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Oscillation Solutions Of Third Order Nonlinear Difference Equations With Delay

Dr. P. Mohankumar[1], V.Ananthan[2] and A.Ramesh[3]

. [1]Professor of Mathematics, Aaarupadaiveedu Institute of Techonology, Vinayaka Mission University, Kanchipuram, Tamilnadu, India

2 Asst.Professor of Mathematics, Aaarupadaiveedu Institute of Techonology, Vinayaka Mission University, Kanchipuram, Tamilnadu, India

[3]Senior Lecturer in Mathematics, District Institute of Education and Training, Uthamacholapuram, Salem-636 010, Tamilnadu India

Abstract

Sufficient conditions for the oscillation of some Third Order nonlinear difference equations of the form

$\Delta\,(r_n\,\Delta^2\,x_n) + q_n\ f\,(x_{n-\tau_n}) = 0, n = 0,1,2,\ldots$ (1)

where Δ denotes the forward difference operator. $\Delta\,v_n = v_{n+1} - v_n$ $\{q_n\}$ is a sequence of real numbers, $\{\tau_n\}$ is a sequence of integers are established.

Keywords: Oscillation, Difference Equations , Neutral, Delay, Schawarz's inequality.

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1. Introduction

In this note we consider the nonlinear difference equation of the form

$\Delta\,(r_n\,\Delta^2\,x_n) + q_n\ f\,(x_{n-\tau_n}) = 0, n = 0,1,2,\ldots$ (1)

where Δ denotes the forward difference operator. $\Delta\,v_n = v_{n+1} - v_n$ $\{q_n\}$ is a sequence of real numbers, $\{\tau_n\}$ is a sequence of integers such that

(C_1): $\lim_{n\to\infty}(n - r_n) = \infty$, where $\{r_n\}$ is a sequence of positive numbers and

(C_2): $R_n = \sum_{k=0}^{n-1}\frac{1}{rk} \to \infty$ as $n \to \infty$.

(C_3): f : $R \to R$ is a continuous with $u\,f\,(u) > 0$ $(u \neq 0)$.

By a solution of Equation (1) we mean a sequence (x_n) which is defined for $N \geq \min_{i\geq 0}(i - r_i)$ and satisfies Equation (1) for all large n. A nontrivial solution (x_n) of (1) is said to be oscillatory if for every $n_0 > 0$ there exists $n \geq n_0$ such $x_n\,x_{n+1} \leq 0$. Otherwise it is called non oscillatory.

In several recent papers the oscillatory behaviour of solution of non linear difference equations has been discussed e.g. see [1] – [8]. Our purpose in this paper is to give the sufficient conditions for the oscillation of solutions of Equation (1). The results obtained here extend those in [8].

2. Objective

- To find the Oscillation Solutions of Third Order Nonlinear Difference Equations with Delay

3. Results and Discussion

Theorem.3.1.

Assume that

(C_4): $q_n \geq 0$ and $\sum_{n=1}^{\infty} q_n = \infty$,

(C_5): $\lim_{|u| \to \infty} \inf |f(u)| > 0$.

Then every solution of equation (1) is oscillatory

Proof:

Assume, that equation (1) has non oscillatory solution $\{x_n\}$, and we assume that (x_n) is eventually positive. Then there is a positive integer n_0 such that

$x_{n-\tau_n} > 0 \; for \; n \geq n_0$ ....(2)

From the Equation (1) we have

$\Delta (r_n \Delta^2 x_n) = -q_n f(x_{n-\tau_n}) \leq 0$, $n \geq n_0$, and so $(r_n \Delta^2 x_n)$ is an eventually non increasing sequence. We first show that $r_n \Delta^2 x_n \geq 0$ for $n \geq n_0$

In fact, if there is an $n_1 \geq n_0$ such that $r_n \Delta^2 x_{n1} = c < 0$ and $r_n \Delta^2 x_n \leq c$ for $n \geq n_1$

that is $\Delta^2 x_n \leq \frac{c}{rn}$ and

hence $\Delta x_n \leq x_{n1} + c \sum_{k=n1}^{n-1} \frac{1}{rk}$

$$x_n \leq \sum_{s=m_1}^{m-1} x\, m_1 + c \sum_{s=m_1}^{m-1} \sum_{k=n}^{n-1} \frac{1}{rks} + x_{n_2} \text{as } n \to \infty,\ m \to \infty$$

which contradicts the fact that $x_n > 0$ for $n \geq n_1$. Hence $r_n \Delta x_n \geq 0$ for $n \geq n_0$

Therefore we obtain $x_n - r_n > 0 \; \Delta^2 x_n \geq 0 \; \Delta (r_n \Delta^2 x_n) \leq 0$ for $n \geq n_0$

Let $L = \lim_{n\to\infty} x_n$

Then $L > 0$ is finite or infinite.

