

# Some Results on Prime Labeling Of Graphs

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## ABSTRACT

A Graph  $G$  with  $n$  vertices is said to admit prime labeling if its vertices can be labeled with distinct positive integers not exceeding  $n$  such that the labels of each pair of adjacent vertices are relatively prime. A graph  $G$  which admits prime labeling is called a prime graph. In this paper we investigate prime labeling of some graphs related to helm  $H_n$ , Gear graph  $G_n$ , Crown  $C_n^*$  and star  $S_n$ . We discuss prime labeling in the context of graph operation namely duplication.

**Keywords:** Graph Labeling, Prime Labeling, Duplication, Prime Graph.

## 1. INTRODUCTION

In this paper, We consider only finite simple undirected graph. The graph  $G$  has vertex set  $V = V(G)$  and edge set  $E = E(G)$ . The set of vertices adjacent to a vertex  $u$  of  $G$  is denoted by  $N(u)$ . For notations and terminology we refer to Bondy and Murthy[1].

The notion of prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout [7]. Two integers  $a$  and  $b$  are said to be relatively prime if their greatest common divisor is 1. Relatively prime numbers play an important role in both analytic and algebraic number theory. Many researchers have studied prime graph. Fu.H [3] has proved that the path  $P_n$  on  $n$  vertices is a prime graph. Deretsky et al [2] have prove that the cycle  $C_n$  on  $n$  vertices is a prime graph. Around 1980 Roger Etringer conjectured that all trees have prime labeling which is not settled till today.

The Prime labeling for planar grid is investigated by Sundaram et al [6], Lee.S.et.al [4] has proved that the wheel  $W_n$  is a prime graph if and only if  $n$  is even.

Duplication of a vertex  $v_k$  of a graph produces a new graph  $G_k$  by adding a vertex  $v'_k$  with  $N(v'_k) = N(v_k)$ .

The graph obtained by duplicating all the vertices of a graph  $G$  is called duplication of  $G$ .

In [8] S. K. Vaidhya and K. K. Kanmani have proved that the graphs obtained by identifying any two vertices, duplicating arbitrary vertex and switching of any vertex in cycle  $C_n$  admit prime labeling.

The helm  $H_n$  is a graph obtained from a wheel by attaching a pendant edge at each vertex of the  $n$ -cycle.

The crown graph  $C_n^*$  is obtained from a cycle  $C_n$  by attaching a pendent edge at each vertex of the  $n$ -cycle.

The gear graph  $G_n$  is, the graph obtained from wheel  $W_n = C_n + K_1$  by subdividing each edge incident with the apex vertex once.

In [5] Meena and Vaithilingam have proved that the graphs obtained by identifying any two vertices, duplicating any arbitrary vertex and switching any vertex in Helm graph admit prime labeling.

In this paper we prove that the graph obtained by duplication of all rim vertices of the Helm  $H_n$ , and all the vertices of the Helm  $H_n$  except the center vertex. The graph obtained by duplicating all the rim vertices of the crown  $C_n^*$  and all the vertices of  $C_n^*$ , the graph obtained by duplicating all the vertices of the gear graph  $G_n$ , except the central vertex and duplication graph of the star graph, are all prime graphs.

## 2. MAIN RESULTS

### Theorem 2.1

The graph  $G$  obtained by duplicating all the vertices in the rim of helm  $H_n$  is a prime graph. if  $n$  is even.

#### Proof.

Let  $V(H_n) = \{c, u_i, v_i / 1 \leq i \leq n\}$

$E(H_n) = \{cu_i, u_i v_i / 1 \leq i \leq n\} \cup \{u_i u_{i+1} / 1 \leq i \leq n - 1\} \cup \{u_1 u_n\}$

Let  $G$  be the graph obtained by duplicating all the rim vertices in  $H_n$  and let the new vertices be  $u'_1, u'_2, \dots, u'_n$ . then,

$$V(G) = \{ c, u_i, v_i, u'_i / 1 \leq i \leq n \}$$

$$E(G) = \{ cu_i, cu'_i, u_i v_i, v_i u'_i / 1 \leq i \leq n \} \cup$$

$$\{ u_i u_{i+1}, u_i u'_{i+1}, u'_i u_{i+1} / 1 \leq i \leq n - 1 \} \cup \{ u_n u_1, u_n u'_1, u'_n u_1 \}$$

$$|V(G)| = 3n + 1, \quad |E(G)| = 7n$$

Define a labeling  $f : V(G) \rightarrow \{1, 2, 3, \dots, 3n + 1\}$  as follows

$$\text{Let } f(c) = 1,$$

$$f(u_i) = 3i - 1, \quad \text{for } 1 \leq i \leq n, \quad i \not\equiv 2 \pmod{5}$$

$$f(v_i) = 3i, \quad \text{for } 1 \leq i \leq n$$

$$f(u'_i) = 3i + 1, \quad \text{for } 1 \leq i \leq n, \quad i \not\equiv 2 \pmod{5}$$

