)$$

The corresponding boundary condition become

$$u_o = 0, u_1 = 0, \theta_1 = 0, C_o \to 1\ C_1 = 1\ \ at\ y = 0$$

$$u_o \to 1, u_1 \to 1, \theta_o \to 0, C_o \to 0, C_1 \to 0\ \ as\ y \to 1 \qquad (3.10)$$

We now solved equations (3.4) – (3.9) under the relevant boundary conditions for the mean flow and unsteady flow separately.

The mean flows are governed by the equations (3.4), (3.6) and (3.8) where u_0, θ_0, C_0 are respectively called the mean velocity, mean temperature and mean species concentration respectively. The unsteady flows are governed by equations (3.5), (3.7) and (3.9) where u_1, $\theta_1$and C_1 are the unsteady components.

These equations are solved analytically under the relevant boundary conditions (3.10) as follows;

Solving equations (3.6) and (3.8) subject to the corresponding relevant boundary conditions in (3.10), we obtain the mean temperature as

$$\theta_0(y) = A_5e^{m_5y} + A_6e^{m_6y} \qquad (3.11)$$

$$C_0(y) = A_1e^{m_1y} + A_2e^{m_2y} \qquad (3.12)$$

Similarly, solving equations (3.7) and (3.9) under the relevant boundary conditions in (3.10), the unsteady temperature becomes

$$\theta_1(y) = A_7e^{m_7y} + A_8e^{m_8y} + B_3e^{m_5y} + B_4e^{m_6y} \qquad (3.13)$$

$$C_1(y) = A_3e^{m_3y} + A_4e^{m_4y} + B_1e^{m_1y} + B_2e^{m_2y} \qquad (3.14)$$

Putting equations (3.11), (3.12) and equations (3.13) and (3.14) into equations (3.4) and (3.5) respectively and using the corresponding boundary conditions in (3.10), we obtain the mean velocity $u_0(y)$ and the unsteady velocity component $u_1(y)$ as follows;

$$u_0(y) = A_9e^{m_9y} + A_{10}e^{m_{10}y} + B_5e^{m_5y} + B_6e^{m_6y} + B_7e^{m_1y} + B_8e^{m_2y} \qquad (3.15)$$

$$u_1(y) = A_{11}e^{m_{11}y} + A_{12}e^{m_{12}y} + B_9e^{m_9y} + B_{10}e^{m_{10}y} + B_{11}e^{m_7y} + B_{12}e^{m_8y} + B_{13}e^{m_3y} + B_{14}e^{m_4y} \quad (3.16)$$

Therefore, the solutions for the velocity, temperature and species concentration profiles are

$$u(y,t) = A_9e^{m_9y} + A_{10}e^{m_{10}y} + B_5e^{m_5y} + B_6e^{m_6y} + B_7e^{m_1y} + B_8e^{m_2y} + \varepsilon[A_{11}e^{m_{11}y} + A_{12}e^{m_{12}y} + B_9e^{m_9y} + B_{10}e^{m_{10}y} + B_{11}e^{m_7y} + B_{12}e^{m_8y} + B_{13}e^{m_3y} + B_{14}e^{m_4y}]e^{i\omega t} \quad (3.17)$$

$$\theta(y,t) = A_5e^{m_5y} + A_6e^{m_6y} + \varepsilon[A_7e^{m_7y} + A_8e^{m_8y} + B_3e^{m_5y} + B_4e^{m_6y}]e^{i\omega t} \quad (3.18)$$

$$C(y,t) = A_1e^{m_1y} + A_2e^{m_2y} + \varepsilon[A_3e^{m_3y} + A_4e^{m_4y} + B_1e^{m_1y} + B_2e^{m_2y}]e^{i\omega t} \quad (3.19)$$

Skin Friction:

The skin friction τ at the plate is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = A_9m_9 + A_{10}m_{10} + B_5m_5 + B_6m_6 + B_7m_1 + B_8m_2 + \varepsilon[A_{11}m_{11} + A_{12}m_{12} + B_9m_9 + B_{10}m_{10} + B_{11}m_7 + B_{12}m_8 + B_{13}m_3 + B_{14}m_4]e^{i\omega t} \quad (3.20)$$

Nusselt number:

The Nusselt number Nu, that is, the rate of heat transfer at the plate is given by

$$Nu = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = -\left[A_5m_5 + A_6m_6 + \varepsilon(A_7m_7 + A_8m_8 + B_3m_5 + B_4m_6)e^{i\omega t}\right] \quad (3.21)$$

Sherwood number:

The Sherwood number, that is, the rate of mass transfer at the plate is given by

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = -\left[A_1m_1 + A_2m_2 + \varepsilon(A_3m_3 + A_4m_4 + B_1m_1 + B_2m_2)e^{i\omega t}\right] \quad (3.22)$$

4. RESULTS AND DISCUSSION

In order to study the effect of heat and mass transfer on an oscillatory MHD mixed convection past an infinite vertical porous plate with variable suction and radiation, the solution for non-dimensional velocity field, temperature and species concentration have been carried out using Matlab by assigning some specific arbitrary value to the different parameters involved in the problem, namely, Hartmann number M, Grashof numbers $Gr\ and\ Gm$, Schmidt number Sc, Prandtl number Pr, chemical parameter K_c, Reynold number Re and Radiation parameter N. The effects of these values are demonstrated through different graphs and table.

