[Volume 2 issue 7 July 2014]
Page No.512-517 ISSN :2320-7167

INTERNATIONAL JOURNAL OF MATHEMATICS AND COMPUTER RESEARCH

Analysis of a Discrete Prey-Predator Model with Prey Harvesting

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Abstract: In this paper, we formulate a discrete Prey – Predator model and introduce harvesting in prey population. The dynamical behavior of the proposed model is investigated through analytical study of the existence of fixed points and their stability on prey harvesting. Stability of the discrete system is investigated Numerical simulations are employed to exhibit the complex dynamics of the discrete model. Bifurcation, period doubling and time plot diagrams are plotted to study the behavior of the model in selected ranges.

Key words: *Harvesting, Fixed points, Stability, Bifurcation and Period doubling.*

INTRODUCTION

The dynamic behavior of the prey-predator interaction is more complex in nature. Population Dynamics has been a prime branch of theoretical ecology. Lotka- Volterra model describes interaction between two species in an ecosystem [5]. Holling introduced more realistic prey-predator models with three kinds of functional responses for different species to model the phenomena of predation. Harvesting has an impact on the population dynamics of a harvested species [3]. The severity of this impact depends on the nature of the implemented harvesting strategy, which may range from extinction to the complete preservation of a population. The study of the population dynamics in harvesting is a subject of mathematical bio-economics [2,9]. In this paper we consider the effect of constant rate of harvesting on the dynamical behavior of interacting species.

DYNAMICAL MODEL AND FIXED POINTS

Discrete time models give rise to more efficient computational models for numerical simulations and they exhibit more complex dynamical behaviors [1,4,7]. In this section, we consider the dynamics of the two-species system consisting of one prey and one predator with harvesting on prey population governed by the following system of difference equations,

$$x(n+1)=rx(n)\left[1-x(n)\right]-\frac{bx(n)y(n)}{c+x(n)}-hx(n)$$

$$y(n+1)=\frac{bx(n)y(n)}{c+x(n)}-ay(n) \qquad ...(1)$$

where r, a, b and $c>0$. Here $x(n)$ and $y(n)$ denote the prey-predator density, r is the intrinsic growth rate of the prey, a is the natural death rate of the predator, h is the harvesting constant and $\frac{bx}{c+x}$ is the predation term in the sense of Holling [5]. All possible non-negative fixed points are,

(i) $E_0=(0,0)$.

(ii) $E_1=\left(\frac{r-(1+h)}{r},0\right)$.

(iii) $E_2=(x^*,y^*)$ where $x^*=\frac{c(1+a)}{b-(1+a)}$ and $y^*=\frac{c}{b-(1+a)}\left[r-(1+h)-r\left(\frac{c(1+a)}{b-(1+a)}\right)\right].$

DYNAMICAL BEHAVIOR OF THE SYSTEM AND STABILITY ANALYSIS

In this section, we analyze the local stability of the non-negative fixed points. The variational matrix of the system (1) at fixed point (x,y) is

$$J(x,y)=\begin{bmatrix} r(1-2x)-\frac{bcy}{(c+x)^2}-h & -\frac{bx}{c+x} \\ \frac{bcy}{(c+x)^2} & \frac{bx}{c+x}-a \end{bmatrix} \qquad ...(2)$$

The characteristic equation of the Jacobian matrix (2) can be written as

$$f_J(\lambda)=\lambda^2-\mathrm{Tr}_J\,\lambda+\mathrm{Det}_J \qquad ...(3)$$

where,

$\mathrm{Tr}_J=r(1-2x)-\frac{b}{c+x}\left[\frac{cy}{c+x}-x\right]-(a+h)$ and $\mathrm{Det}_J=\left[r(1-2x)-\frac{bcy}{(c+x)^2}-h\right]\left[\frac{bx}{c+x}-a\right]+\frac{b^2cxy}{(c+x)^3}.$ The system (1) is dissipative dynamical system if $|\mathrm{Det\,J}|<1,$ conservative dynamical system if and only if $|\mathrm{Det\,J}|=1.$ Otherwise, the system (1) undissipated dynamical system [1,6].