$$f(u_i) = 3i + 1, \quad \text{for } 1 \leq i \leq n, \quad i \equiv 2 \pmod{5}$$

$$f(u'_i) = 3i - 1, \quad \text{for } 1 \leq i \leq n, \quad i \equiv 2 \pmod{5}$$

$$\text{since } f(c) = 1,$$

$$\gcd(f(c), f(u_i)) = 1, \quad \text{for } 1 \leq i \leq n$$

$$\gcd(f(c), f(u'_i)) = 1, \quad \text{for } 1 \leq i \leq n$$

clearly,

$$\gcd(f(u_i), f(v_i)) = \gcd(3i - 1, 3i) = 1 \quad \text{for } 1 \leq i \leq n$$

$$\gcd(f(v_i), f(u'_i)) = \gcd(3i, 3i + 1) = 1 \quad \text{for } 1 \leq i \leq n$$

$$\gcd(f(u_i), f(u_{i+1})) = \gcd(3i - 1, 3i + 2) = 1 \quad \text{for } 1 \leq i \leq n - 1$$

as one of these numbers is even then the other number is odd. Also the difference of these two numbers is 3.

$$\gcd(f(u_1), f(u_n)) = \gcd(2, 3n - 1) = 1 \quad \text{since } n \text{ is even and } 3n - 1 \text{ is odd.}$$

$$\gcd(f(u'_i), f(u_{i+1})) = \gcd(3i + 1, 3i + 2) = 1 \quad \text{for } 1 \leq i \leq n - 1.$$

$$\gcd(f(u_i), f(u'_{i+1})) = \gcd(3i - 1, 3i + 4) = 1 \quad \text{if } i \not\equiv 2 \pmod{5}$$

then  $3i - 1$  is not multiple of 5.

$$\gcd(f(u_i), f(u'_{i+1})) = \gcd(3i + 1, 3(i + 1) + 1)$$

$$= \gcd(3i + 1, 3i + 4) = 1 \quad \text{if } i \equiv 2 \pmod{5}$$

$$\gcd(f(u'_1), f(u_n)) = \gcd(4, 3n - 1) = 1 \quad \text{since } 3n - 1 \text{ is odd.}$$

$$\gcd(f(u'_n), f(u_1)) = \gcd(3n + 1, 2) = 1 \quad \text{since } n \text{ is even and } 3n + 1 \text{ is odd.}$$

then  $f$  is a prime labeling

thus  $G$  is a prime graph.

## Theorem 2.2

The graph  $G$  obtained by duplicating all the vertices of the helm  $H_n$ , except the apex vertex, is a prime graph.

**proof:**

Let  $V(H_n) = \{c, u_i, v_i / 1 \leq i \leq n\}$

$E(H_n) = \{cu_i, u_i v_i / 1 \leq i \leq n\} \cup \{u_i u_{i+1} / 1 \leq i \leq n - 1\} \cup \{u_1 u_n\}$

Let  $G$  be the graph obtained by duplicating all the vertices in the helm  $H_n$ , except the apex vertex  $c$ .

Let  $u'_1, u'_2, \dots, u'_n$ , and  $v'_1, v'_2, \dots, v'_n$  be the new vertices of  $G$  by

duplicating  $u_1, u_2, \dots, u_n$  and  $v_1, v_2, \dots, v_n$ , then

$V(G) = \{c, u_i, v_i, u'_i, v'_i / 1 \leq i \leq n\}$

$E(G) = \{cu_i, cu'_i, u_i v_i, v_i u'_i, u_i v'_i / 1 \leq i \leq n\} \cup \{u_i u_{i+1} / 1 \leq i \leq n - 1\} \cup$   
 $\{u_i u'_{i+1}, u'_{i-1} u_i / 2 \leq i \leq n - 1\} \cup \{u_n u_1, u_n u'_1, u'_n u_1, u'_n u'_1\}$

$|V(G)| = 4n + 1, \quad |E(G)| = 8n$

Define a labeling  $f : V(G) \rightarrow \{1, 2, 3, \dots, 4n + 1\}$  as follows

Let  $f(c) = 1, \quad f(u_1) = 4, \quad f(u'_1) = 2$  and  $f(v_1) = 3$

$f(u_i) = 4i - 1, \quad \text{for } 2 \leq i \leq n, \quad i \not\equiv 1 \pmod{3}$

$f(v_i) = 4i - 2, \quad \text{for } 2 \leq i \leq n$

$f(u'_i) = 4i - 3, \quad \text{for } 2 \leq i \leq n, \quad i \not\equiv 1 \pmod{3}$

$f(v'_i) = 4i, \quad \text{for } 2 \leq i \leq n,$

$f(v'_1) = 4n + 1,$

$f(u_i) = 4i - 3, \quad \text{for } 2 \leq i \leq n, \quad i \equiv 1 \pmod{3}$

$f(u'_i) = 4i - 1, \quad \text{for } 2 \leq i \leq n, \quad i \equiv 1 \pmod{3}$

Since  $f(c) = 1,$

$\gcd(f(c), f(u_i)) = 1, \quad \text{for } 1 \leq i \leq n$

$\gcd(f(c), f(u'_i)) = 1, \quad \text{for } 1 \leq i \leq n$

$\gcd(f(u_i), f(v_i)) = \gcd(4i - 1, 4i - 2) = 1, \quad \text{for } 1 \leq i \leq n$