Figure 1 illustrates the effect of modified Grashof number Gm on velocity u. it is simulated from this figure that an increase in the modified Grashof number Gm on velocity profile, that is, the fluid motion is accelerated for increasing the permeability of the medium.

Figure 2 demonstrates the effect Grashof number Gr on velocity u. It is inferred from the figure that under the action of the Grashof number Gr, the fluid motion is accelerated. We recall that the Grashof parameter increases means that the free stream velocity is enhanced due to increase in the free stream velocity i.e. the boundary layer velocity is enhanced due to increase in the free stream velocity.

Figure 3 presents the effect of Hartman number M on velocity u. it is inferred from this figure that an increase in the Hartman number M lead to the decrease of the fluid velocity.

Figure 4 exhibits the effect of Radiation parameter N on velocity u. It is simulated from this figure that an increase in the Radiation parameter N lead to the decrease of the fluid motion.

Figure 5 shows the effect of Reynold number Re on velocity u. It means that if the Reynold number Re decrease the fluid velocity will decrease.

Figure 6 illustrates the effect of Radiation parameter N on temperature θ. It is inferred that an increase in the Radiation parameter N will lead to the decrease in the temperature θ.

Figure 7 show the effect of Prandtl number Pr on temperature θ. It is simulated from this figure that an increase to the Prandtl number lead to the decrease of the temperature θ.

Figures 8 and 9 illustrate the effect of Chemical parameter K_c and the Schmidt number Sc on the Species Concentration. It is simulated from this figure that an increase in the Chemical parameter K_c and Schmidt number Sc lead to the decrease of the Species Concentration.

Table 1 shows the variation of Skin friction τ, Nusselt number Nu and Sherwood number Sh with time t. It can be observed that the skin friction increases with increase in time, the Nusselt number decreases with increase in time and the Sherwood number decreases with increases with increase in time.

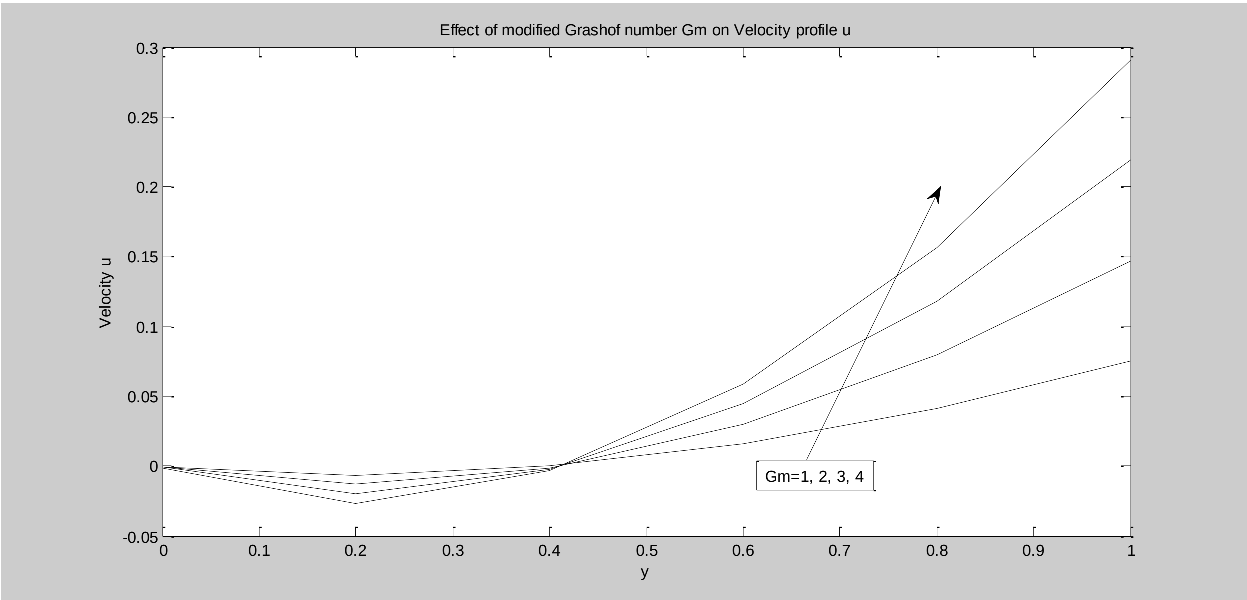

Figure 1: Effect of modified Grashof number Gm on Velocity u $with$ $Gr = 1, Re = 1, Ha = 1, \varepsilon = 0.02, t = 1,\ Sc = 1, N = 1, M = 1\ Kc = 1\ and\ w = 1$.

