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GRACEFUL LABELING FOR ONE POINT UNION OF PATH AND BARYCENTRIC SUBDIVISION OF A GRID GRAPH

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Abstract

In this paper we have obtained some grid related family of connected graceful graphs. We obtained graceful labeling for one point union of path and barycentric subdivision of grid graph.

Key words: Graceful labeling, one point union of path, barycentric subdivision of grid graph.

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1 Introduction:

Acharya and Gill [1] have investigated graceful labeling for the grid graph $(P_n \times P_{m)}$. Kaneria and Makadia [4] discussed gracefulness of $(P_n \times P_m) \cup (P_r \times P_s), (P_n \times P_m) \cup (P_r \times P_s) \cup C_{2f+3}$, tensor product $P_2(T_p)P_n$ and star of cycle $C_n^*(n \equiv 0(mod\, 4))$. For a dynamic survey on graph labeling one can refer to Gallian [2].

We begin with a simple undirected finite graph $G = (V, E)$ on p vertices and q edges. For all terminology and notations we follows Harary [3]. First of all we shall recall some definitions which are useful for this paper.

Definition – 1.1: A function f is called graceful labeling of a graph $G = (V, E)$ if $f: V \to \{0,1, \dots, q\}$ is injective and the induced function $f^*: E \to \{1,2, \dots, q\}$ defined as $f^*(e) = |f(u) - f(v)|$ is bijective for every edge $e = (u, v) \in E$. A graph G is called graceful graph if it admits a graceful labeling.

Definition–1.2: A graph G obtained by replacing each edge of $k_{1,t}$ by a path $P_n$of length n on

$n+1$ vertices is called one point union for t copies of path P_n. We shall denote such graph G by P_n^t.

Definition–1.3: If every edge a graph G is subdivided by a new vertex then the resulting graph is called barycentric subdivision of graph G. In other word barycentric subdivision is the graph obtained by inserting a new vertex of degree 2 into every edge of the original graph G.

In this paper we have discussed graceful labeling for barycentric subdivision of the grid graph $(P_n \times P_m)$ and P_n^t, one point union of t copies of path P_n.

2 Main Results:

Theorem– 2:1: Barycentric subdivision of $(P_n \times P_m)$ is graceful.

Proof: Let G be a graph obtained by barycentric subdivision for $(P_n \times P_m)$. We know that the number of vertices for $(P_n \times P_m)$ is $p = mn$and the number of edges$q = 2mn - (m+n)$. Also we see that the number of vertices for G is $P = |V(E)| = p + q$ and the number of edges$Q = |E(G)| = 2q$.

Let $u_{i,j}(j = 1,2, \ldots, 2n-1, i = 1,2, \ldots, m)$ be the vertices for vertical edges in G and $v_{i,j}(j = 1,3, \ldots, 2n-1, i = 1,2, \ldots, m-1)$ be the vertices which divide to the horizontal edges in G. We define labeling function $f: V(G) \rightarrow \{0,1, \ldots, Q\}$ as follow.

$$f(u_{i,j}) = Q - (i-1)n - \frac{j-1}{2} \qquad \forall i - 1,2, \ldots, m, \quad \forall j = 1,3, \ldots, 2n-1$$

$$= (i-1)(3n-2) + \frac{j-2}{2}, \qquad \forall i - 1,2, \ldots, m, \qquad \forall j = 2,4, \ldots, 2n-2;$$

$$f(v_{i,j}) = (3n-2)i - j, \qquad \forall i - 1,2, \ldots, m, -1, \forall j = 1,3, \ldots, 2n-1.$$

Above labeling pattern give rise graceful labeling to the graph G and so it is a graceful graph.

Illustration– 2:2**:** Barycentric subdivision of $(P_3 \times P_4)$and its graceful labeling shown