$\gcd(f(v_i), f(u'_i)) = \gcd(4i - 2, 4i - 3) = 1, \quad \text{for } 1 \leq i \leq n$

$\gcd(f(u_1), f(u_n)) = \gcd(4, 4n - 1) = 1, \quad \text{since } 4n - 1 \text{ is odd}$

$\gcd(f(u_i), f(u'_{i+1})) = \gcd(4i - 1, 4i + 1) = 1, \quad \text{for } 2 \leq i \leq n - 1.$

as these two numbers are odd and difference is 2.

$$\gcd(f(u_i), f(u'_{i-1})) = \gcd(4i - 1, 4i - 7) = 1 \quad \text{if } i \not\equiv 1 \pmod{3}$$

$$\gcd(f(u_i), f(u_{i-1})) = \gcd(4i - 3, 4i - 5) = 1 \quad \text{if } i \equiv 1 \pmod{3}$$

$\gcd(f(u_i), f(u_{i+1})) = \gcd(4i - 1, 4i + 3) = 1$  as these two numbers are odd and difference is 4.

$$\gcd(f(u_1), f(u'_n)) = \gcd(4, 4n - 3) = 1, \quad \text{since } 4n - 3 \text{ is odd.}$$

$$\gcd(f(u_n), f(u'_1)) = \gcd(4n - 1, 2) = 1, \quad \text{since } 4n - 1 \text{ is odd.}$$

$$\gcd(f(u_i), f(v'_i)) = \gcd(4i - 1, 4i) = 1.$$

Then  $f$  is a prime labeling.

Thus  $G$  is a prime graph.

### Theorem 2.3

The graph obtained by duplicating all the rim vertices in crown  $C_n^*$  is a prime graph, if  $n$  is even.

**Proof:**

$$\text{Let } V(C_n^*) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$$

$$E(C_n^*) = \{u_i v_i / 1 \leq i \leq n\} \cup \{u_i u_{i+1} / 1 \leq i \leq n - 1\} \cup \{u_n u_1\}$$

Let  $G$  be the graph obtained by duplicating all the rim vertices in  $C_n^*$

Let  $u'_1, u'_2, \dots, u'_n$  be the new vertices of  $G$  by duplicating  $u_1, u_2, \dots, u_n$  then

$$V(G) = \{u_i, v_i, u'_i / 1 \leq i \leq n\}$$

$$E(G) = \{u_i v_i, v_i u'_i, u_i u'_{i+1} / 1 \leq i \leq n\} \cup \{u_i u_{i+1}, u'_i u'_{i+1} / 1 \leq i \leq n - 1\} \\ \cup \{u_n u_1, u_1 v_1, u'_n u'_1\}$$

$$|V(G)| = 3n, \quad |E(G)| = 5n.$$

Define a labeling  $f : V(G) \rightarrow \{1, 2, 3, \dots, 3n\}$  as follows

$$\text{Let } f(u_1) = 1, \quad f(u'_1) = 3n - 1 \quad \text{and} \quad f(v_1) = 3n$$

$$f(u_i) = 3i - 2, \quad \text{for } 2 \leq i \leq n, \quad i \not\equiv 4 \pmod{5}$$

$$f(v_i) = 3i - 3, \quad \text{for } 2 \leq i \leq n$$

$$f(u'_i) = 3i - 4, \quad \text{for } 2 \leq i \leq n, \quad i \not\equiv 4 \pmod{5}$$

$$f(u_i) = 3i - 4, \quad \text{for } 2 \leq i \leq n, \quad i \equiv 4 \pmod{5}$$

$$f(u'_i) = 3i - 2, \quad \text{for } 2 \leq i \leq n, \quad i \equiv 4 \pmod{5}$$

$$\gcd(f(u_i), f(u_{i+1})) = \gcd(3i - 2, 3i + 1) = 1 \quad \text{for } 2 \leq i \leq n - 1$$

as these two numbers has one is even and other is odd and their difference is 3.