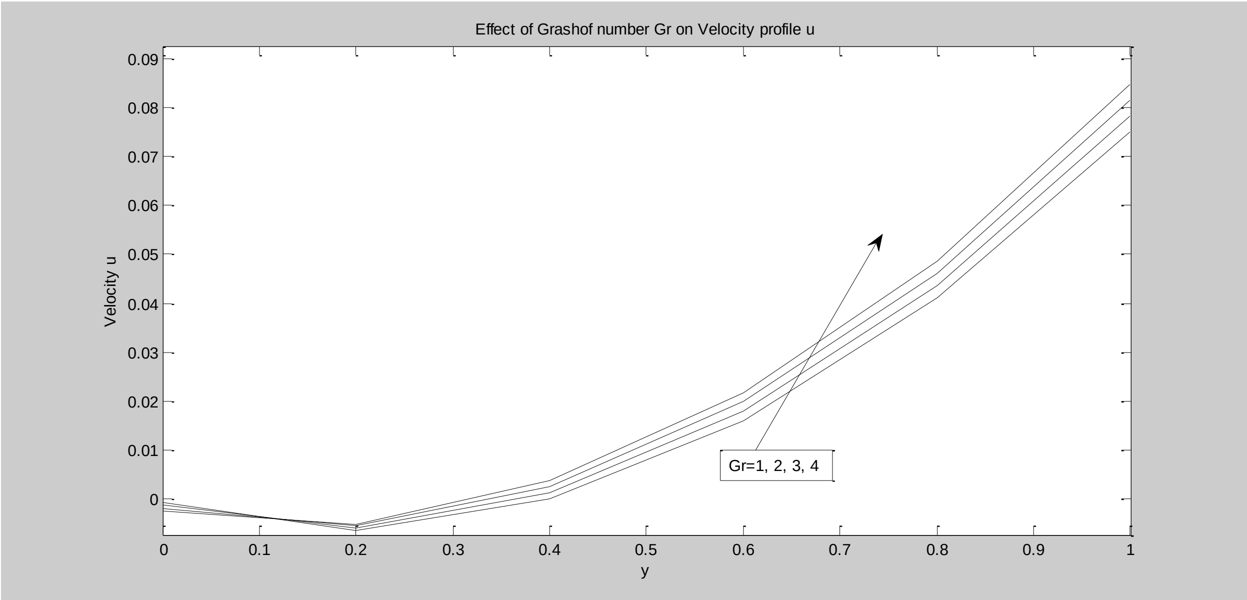

Figure2: Effect of Grashof number G_r on velocity profile u $with$ $G_m = 1, Re = 1, Ha = 1, \varepsilon = 0.02, t = 1,$ $Sc = 1, N = 1, M = 1, Kc = 1$ and $w = 1.$