Case 1.

$L > 0$ is finite.

From the continuity of function f (u) we have $\lim_{n\to\infty} f(x_n - r_n) = f(L) > 0$. Thus we may choose a positive integer $n_3 \geq n_0$ such that

$f(x_n - r_n) > \frac{1}{2}\ f(L)\ n \geq n_3$............................(3)

By substituting (3) into Equation (1) we obtain

$\Delta(r_n \Delta^2 x_n) + \frac{1}{2}\ f(L)\ q_n \leq 0\ n \geq n_3$. .... (4)

Summing up both sides of (4) from n_3 to n ($\geq n_3$),

we obtain $r_{n+1} \Delta x_{n+1} - r_{n_3}\ \Delta x_{n_3} + \frac{1}{2}\ f(L) \sum_{i=n_3}^{n} q_i \leq 0$

and so $\frac{1}{2}\ f(L)\ \sum_{i=n_3}^{n} q_i \leq r_{n_3}\ \Delta^2 x_{n_3}\ \ n \geq n_3$ contradicts

Case 2.

$L = \infty$

For this case, from the condition (C_1)

we have $\lim_{n\to\infty} inf\ (x_n - r_n) > 0$ and so we may choose a positive constant c and a positive integer n_4 sufficiently large such that

$f(x_n - r_n) \geq c$ for $n \geq n_4$. ....(5)

Substituting (5) into Equation (1) we have $\Delta(\ r_n\ \Delta^2 x_n) + cq_n \leq 0\ n \leq n_4$.

Using the similar argument as that of Case 1 we may obtain a contradiction to the condition (C_1). This completes the proof.

Theorem 3.2:

Assume, that

(C_6): $q_n \geq 0$ and $\sum_{n=0}^{\infty} R_n q_n = \infty$, then every bounded solution of (1) is oscillatory.

Proof:

Proceeding as in the proof of Theorem 1 with assumption that (x_n) is a Bounded non oscillatory solution of (1) we get the inequality (4) and so we obtain

$R_n\ \Delta(\ r_n\ \Delta^2 x_n) + \frac{1}{2}\ f(L) R_n q_n \leq 0\ n \geq n_3$.............................(6)

It is easy to see that

$R_n\ \Delta(r_n\ \Delta^2 x_n) \geq \Delta(\ R_n r_n\ \Delta^2 x_n) - r_n\ \Delta^2 x_n\ \Delta R_n$......................(7)

From inequalities (6) and (7) we deduce

$\sum_{k=n3}^{n} \Delta\ (R_k r_k \Delta^2 x_k) - \sum_{k=n3}^{n} \Delta^2 x_k + \frac{1}{2} f(L) \sum_{k=n3}^{n} R_k q_k \leq 0\ n \geq n_3$

which implies $\frac{1}{2}\ f(L)\sum_{k=n_3}^{n} R_k q_k \leq x_{n+1} + R_{n3}\, r_{n_3}\ \ \Delta^2 x_{n_3} - x_{n_3}$ $n \geq n_3$ Hence there exists a constant c such that $\sum_{k=n_3}^{n} R_k q_k \leq c$ for all $n \geq n_3$. contrary to the assumption of the theorem.

Theorem 3.3: Assume that

(C_7): ($n- r_n$) is non decreasing, where $r_n \in \{0,1,2, ...\}$, there is a subsequence of (r_n),

say (r_{n_k}) such that $r_{n_k} \leq 1$ for k = 0,1,2,....,

(C_8): $\sum_{n=0}^{\infty} q_n = \infty$,

(C_9): f is non decreasing and there is a nonnegative constant M such that

$\lim_{u\to 0} sup \frac{u}{f(u)}$ = M...................... (8)

Then the difference ($\Delta^2 x_n$) of every solution (x_n) of Equation (1) oscillates.