$$\begin{aligned} \gcd(f(u_n), f(u_1)) &= \gcd(3n - 2, 1) = 1 \\ \gcd(f(u_i), f(v_i)) &= \gcd(3i - 2, 3i - 3) = 1 \quad \text{for } 2 \leq i \leq n \\ \gcd(f(v_i), f(u'_i)) &= \gcd(3i - 3, 3i - 4) = 1 \quad \text{for } 2 \leq i \leq n \\ \gcd(f(u'_i), f(u_{i+1})) &= \gcd(3i - 4, 3i + 1) = 1 \quad \text{if } i \not\equiv 4 \pmod{5} \\ \gcd(f(u'_i), f(u_{i+1})) &= \gcd(3i - 2, 3(i + 1) - 4) \\ &= \gcd(3i - 2, 3i - 1) = 1 \quad \text{if } i \equiv 4 \pmod{5} \\ \gcd(f(u'_n), f(u_1)) &= \gcd(3n - 4, 1) = 1 \\ \gcd(f(u_i), f(u'_{i+1})) &= \gcd(3i - 2, 3i - 1) = 1 \quad \text{for } 1 \leq i \leq n \\ \gcd(f(u_1), f(v_1)) &= \gcd(1, 3n) = 1 \\ \gcd(f(v_1), f(u'_1)) &= \gcd(3n, 3n - 1) = 1 \\ \gcd(f(u_n), f(u'_1)) &= \gcd(3n - 2, 3n - 1) = 1 \end{aligned}$$

Thus  $f$  is a prime labeling.

Hence  $G$  is a prime Graph.

#### Theorem 2.4

The graph obtained by duplicating all the vertices in crown  $C_n^*$  is a prime graph.

**Proof :**

$$\text{Let } V(C_n^*) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_n\}$$

$$E(C_n^*) = \{u_i v_i / 1 \leq i \leq n\} \cup \{u_i u_{i+1} / 1 \leq i \leq n - 1\} \cup \{u_n u_1\}$$

Let  $G$  be the graph obtained by duplicating all the vertices in  $C_n^*$

Let  $u'_1, u'_2, \dots, u'_n$  and  $v'_1, v'_2, \dots, v'_n$  be the new vertices of  $G$  by duplicating  $u_1, u_2, \dots, u_n$  and  $v_1, v_2, \dots, v_n$  then

$$V(G) = \{u_i, v_i, u'_i, v'_i / 1 \leq i \leq n\}$$

$$\begin{aligned} E(G) &= \{u_i v_i, v_i u'_i, u_i v'_i / 1 \leq i \leq n\} \cup \{u_i u_{i+1}, u'_i u_{i+1}, u_i u'_{i+1} / 1 \leq i \leq n - 1\} \\ &\quad \cup \{u_n u_1, u_n u'_1, u'_n u_1\} \end{aligned}$$

$$|V(G)| = 4n, \quad |E(G)| = 6n.$$

Define a labeling  $f : V(G) \rightarrow \{1, 2, 3, \dots, 4n\}$  as follows

Let  $f(u_1) = 1, f(u'_1) = 2$  and  $f(v_1) = 3$ .

$$f(u_i) = 4i - 1, \quad \text{for } 2 \leq i \leq n, \quad i \not\equiv 1 \pmod{3}$$

$$\begin{aligned}
f(v_i) &= 4i - 2, & \text{for } 2 \leq i \leq n \\
f(u'_i) &= 4i - 3, & \text{for } 2 \leq i \leq n, \quad i \not\equiv 1 \pmod{3} \\
f(v'_i) &= 4i, & \text{for } 1 \leq i \leq n, \\
f(u_i) &= 4i - 3, & \text{for } 2 \leq i \leq n, \quad i \equiv 1 \pmod{3} \\
f(u'_i) &= 4i - 1, & \text{for } 2 \leq i \leq n, \quad i \equiv 1 \pmod{3}
\end{aligned}$$

$$\gcd(f(u_i), f(u_{i+1})) = \gcd(4i - 1, 4i + 3) = 1, \text{ for } 1 \leq i \leq n$$

as these two numbers are odd and their difference is 4

$$\gcd(f(u_n), f(u_1)) = \gcd(4n - 1, 1) = 1$$

$$\gcd(f(u_i), f(v_i)) = \gcd(4i - 1, 4i - 2) = 1 \text{ for } 2 \leq i \leq n$$

$$\gcd(f(v_i), f(u'_i)) = \gcd(4i - 2, 4i - 3) = 1 \text{ for } 2 \leq i \leq n$$

$$\gcd(f(u_i), f(v'_i)) = \gcd(4i - 1, 4i) = 1 \text{ for } 2 \leq i \leq n$$

$$\gcd(f(u_i), f(u'_{i-1})) = \gcd(4i - 1, 4i - 7) = 1 \text{ for } 2 \leq i \leq n-1, i \not\equiv 1 \pmod{3}$$

as their difference is 6 and they are odd.

$$\gcd(f(u_i), f(u'_{i-1})) = \gcd(4i - 3, 4i - 5) = 1 \text{ if } i \equiv 1 \pmod{3}$$

since they are two consecutive odd numbers.

$$\gcd(f(u_i), f(u'_{i+1})) = \gcd(4i - 1, 4i + 1) = 1 \text{ for } 2 \leq i \leq n-1$$

$$\gcd(f(u_1), f(u'_n)) = \gcd(1, 4n - 3) = 1$$

$$\gcd(f(u_n), f(u'_1)) = \gcd(4n - 1, 2) = 1$$

Then  $f$  is a prime labeling.

Hence  $G$  is a prime graph.