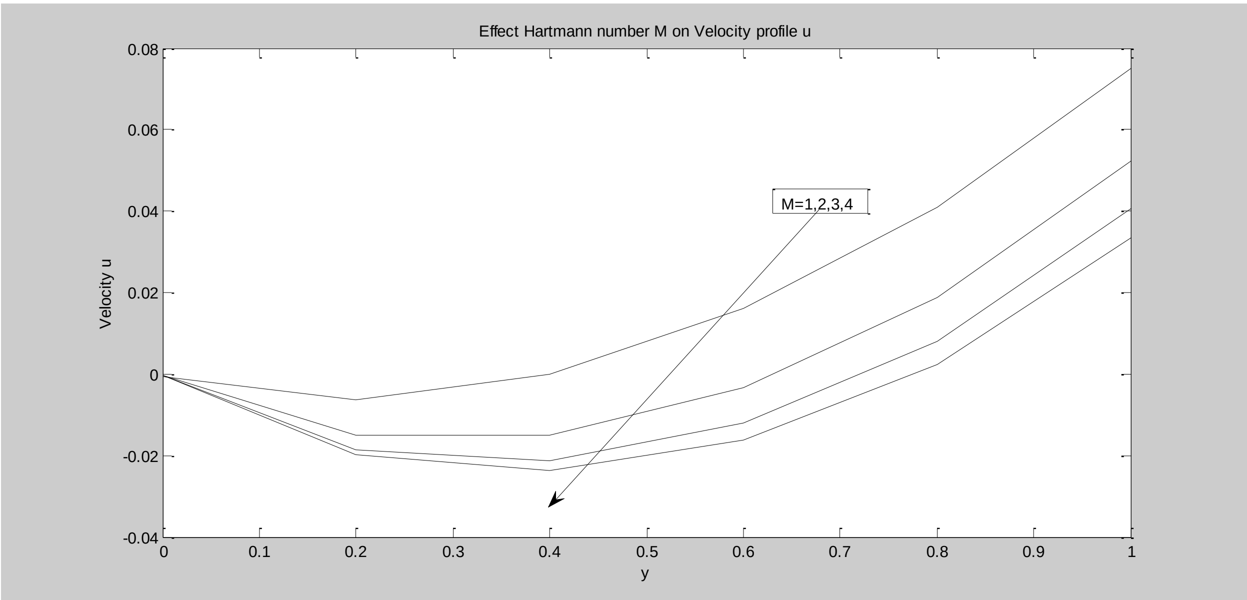

Figure 3: Effect of Hartman number M on velocity profile u $with$ $Gr = 1, G_m = 1, Re = 1, Ha = 1, \varepsilon = 0.02, t = 1,$ $Sc = 1, N = 1, Kc = 1$ and $w = 1.$

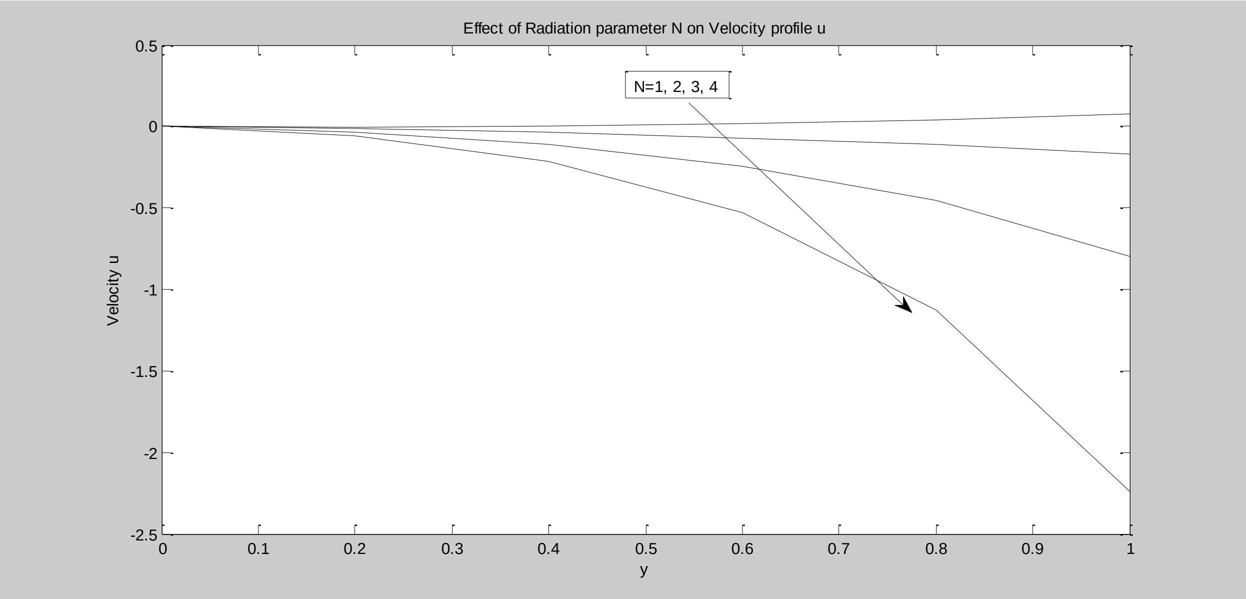

Figure 4: Effect of Radiation parameter N on velocity profile $u\ with\ Gr = 1, G_m = 1, Re = 1, Ha = 1, \varepsilon = 0.02, t = 1,\ Sc = 1, M = 1, Kc = 1\ and\ w = 1.$