Proof:

If not, then Equation (1) has a solution (x_n) such that its difference ($\Delta^2 x_n$) is non oscillatory. Assume that the sequence ($\Delta^2 x_n$) is eventually negative.

Then there is positive integer n_0 such that $\Delta^2 x_n < 0$ $n > n_0$. and so (x_n) decreasing for $n \geq n_0$ which implies that (x_n) is also non oscillatory.

Set

$w_n = \frac{r_n \Delta^2 x_n}{f(x_n - \tau_n)}$ $n \geq n_1 \geq n_0$.............................(9) then

$\Delta w_n = \frac{r_{n+1}\Delta^2 x_{n+1}}{f(x_{n+1} - \tau_{n+1})} - \frac{r_n \Delta^2 x_n}{f(x_n - \tau_n)}$

$= \frac{\Delta r_n \Delta^2 x_n}{f(x_n - \tau_n)} + r_{n+1}\, \Delta^2 x_{n+1} \frac{f(x_n - \tau_n) - f(x_{n+1} - \tau_{n+1})}{f(x_{n+1} - \tau_{n+1}) f(x_n - \tau_n)}$.......................(10)

$\leq \frac{\Delta r_n \Delta^2 x_n}{f(x_n - \tau_n)} = q_n$, $n \geq n_1$.

Summing up both sides of (10) from n_1 to n, we have

$w_{n+1} - w_{n_1} \leq \sum_{i=n_1}^{n} q_i$ and, by (vi) we get

$\lim_{n\to\infty} w_n = -\infty$,............................(11) Which implies that eventually

$f(x_n - r_n) > 0$ ....(12)

and therefore $x_n - r_n > 0$. By (11), we can choose n_2 ($\geq n_1$)

such that $W_n \leq -(M+1)$, $n \geq n_2$.

$r_n \Delta^2 x_n + (M+1) f(x_n - r_n) \leq 0$, $n \geq n_2$...........................(13)

Set $\lim_{n\to\infty} x_n = L$

Then $L \geq 0$. Now we prove that $L = 0$. If $L > 0$ then we have

$\lim_{n\to\infty} f(x_n - r_n) = f(L) > 0$

By the continuity of $f(u)$. Choosing an n_3 sufficiently large, such that

$f(x_n - r_n) > \frac{1}{2} f(L)$ $n \geq n_3$ ....(14)

and substituting (14) into (13) we have

$\Delta^2 x_n + \frac{1}{2rn}(M+1) f(L) \leq 0$ $n \geq n_3$................................. (15)

Summing up both sides of (15) from n_3 to n we get

$x_{n+1} - x_{n_3} + \frac{1}{2}(M+1) f(L) \sum_{i=n_3}^{n} \frac{1}{r_i} \leq 0$ which implies that $\lim_{n\to\infty} x_n = -\infty$ This contradicts (12). Hence $\lim_{n\to\infty} x_n = 0$.

By the assumptions we have

$\lim_{n\to\infty} sup \frac{x_n - \tau_n}{f(x_n - \tau_n)} \leq M$. From this we can choose n_4 such that

$\frac{x_n - \tau_n}{f(x_n - \tau_n)} < M+1$, $n \geq n_4$ That is $x_n - r_n < (M+1) f(x_n - r_n)$ $n \geq n_4$ and

so from (13) we get

$r_n \Delta^2 x_n + x_n - r_n < 0, n \geq n_4$........................(16)

In particular, from (16) for a subsequence (r_{nk}) satisfying the condition (v),

we have $x_{n_k+1} - x_{n_k} + x_{n_k} - r_{n_k} \leq r_{n_k}(x_{n_k+1} - x_{n_k}) + x_{n_k} - r_{n_k} < 0$,

for k sufficiently large, which implies that $0 < x_{n_k+1} + (x_{n_k} - r_{n_k} - x_{n_k}) < 0$ for all large k. This is a contradiction. The case that ($\Delta^2 x_n$) is eventually positive can be treated in a similar fashion and so the proof of Theorem 3.3 is completed.

4. CONCLUSION

The Oscillatory Properties Third Order Nonlinear Neutral Delay Difference Equation it become Oscillate using Schwarz's Inequality

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