### Theorem 2.5

The graph obtained by duplicating all the vertices in Gear graph  $G_n$ , except the apex vertex, is a prime graph.

**Proof :**

$$\text{let } V(G_n) = \{c, u_i, v_i / 1 \leq i \leq n\}$$

$$E(G_n) = \{cu_i, u_i v_i / 1 \leq i \leq n\} \cup \{v_i v_{i+1} / 1 \leq i \leq n-1\} \cup \{v_n v_1\}$$

Let  $G$  be the graph obtained by duplicating all the vertices in Gear graph  $G_n$ , except the apex vertex.

Let  $u'_1, u'_2, \dots, u'_n$  and  $v'_1, v'_2, \dots, v'_n$  be the new vertices of  $G$  by duplicating  $u_1, u_2, \dots, u_n$  and  $v_1, v_2, \dots, v_n$  then

$$V(G) = \{c, u_i, v_i, u'_i, v'_i / 1 \leq i \leq n\}$$

$$E(G) = \{cu_i, cu'_i, u_i v_i, v_i u'_i, u_i v'_i / 1 \leq i \leq n\} \cup \{v_i v_{i+1}, v_i v'_{i+1}, v'_i v_{i+1} / 1 \leq i \leq n-1\}$$

$$\cup \{v_n v'_1, v'_n v_1, v_n v_1\}$$

$$|V(G)| = 4n + 1, \quad |E(G)| = 8n.$$

Define a labeling  $f : V(G) \rightarrow \{1, 2, 3, \dots, 4n + 1\}$  as follows

Let  $f(c) = 1, f(u_1) = 3, f(v_1) = 2, f(v'_1) = 4$  and  $f(u'_1) = 5$ .

$$\begin{aligned} f(u_i) &= 4i, & \text{for } 2 \leq i \leq n \\ f(v_i) &= 4i - 1, & \text{for } 2 \leq i \leq n, \quad i \not\equiv 1 \pmod{3} \\ f(u'_i) &= 4i - 2, & \text{for } 2 \leq i \leq n \\ f(v'_i) &= 4i + 1, & \text{for } 2 \leq i \leq n, \quad i \not\equiv 1 \pmod{3} \\ f(v_i) &= 4i + 1, & \text{for } 4 \leq i \leq n, \quad i \equiv 1 \pmod{3} \\ f(v'_i) &= 4i - 1, & \text{for } 4 \leq i \leq n, \quad i \equiv 1 \pmod{3} \end{aligned}$$

Clearly,

$$\gcd(f(c), f(u_i)) = 1, \quad \text{for } 2 \leq i \leq n.$$

$$\gcd(f(c), f(u'_i)) = 1, \quad \text{for } 2 \leq i \leq n.$$

$$\gcd(f(u_i), f(v_i)) = \gcd(4i, 4i - 1) = 1, \quad \text{for } 2 \leq i \leq n$$

$$\gcd(f(v_i), f(u'_i)) = \gcd(4i - 1, 4i - 2) = 1, \quad \text{for } 2 \leq i \leq n$$

$$\gcd(f(u_i), f(v'_i)) = \gcd(4i, 4i + 1) = 1, \quad \text{for } 2 \leq i \leq n$$

$$\gcd(f(v_i), f(v_{i+1})) = \gcd(4i - 1, 4i + 3) = 1, \quad \text{for } 1 \leq i \leq n - 1$$

as these two numbers are odd and difference is 4.

$$\gcd(f(v_i), f(v'_{i+1})) = \gcd(4i - 1, 4i + 5) = 1 \quad \text{if } i \not\equiv 1 \pmod{3}$$

$$\begin{aligned} \gcd(f(v_i), f(v'_{i+1})) &= \gcd(4i + 1, 4(i + 1) + 1) \\ &= \gcd(4i + 1, 4i + 5) = 1 \quad \text{if } i \equiv 1 \pmod{3} \end{aligned}$$

$$\gcd(f(v'_i), f(v_{i+1})) = \gcd(4i + 1, 4i + 3) = 1 \quad \text{if } i \not\equiv 1 \pmod{3} \text{ for } 2 \leq i \leq n - 1$$

as these two numbers are consecutive odd.

$$\gcd(f(v_1), f(v'_n)) = \gcd(2, 4n + 1) = 1, \quad \gcd(f(v_1), f(v_n)) = \gcd(2, 4n - 1) = 1,$$

$$\gcd(f(v'_1), f(v_n)) = \gcd(4, 4n - 1) = 1$$

Then  $f$  is a prime labeling.

Thus  $G$  is a prime graph.

### Theorem 2.6

The graph obtained by duplicating all the vertices of the star  $S_n = K_{1,n}$  is a prime graph.

**Proof:**

$$\text{Let } V(S_n) = \{c, v_1, v_2, \dots, v_n\}$$



$$E(S_n) = \{cv_i / 1 \leq i \leq n\}$$

Let  $G$  be the graph obtained by duplicating all the vertices of the star  $S_n = K_{1,n}$ .