Proposition 1: The Prey-Predator co-extinction fixed point E_0 is stable if $r<1+h$ and $a<1.$

Proof: At $E_0,$ the Jacobian matrix (2) is of the form $J(E_0)=\begin{bmatrix} r-h & 0 \\ 0 & -a \end{bmatrix}.$

The corresponding eigenvalues of the Jacobian matrix $J(E_0)$ are $\lambda_1=r-h$ and $\lambda_2=-a.$ Fixed point E_0 is stable if $|\lambda_{1,2}|<1.$ From this, we obtain $|r-h|<1$ and $|-a|<1,$ so that $r<1+h$ and $a<1.$ Fixed point E_0 is unstable if the following conditions hold.

1. $r>1+h$ and $a>1$ (Source).
2. $r>1+h$ and $a<1$ $($or $r<1+h$ and $a>1)$ (Saddle).
3. Either $r=1+h$ or $a=1$ (Non-hyperbolic).

Proposition 2: The Predator extinction fixed point E_1 is stable if $1+h<r<3+h$ and $r<\frac{(1+h)[b-(1+a)]}{b-(1+a)(1+c)}.$

Proof: At E_1, the Jacobian Matrix (2) is of the form

$$J(E_1)=\begin{bmatrix} 2+h-r & -\dfrac{b[r-(1+h)]}{r(1+c)-(1+h)} \\ 0 & \left[\dfrac{b[r-(1+h)]}{r(1+c)-(1+h)}\right]-a \end{bmatrix}.$$

Hence the eigenvalues of the above Jacobian matrix $J(E_1)$ are $\lambda_1=2+h-r$ and $\lambda_2=\dfrac{b[r-(1+h)]}{r(1+c)-(1+h)}-a$. From the condition of sink, we obtain $|2+h-r|<1$ and $\left|\dfrac{b-[r-(1+h)]}{r(1+c)-(1+h)}-a\right|<1$, so that $1+h<r<3+h$ and $r<\dfrac{(1+h)[b-(1+a)]}{b-(1+a)(1+c)}$. Fixed point E_1 is unstable if the following conditions hold.

1. $r>3+h$ and $r>\dfrac{(1+h)[b-(1+a)]}{b-(1+a)(1+c)}$ (Source).
2. $r>3+h$ and $r<\dfrac{(1+h)[b-(1+a)]}{b-(1+a)(1+c)}$ or $r<3+h$ and $r>\dfrac{(1+h)[b-(1+a)]}{b-(1+a)(1+c)}$ (Saddle).
3. Either $r=3+h$ or $r=\dfrac{(1+h)[b-(1+a)]}{b-(1+a)(1+c)}$ (Non-hyperbolic).

Proposition 3: In this proposition, we discuss the stability of the interior point E_2.

At E_2, the Jacobian matrix (2) is of the form $J(E_2)=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & 1 \end{bmatrix}$, where

$$a_{11}=r\left[1-2\left\{\frac{c(1+a)}{b-(1+a)}\right\}\right]-\left\{1-\frac{(1+a)}{b}\right\}\left[r-(1+h)-r\left\{\frac{c(1+a)}{b-(1+a)}\right\}\right]-h,$$

$$a_{12}=-(1+a) \text{ and } a_{21}=\left\{1-\frac{(1+a)}{b}\right\}\left[r-(1+h)-r\left\{\frac{c(1+a)}{b-(1+a)}\right\}\right].$$

By (3), the characteristic equation of the Jacobian matrix $J(E_2)$ is

$$f_{J(E_2)}(\lambda)=\lambda^2-\mathrm{Tr}_{J(E_2)}\lambda+\mathrm{Det}_{J(E_2)} \qquad ...(4)$$

where,

$$\mathrm{Tr}_{J(E_2)}=r\left[1-2\left\{\frac{c(1+a)}{b-(1+a)}\right\}\right]-\left\{1-\frac{(1+a)}{b}\right\}\left[r-(1+h)-r\left\{\frac{c(1+a)}{b-(1+a)}\right\}\right]-h+1 \text{ and}$$

$$\mathrm{Det}_{J(E_2)}=r\left[1-2\left\{\frac{c(1+a)}{b-(1+a)}\right\}\right]+a\left\{1-\frac{(1+a)}{b}\right\}\left[r-(1+h)-r\left\{\frac{c(1+a)}{b-(1+a)}\right\}\right]-h.$$