In *figure*–*1.*

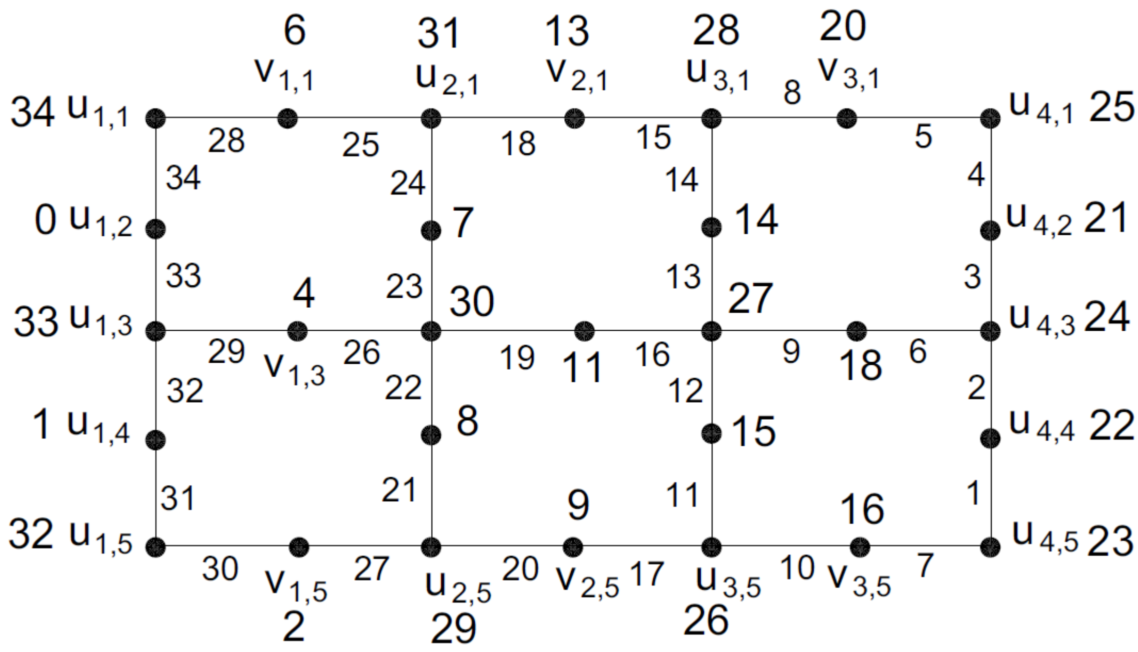

Figure –1 A graph obtained by barycentric subdivision of $(P_3 \times P_4)$ and its graceful labeling.

Theorem –2:3: P_n^t, one point union of path P_n on $n+1$ vertices is graceful.

Proof: Let G be a graph obtained by replacing each edge of $K_{1,t}$ by a path P_n of length n. Let v_0 be the central vertex for P_n^t and $v_{i,j}(1 \le i \le t, 1 \le j \le n)$ be the consecutive vertices of each branch of P_n^t from v_0.

To define labeling function $f: V(G) \to \{0,1,\ldots,q\}$, where $q = tn$,, we shall take following four cases.

Case– I : t and n both are even

$$f(v_{1,j}) = q - \left(\frac{n-j}{2}\right), \qquad \forall j = 2,4,\ldots,n$$
$$= \left(\frac{n-1-j}{2}\right), \qquad \forall j = 1,3,\ldots,n-1;$$

$$f(v_0) = f(v_{1,2}) - 1;$$
$$f(v_{2,j}) = f(v_{1,1}) + \left(\frac{j+1}{2}\right), \qquad \forall\, j = 1,3,\ldots,n-1$$
$$= f(v_0) - \frac{j}{2}, \qquad \forall\, j = 2,4,\ldots,n;$$
$$f(v_{i,j}) = f(v_{i-1,n-1}) + \left(\frac{n+2-j}{2}\right), \quad \forall\, i = 3,5,\ldots,t-1, \quad \forall\, j = 2,4,\ldots,n$$
$$= f(v_{i-1,n}) - \left(\frac{n+1-j}{2}\right), \quad \forall\, i = 3,5,\ldots,t-1, \quad \forall\, j = 1,3,\ldots,n-1$$
$$= f(v_{i-1,2}) + \left(\frac{j+1}{2}\right), \quad \forall\, i = 4,6,\ldots,t, \quad \forall\, j = 1,3,\ldots,n-1$$
$$= f(v_{i-1,1}) - \left(\frac{j}{2}\right), \quad \forall\, i = 4,6,\ldots,t, \quad \forall\, j = 2,4,\ldots,n.$$

Case −II: t is even and n is odd

$$
\begin{aligned}
f(v_{1,j}) &= q-\left(\tfrac{n-j}{2}\right), && \forall j = 1,3,\ldots,n\\
&= \left(\tfrac{n-1-j}{2}\right), && \forall j = 2,4,\ldots,n-1;\\
f(v_0) &= f(v_{1,2})+1;\\
f(v_{2,j}) &= f(v_{1,1})-\left(\frac{j+1}{2}\right), && \forall j = 1,3,\ldots,n\\
&= f(v_0)+\tfrac{j}{2}, && \forall j = 2,4,\ldots,n-1;\\
f(v_{i,j}) &= f(v_{i-1,n-1})+\left(\frac{n+2-j}{2}\right), && \forall i = 3,5,\ldots,t-1, \forall j = 1,3,\ldots,n\\
&= f(v_{i-1,n})-\left(\tfrac{n+1-j}{2}\right), && \forall i = 3,5,\ldots,t-1,\ \forall j = 2,4,\ldots,n-1\\
&= f(v_{i-1,2})-\left(\tfrac{j+1}{2}\right), && \forall i = 4,6,\ldots,t,\quad \forall j = 1,3,\ldots,n\\
&= f(v_{i-1,1})+\left(\tfrac{j}{2}\right), && \forall i = 4,6,\ldots,t,\quad \forall j = 2,4,\ldots,n-1.
\end{aligned}
$$