Let  $v'_1, v'_2, \dots, v'_n$  and  $c'$  be the new vertices of  $G$  by duplicating  $v_1, v_2, \dots, v_n$  and  $c$  respectively.

$$\text{Then } V(G) = \{c, c', v_1, v_2, \dots, v_n, v'_1, v'_2, \dots, v'_n\}$$

$$E(G) = \{cv_i, c'v_i, cv'_i / 1 \leq i \leq n\}$$

$$|V(G)| = 2n + 2 \text{ and } |E(G)| = 3n.$$

Define a labeling  $f : V(G) \rightarrow \{1, 2, \dots, 2n + 2\}$

$$f(c) = 1, \quad f(c') = 2,$$

$$f(v_i) = 2i + 1, \quad \text{for } 1 \leq i \leq n$$

$$f(v'_i) = 2i, \quad \text{for } 1 \leq i \leq n$$

$$\text{Then } \gcd(f(c), f(v_i)) = \gcd(1, 2i + 1) = 1, \quad \text{for } 1 \leq i \leq n$$

$$\gcd(f(c'), f(v_i)) = \gcd(2, 2i + 1) = 1, \quad \text{for } 1 \leq i \leq n$$

$$\gcd(f(c), f(v'_i)) = \gcd(1, 2i) = 1, \quad \text{for } 1 \leq i \leq n$$

Then  $f$  is a prime labeling.

Thus  $G$  is a prime graph.