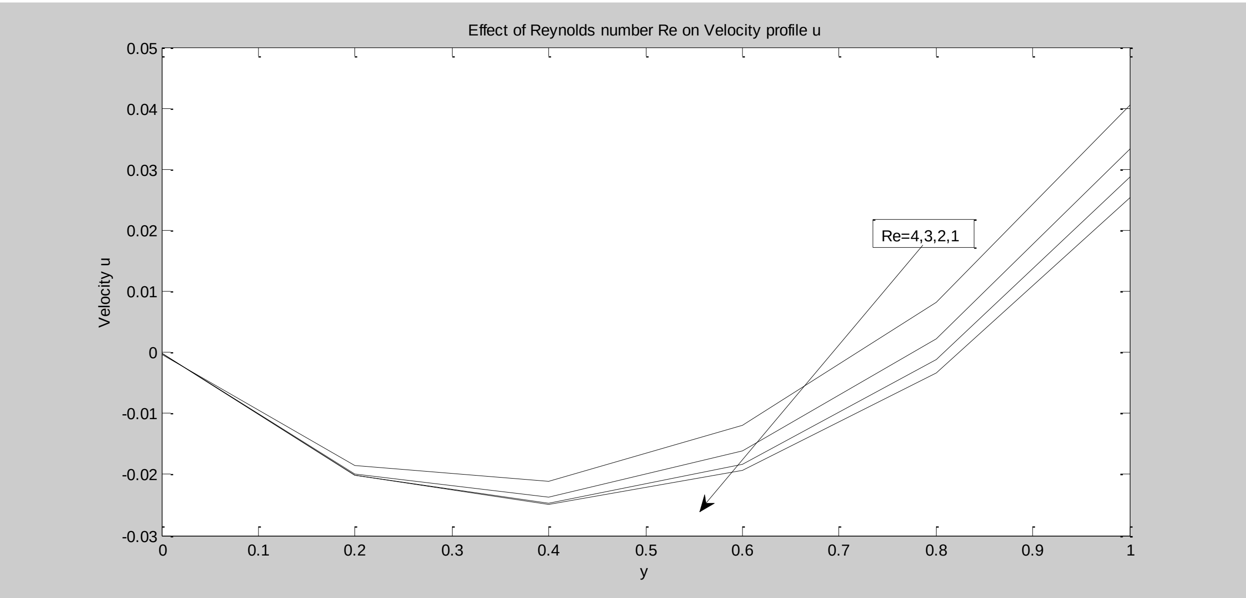

Figure 5: Effect of Reynold number on velocity profile $u\ with\ Gr = 1, G_m = 1, Ha = 1, \varepsilon = 0.02, t = 1,\ Sc = 1, N = 1, M = 1, Kc = 1\ and\ w = 1.$

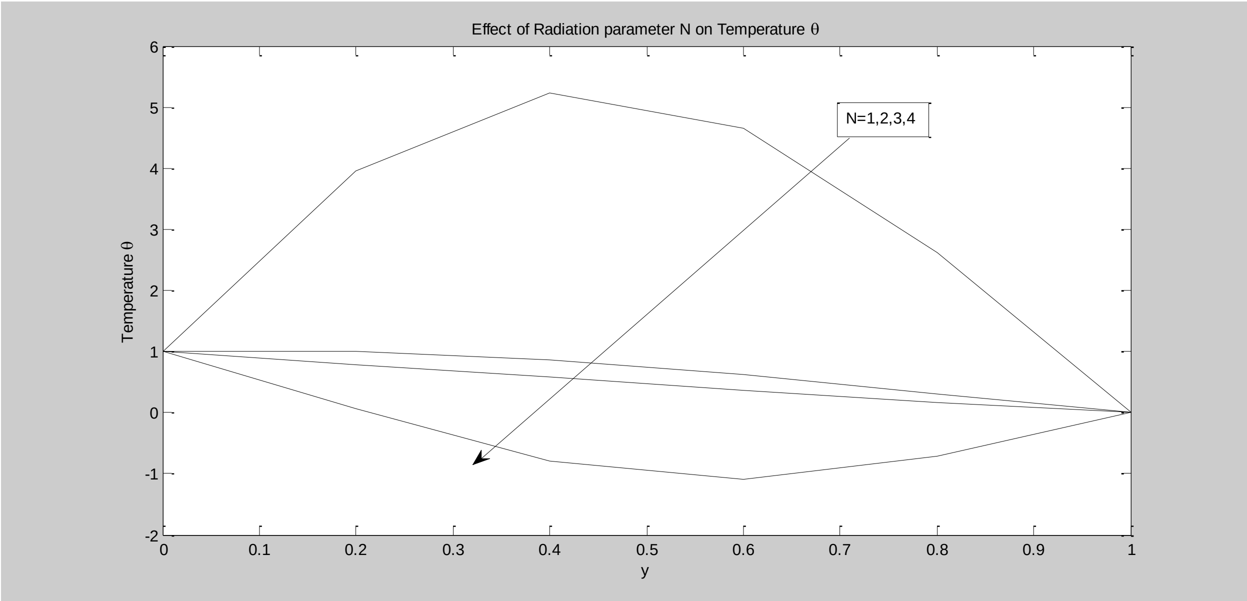

Figure 6: Effect of Radiation parameter N on Temperature θ $with$ $Gr = 1, G_m = 1, Re = 1, Ha = 1, \varepsilon = 0.02, t = 1,$ $Sc = 1, M = 1, Kc = 1$ and $w = 1$.