By using Jury conditions, the interior point E_2 is sink if $r>\dfrac{A}{B}$ and $r<\dfrac{C}{D}$; Source if $r>\dfrac{A}{B}$ and $r>\dfrac{C}{D}$; Saddle if $r<\dfrac{A}{B}$; and Non-hyperbolic if $r=\dfrac{A}{B}$; where,

$$A = \left[b-(1+a)\right]\left[2b(h-1)+(1+h)(a-1)\{b-(1+a)\}\right];$$
$$B = \left[b-(1+a)(1+c)\right]\left[2b+(a-1)\{b-(1+a)\}\right]-2bc(1+a);$$
$$C = \left[b-(1+a)\right]\left[(1+h)\{b+a\left[b-(1+a)\right]\}\right]; \text{ and}$$
$$D = \left[b-(1+a)(1+c)\right]\{b+a\left[b-(1+a)\right]\}-bc(1+a).$$

I. Bifurcation Analysis

In this section, we provide bifurcation diagram and time plots of the system (1) in particular ranges which are plotted by using the software MATLAB [8].

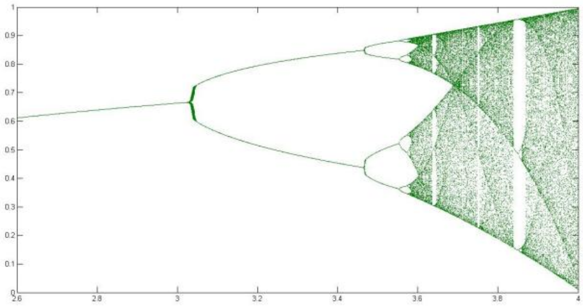

Figure-1

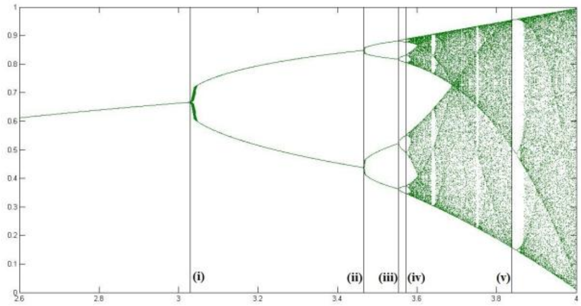

Figure-2

The bifurcation diagram is plotted in the range $r_0 < r < r_S$, where bounded solutions can occur. Here we fix values $a = 0.01, b = 1.75, c = 0.52,\ h = 0.01$ and the intrinsic growth rate 'r' is taken between 2.6 and 4. Between $r_0 = 2.6$ and $r_1 = 3.02$, stationary regimes should be considered as a periodic-1 period orbit. This limit set from r_0 to r_1 consists of a single value. At $r_1 = 3.02$ a bifurcation occurs and the population density oscillates between two values when 'r' changes from $r = 3.02$ to $r = 3.465$. So that $r = r_1$, gives birth to a periodic-2 period orbit and the bifurcation diagram has two branches.

At $r = r_2 = 3.465$, the number of bifurcation doubled and there are four branches occurred in the bifurcation diagram. So that the population density oscillates between four values. Hence we obtain, a periodic-4 period orbit at $r = 3.465$. At $r = r_1$ and $r = r_2$ are the first two members of an infinite series where the periodic doubling bifurcation occurred are known as the period-doubling cascade. The bifurcation at $r_3 = 3.553$, gives the birth to a periodic-8 period orbit and $r_4 = 3.574$ leading to a periodic-16 period orbit which is easily seen in the bifurcation figure-2. At $r = r_5$ is hardly visible and the following ones are completely indiscernible to the naked eye. For every integer $n \geq 0$, the period orbit 2^n can be created at every period-doubling cascade.