### EXAMPLES

#### Illustration of 2.1

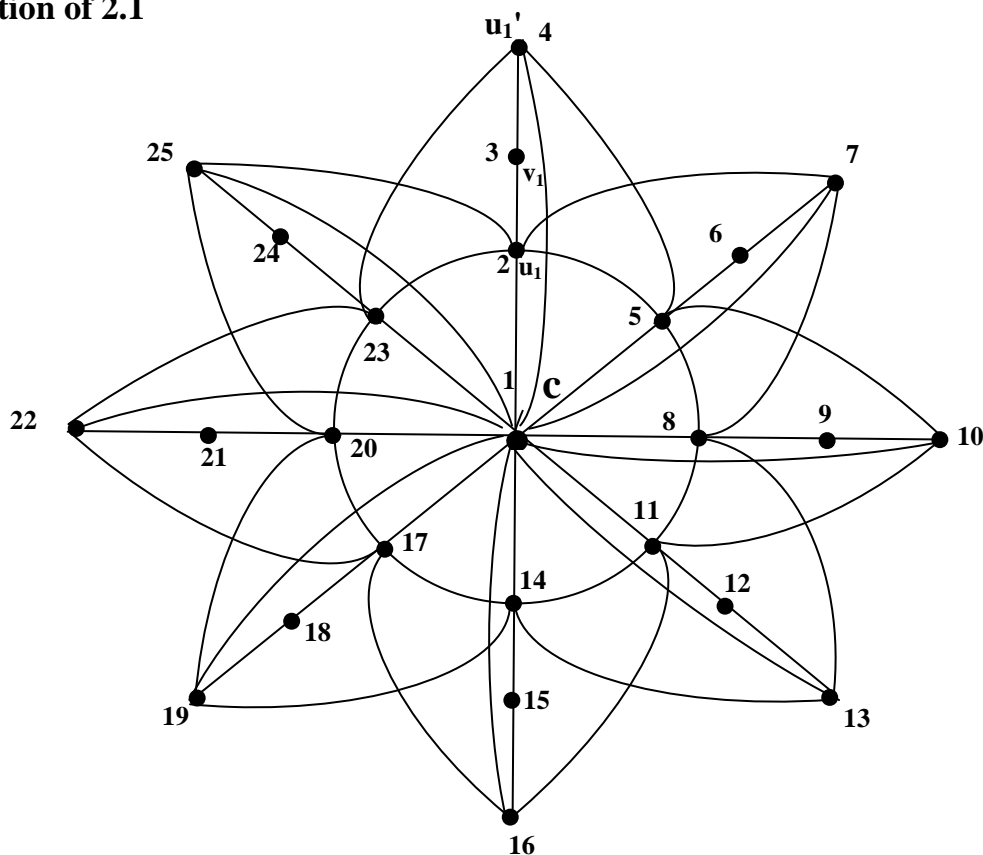


Figure 1. Prime Labeling of duplication of all the rim vertices of Helm  $H_8$



Illustration of 2.2

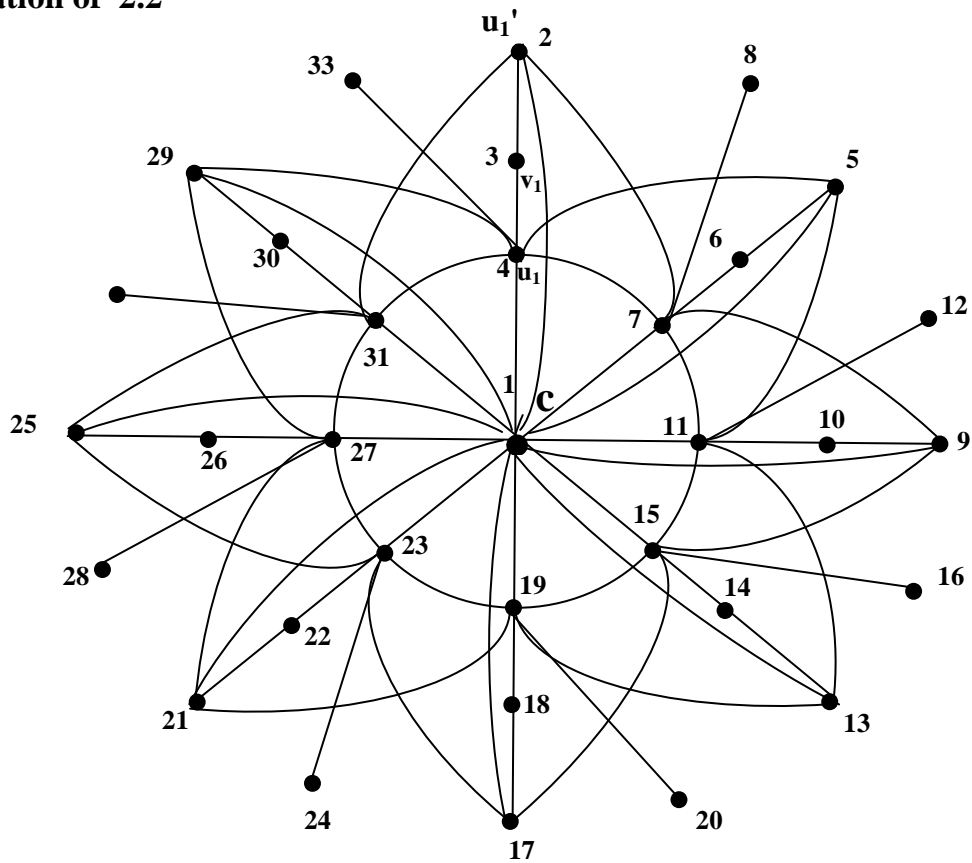


Figure 2. Prime Labeling of duplication of all the vertices of Helm  $H_8$

Illustration of 2.3

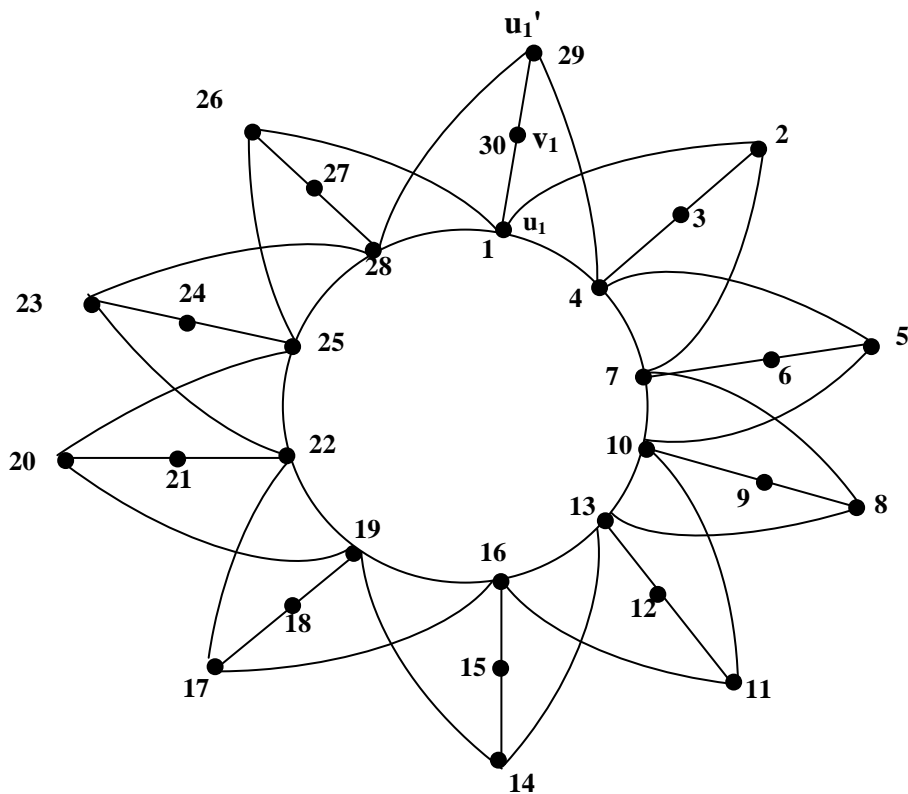


Figure 3. Prime Labeling of duplication of all the rim vertices of crown  $C_{10}^*$