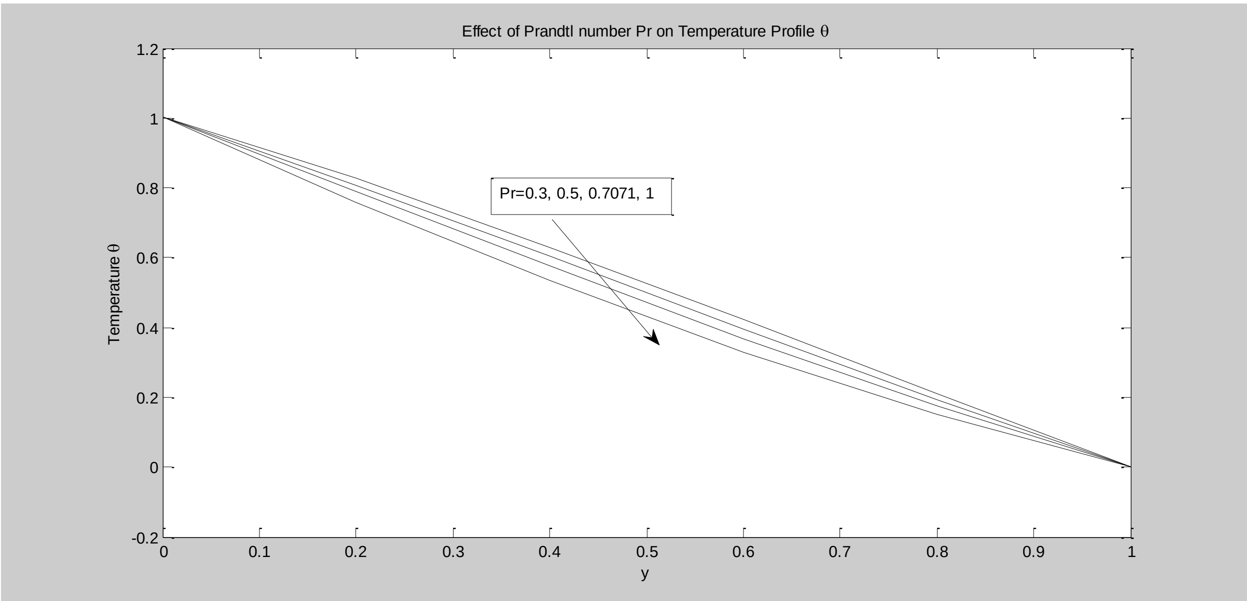

Figure 7: Effect of Prandtl number P_r on temperature profile θ $with$ $Gr = 1, G_m = 1$ $Re = 1, Ha = 1, \varepsilon = 0.02, t = 1,$ $Sc = 1, M = 1, N = 1, Kc = 1$ and $w = 1$.