In figure-2, from left to right, the vertical lines denote the birth of (i) a period-2 orbit (ii) a period-4 orbit, (iii) a period-8 orbit, (iv) a period-16 orbit and (v) the starting point of a period-3 window.

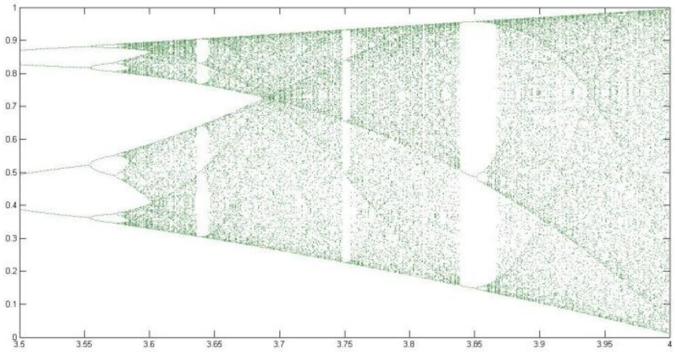

Figure-3

The period-doubling cascade is one of the best known routes to chaos and it can be observed in many low dimensional systems. In figure-3, the large periodic window, which corresponds to the domain of stability of a period-3 orbit, is clearly seen to begin at $r = 3.838,$ well inside the chaotic zone. In figure-3, we can see that periodic windows are clearly visible to the naked eye only for very low periods.

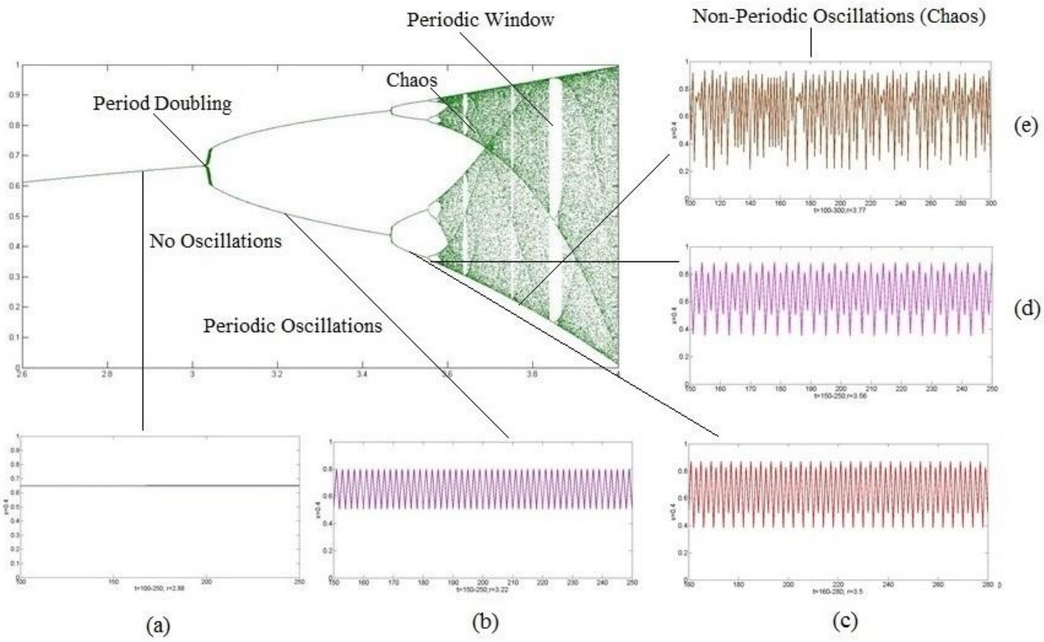

Figure-4

Orbit diagram

In figure-4, we observed that (a) Stationary regime at $r = 2.88$, (b) Periodic regime of period-1 (Uniform oscillations) at $r = 3.22$, (c) Periodic regime of period-3 at $r = 3.5$ (d) Periodic regime of period-7 at $r = 3.56$ and (e) Non-periodic oscillations occur in chaotic region at $r = 3.77$.

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