Case −III: t and n both are odd

$$
\begin{aligned}
f(v_{1,j}) &= q-\left(\tfrac{n-j}{2}\right), && \forall j = 1,3,\ldots,n\\
&= \left(\tfrac{n-1-j}{2}\right), && \forall j = 2,4,\ldots,n-1;\\
f(v_0) &= f(v_{1,2})+1;\\
f(v_{2,j}) &= f(v_{1,1})-\left(\frac{j+1}{2}\right), && \forall j = 1,3,\ldots,n\\
&= f(v_0)+\tfrac{j}{2}, && \forall j = 2,4,\ldots,n-1;\\
f(v_{i,j}) &= f(v_{i-1,n-1})+\left(\tfrac{n+2-j}{2}\right), && \forall i = 3,5,\ldots,t,\quad \forall j = 1,3,\ldots,n\\
&= f(v_{i-1,n})-\left(\tfrac{n+1-j}{2}\right), && \forall i = 3,5,\ldots,t,\quad \forall j = 2,4,\ldots,n-1\\
&= f(v_{i-1,2})-\left(\tfrac{j+1}{2}\right), && \forall i = 4,6,\ldots,t-1,\ \forall j = 1,3,\ldots,n\\
&= f(v_{i-1,1})+\left(\tfrac{j}{2}\right), && \forall i = 4,6,\ldots,t-1,\ \forall j = 2,4,\ldots,n-1.
\end{aligned}
$$

Case−IV: t is odd and n is even

$$
\begin{aligned}
f(v_0) &= 0;\\
f(v_{1,j}) &= \ q-\left(\tfrac{j-1}{2}\right), && \forall j1,3,\ldots,n-1\\
&= \left(\tfrac{j}{2}\right), && \forall j = 2,4,\ldots,n;\\
f(v_{2,j}) &= f(v_{1,n})+\left(\tfrac{n+1-j}{2}\right), && \forall i = 1,3,\ldots,n-1\\
&= f(v_{1,n-1})-\left(\tfrac{n+2-j}{2}\right), && \forall j = 2,4,\ldots,n\\
f(v_{i,j}) &= f(v_{i-1,1})+\left(\tfrac{j}{2}\right), && \forall i = 3,5,\ldots,t,\quad \forall j = 2,4,\ldots,n\\
&= f(v_{i-1,2})-\left(\tfrac{j+1}{2}\right), && \forall i = 3,5,\ldots,t,\quad \forall j = 1,3,\ldots,n-1\\
&= f(v_{i-1,n})+\left(\tfrac{n+1-j}{2}\right), && \forall i = 4,6,\ldots,t-1,\ \forall j = 1,3,\ldots,n-1
\end{aligned}
$$

$$= f\left(v_{i-1,n-1}\right) - \left(\frac{n+2-j}{2}\right), \qquad \forall\, i = 4,6,\ldots,t-1, \quad \forall\, j = 2,4,\ldots,n.$$

Above labeling pattern give rise graceful labeling to the graph P_n^t and so it is a graceful graph.

Illustration–2.4: P_4^5 and its graceful labeling shown in *figure-2.*

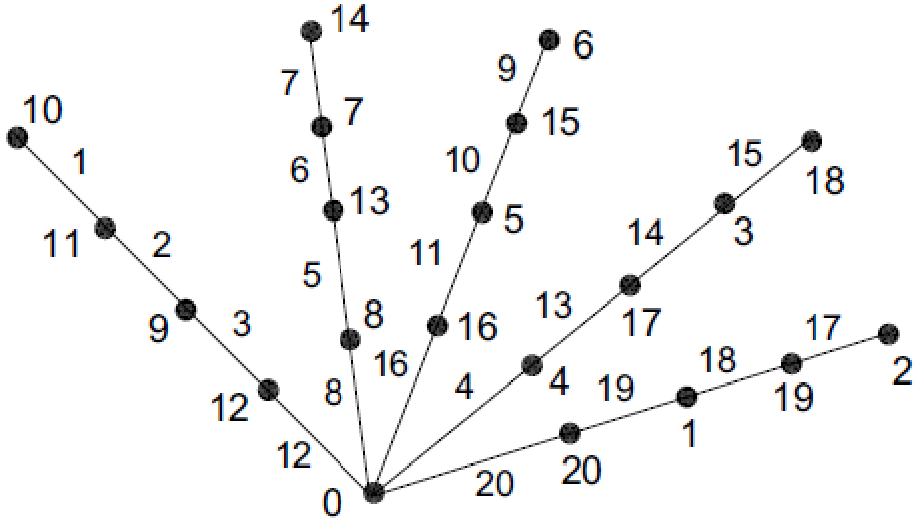

Figure –2 One point union of five copies of P_4 and its graceful labeling.

3 Concluding Remarks:

Here we have given graceful labeling to barycentric subdivision of $(P_n \times P_m)$ and P_n^t, one point union of path. The results obtained here are new and of very general. This work contributes two results to the families of graceful labeling. The labeling pattern is demonstrated by means of illustrations.

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