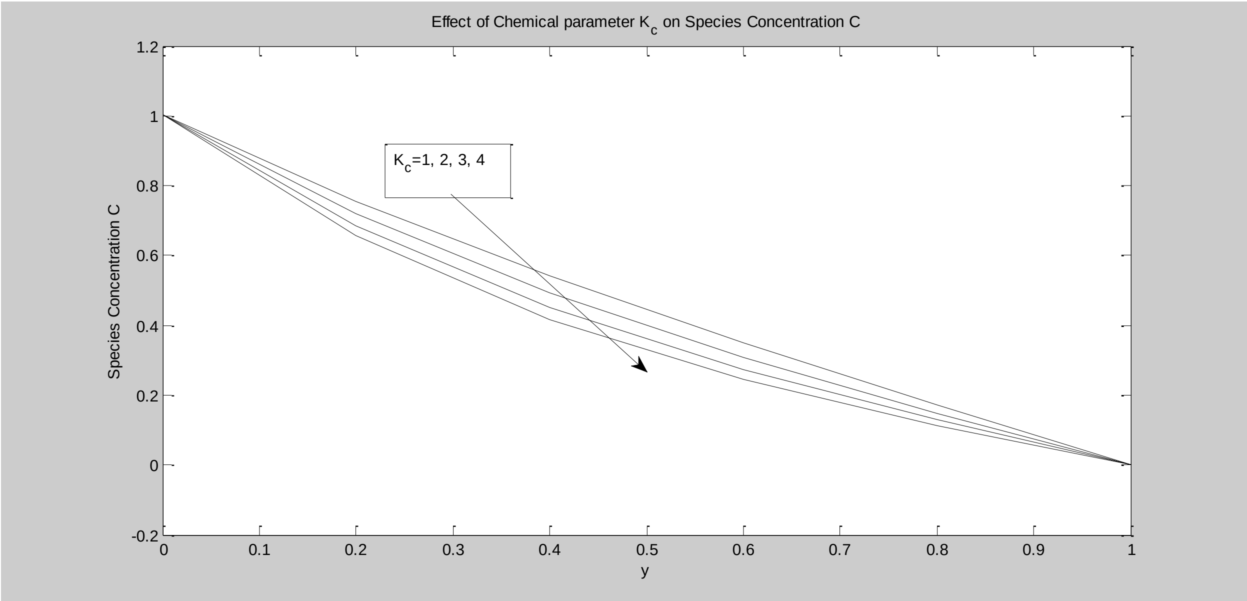

Figure 8: Effect of Chemical parameter K_c on Species Concentration C $with\ Gr = 1, G_m = 1\ Re = 1, Ha = 1, \varepsilon = 0.02, t = 1,\ Sc = 1, N = 1, M = 1, and\ w = 1.$

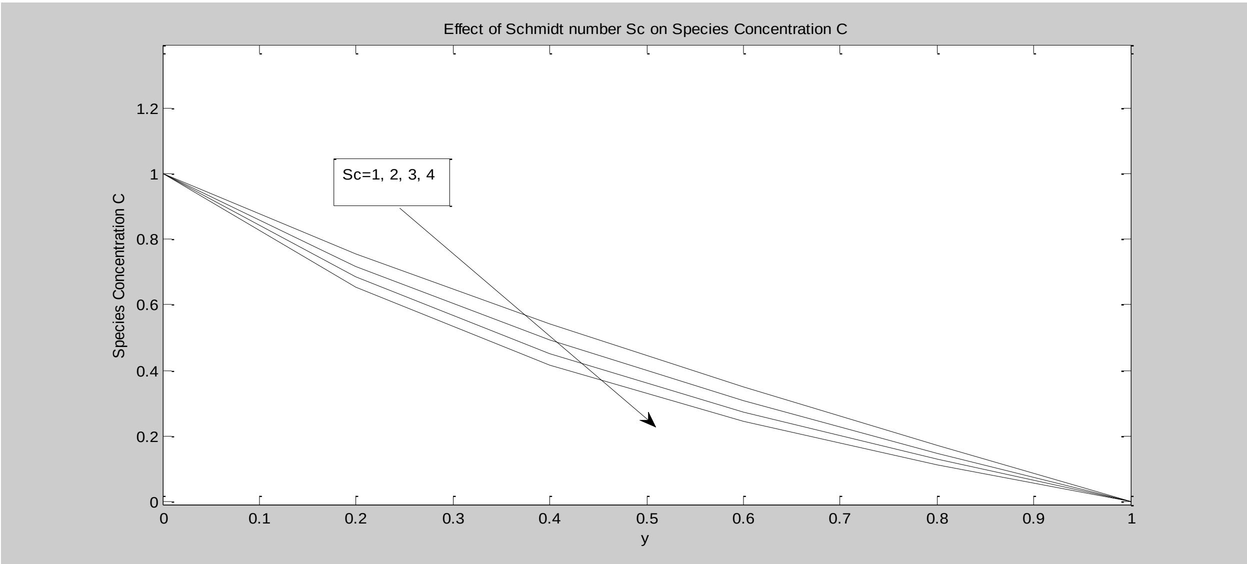

Figure 9: Effect of Schmidt number S_c on Species Concentration C $with\ Gr = 1, G_m = 1, Re = 1, Ha = 1, \varepsilon = 0.02, t = 1, M = 1, N = 1, Kc = 1\ and\ w = 1.$

Table 1: Variation of Skin friction τ, Nusselt number Nu and Sherwood number Sh with time t

t	τ	Nu	Sh
0.0	0.5138	1.0486	1.7002
0.1	0.5143	1.0485	1.7002
0.2	0.5148	1.0484	1.7001
0.3	0.5152	1.0483	1.7001
0.4	0.5157	1.0482	1.6999
0.5	0.5160	1.0481	1.6996
0.6	0.5164	1.0480	1.6994
0.7	0.5167	1.0478	1.6991
0.8	0.5169	1.0476	1.6988
0.9	0.5171	1.0473	1.6984
1.1	0.5172	1.0470	1.6981

5. SUMMARY AND CONCLUSION

In this section, we studied the effect of heat and mass transfer on an oscillatory mixed convective flow in an infinite vertical porous plate with variable suction and radiation. The governing equations, the momentum, energy and species concentration equations have been written in dimensionless form using dimensionless parameter. The regular perturbation technique has been employed to evaluate and solve the dimensionless velocity u, temperature θ, the species concentration C, Skin friction τ, Nusselt number Nu and Sherwood number Sh.