Illustration of 2.4

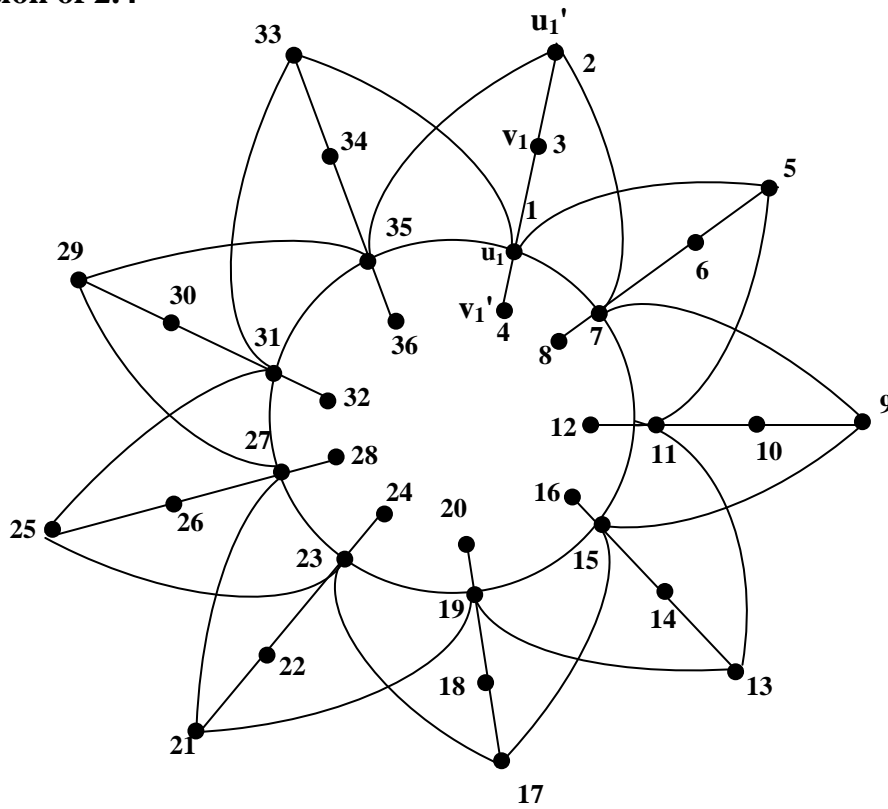


Figure 4. Prime Labeling of duplication of all the vertices of crown  $C_9^*$

Illustration of 2.5

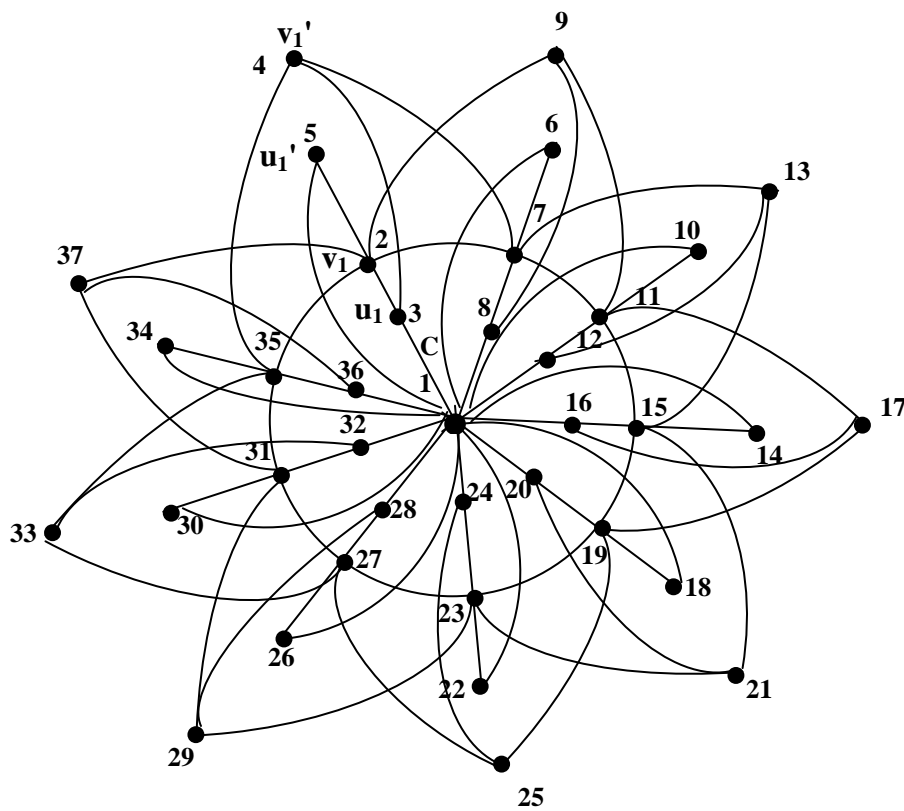


Figure 5. Prime Labeling of duplication of all the vertices of Gear  $G_9$

### 3. CONCLUSION

Labeled graph is the topic of current due to its diversified application. We investigate six new results on prime labeling. It is an effort to relate the prime labeling and some graph operations. This approach is novel as it provides prime labeling for the larger graph resulted due to certain graph operations on a given graph.

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