However our investigation of the problem setup leads to the following conclusions:

- The fluid motion is decelerated under the action of transverse magnetic field and radiation conduction parameter.
- Temperature falls due to effect of radiation and Prandtl numbers.
- The concentration distribution reduced with increase in Schmidt number and chemical parameter.

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Appendix

$m_1 = \sqrt{b_1}, \quad m_2 = -\sqrt{b_1}\ , A_1 = -\frac{e^{m_2}}{e^{m_1}-e^{m_2}}, \ A_2 = 1 - A_1, \ m_3 = \frac{-Sc+\sqrt{Sc^2+4b_2}}{2}\ , \quad m_4 = \frac{-Sc-\sqrt{Sc^2+4b_2}}{2}$ ‘$B_1 = \frac{A_{11}}{m_1^2+Scm_1-b_2}\ , B_2 = \frac{A_{21}}{m_2^2+Scm_2-b_2}\ , D_1 = 1-(B_1+B_2), D_{2=} -(B_1 e^{m_1} + B_2 e^{m_2}), D_3 = D_2 - D_1 e^{m_4}\ , A_3 = \frac{D_3}{e^{m_3}-e^{m_4}}\ , A_4 = D_1 - A_3$

$m_5 = \frac{-Pr+\sqrt{Pr^2-4N^2}}{2}, \ m_6 = \frac{-Pr-\sqrt{Pr^2-4N^2}}{2}, A_5 = \frac{-e^{m_6}}{e^{m_5}-e^{m_6}}, A_6 = 1 - A_5$

$m_7 = \frac{-Pr+\sqrt{Pr^2-4b_3}}{2}\quad , \quad m_8 = \frac{-Pr-\sqrt{Pr^2-4b_3}}{2}, \quad A_{23} = -APrA_5m_5, \quad A_{24} = -APrA_6m_6,$
$B_3 = \frac{A_{23}}{m_5^2+Prm_5-b_3}, B_4 = \frac{A_{24}}{m_6^2+Prm_6-b_3}, D_4 = 1-(B_3+B_4)$

$D_5 = -(B_3 e^{m_5} + B_4 e^{m_6}), A_7 = \frac{D_5 - D_4 e^{m_8}}{e^{m_7}-e^{m_8}}, D_6 = D_5 - D_4 e^{m_8}, A_7 = \frac{D_6}{e^{m_7}-e^{m_8}}$

$A_8 = D_4 - A_7, m_9 = \frac{-1+\sqrt{1-4B}}{2}, \ m_{10} = \frac{-1-\sqrt{1-4B}}{2}, B_5 = \frac{-A_5G_r}{(m_5^2+m_5-B}$

$B_6 = \frac{-A_6G_r}{m_6^2+m_6-B}\ , B_7 = \frac{-A_1G_r}{m_1^2+m_1-B}, \ B_8 = \frac{-A_2G_r}{(m_2^2+m_2-B}\ , D_7 = -(B_5+B_6+B_7+B_8)$

$D_8 = 1-(B_5 e^{m_5} + B_6 e^{m_6} + B_7 e^{m_1} + B_8 e^{m_2}),$

$A_9 = \frac{D_9}{e^{m_9}-e^{m_{10}}}, D_9 = D_8 - D_7 e^{m_{10}}, m_{11} = \frac{-1+\sqrt{1^2+4b_4}}{2}, m_{12} = \frac{-1-\sqrt{1^2+4b_4}}{2}$

$B_9 = \frac{-AA_9m_9}{m_9^2+m_9-b_4}, B_{10} = \frac{-AA_{10}m_{10}}{m_{10}^2+m_{10}-b_4}, B_{11} = \frac{-A_7G_r}{m_7^2+m_7-b_4}\ , B_{12} = \frac{-A_8G_r}{m_8^2+m_8-b_4}$

$B_{13} = \frac{-A_3G_m}{m_3^2+m_3-b_4}, B_{14} = \frac{-A_4G_m}{m_4^2+m_4-b_4}, D_{10} = -(B_9+B_{10}+B_{11}+B_{12}+B_{13}+B_{14})$

$A_{12} = D_{10} - A_{11}\ , D_{11} = 1-(B_9 e^{m_9} + B_{10} e^{m_{10}} + B_{11} e^{m_7} + B_{12} e^{m_8} + B_{13} e^{m_3} + B_{14} e^{m_4})$

$D_2 = D_{11} - D_{10}\ e^{m_{12}}, A_{11} = \frac{D_{12}}{e^{m_{11}}+e^{m_{12}}}\ , A_{12} = D_{10} - A_{11}$