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Transfer of Thermal Energy in Second Order Fluids

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Abstract : *In this paper, we have considered transfer of thermal energy in second order fluids. We have seen the fluids elasticity has some rather unexpected results on momentum transfer, and, hence, one might expect anomalies for heat transfer as well. A review of relevant work is given by Showalter is extended to transfer of thermal energy in second order fluids. A variety of steady and unsteady problems is presented.*

Key words : *Second Order Fluid, Boundary Layer Flow*

I. Introduction

Drag reducing dilute polymer can also reduce heat transfer, but the effect is not in complete analogy to the momentum transfer phenomenon. Transport of thermal energy in two-dimensional flows of the second order fluid is discussed.

For this problem the laminar boundary layer equation for heat convection on a vertical plat are

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\theta+\frac{\partial^2 u}{\partial y^2}+\overline{\mu_1}\left(\frac{\partial^3 u}{\partial y^2\partial t}+u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}+\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}\right)$$
$$+(3\overline{\mu_1}+2\overline{\mu_2})\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y} \qquad (1.1a)$$

$$\frac{\partial \theta}{\partial t}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\sigma\frac{\partial^2 \theta}{\partial y^2} \qquad (1.1b)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1.1c)$$

Where θ is a dimensionless temperature difference and σ is the coefficient of diffusivity.

The boundary conditions are

At $y=0$: $u=v=0, \theta=W$ (The wall temperature) (1.2a)

At $y\to\infty: u\to U, \theta\to 0.$ (1.2b)

The solution of (1.1c) may be written in terms of a stream function ψ defined as follows:

$$u=\frac{\partial \psi}{\partial y}, \qquad v=-\frac{\partial \psi}{\partial x} \qquad (1.3)$$

Employing (1.3) in equations (1.1) we obtain

$$\frac{\partial^2 \psi}{\partial y\partial t}+\frac{\partial \psi}{\partial y}\frac{\partial^2 \psi}{\partial x\partial y}-\frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial y^2}=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}+\theta+\frac{\partial^3 \psi}{\partial y^3}+\bar{\mu}_1\left[\frac{\partial^4 \psi}{\partial y^3\partial t}+\frac{\partial \psi}{\partial y}\frac{\partial^4 \psi}{\partial x\partial y^3}\right.$$

$$-\frac{\partial\psi}{\partial x}\frac{\partial^4\psi}{\partial y^4}+\frac{\partial^2\psi}{\partial x\partial y}\frac{\partial^3\psi}{\partial y^3}\bigg]+(3\bar{\mu}_1+2\bar{\mu}_2)\frac{\partial^2\psi}{\partial y^2}\frac{\partial^3\psi}{\partial x\partial y^2} \tag{1.4a}$$

$$\frac{\partial\theta}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}=\sigma\frac{\partial^2\theta}{\partial y^2} \tag{1.4b}$$

II. The Similarity Transformation

Now we introduce the similarity transformations

$$\eta = y\,\phi\,(x,t) \tag{1.5a}$$
$$\theta = W\,(x,t)\,K\,(\eta) \tag{1.5b}$$
$$\psi = H\,(x,t)\,F\,(\eta) \tag{1.5c}$$

The partical derivatives in equations (1.4) transform according to:

$$\frac{\partial\psi}{\partial x}=\frac{\partial H}{\partial x}F+H\frac{\partial F}{\partial x}=\frac{\partial H}{\partial x}F+H\left(\frac{\partial F}{\partial\eta}\frac{\partial\eta}{\partial x}\right)\;=\frac{\partial H}{\partial x}\,F+\frac{\partial\ell n\phi}{\partial x}\,H\,\eta\,F'$$

$$\frac{\partial\psi}{\partial y}=H\,\frac{\partial F}{\partial\eta}\frac{\partial\eta}{\partial y}=\phi\,H\,F'$$

$$\frac{\partial^2\psi}{\partial y^2}=\frac{\partial}{\partial y}\,(\phi\,H\,F')=\phi^2\,H\,F''$$

$$\frac{\partial^4\psi}{\partial y^3}=\frac{\partial}{\partial y}\,(\phi^2\,H\,F'')=\phi^2\,H\,F'''$$

$$\frac{\partial^4\psi}{\partial y^2}=\frac{\partial}{\partial y}\,(\phi^3\,H\,F'''=\phi^4\,H\,F^{\mathrm{iv}}$$

$$\frac{\partial^2\psi}{\partial x\partial y}=\frac{\partial}{\partial x}\,(\phi H\,F')=\left(\frac{\partial\phi}{\partial x}\,H+\phi\,\frac{\partial H}{\partial x}\right)F'+\frac{\partial\phi}{\partial x}H\,\eta\,F''$$

$$\frac{\partial^4\psi}{\partial x\partial y^3}=\frac{\partial}{\partial x}(\phi^3\,H\,F''')$$

$$\frac{\partial^4\psi}{\partial x\partial y^3}=\left(3\phi^2\frac{\partial\phi}{\partial x}\,H+\phi^3\frac{\partial H}{\partial x}\right)F'''+\phi^2\frac{\partial\phi}{\partial x}H\,\eta\,F^{\mathrm{iv}}$$

$$\frac{\partial^3\psi}{\partial x\partial y^2}=\left(2\phi\frac{\partial\phi}{\partial x}\,H+\phi^2\frac{\partial H}{\partial x}\right)F''+\phi\frac{\partial\phi}{\partial x}H\,\eta\,F'''$$

$$\frac{\partial^2\psi}{\partial y\partial t}=\frac{\partial}{\partial t}\,(\phi HF')=\left(\frac{\partial\phi}{\partial t}\,H+\phi\frac{\partial H}{\partial t}\right)F'+H\frac{\partial\phi}{\partial t}\eta\,F''$$

$$\frac{\partial^4\psi}{\partial t^4}=\left(3\phi\frac{\partial\phi}{\partial t}\,H+\phi^3\frac{\partial H}{\partial t}\right)F'''+\phi^2H\frac{\partial\phi}{\partial t}\eta\,F^{\mathrm{iv}}$$

$$\frac{\partial\theta}{\partial x}=\frac{\partial W}{\partial x}\,K+\frac{\partial\ell n\phi}{\partial x}W\eta K'$$

$$\frac{\partial\theta}{\partial t}=\frac{\partial W}{\partial t}\,K+W\frac{\partial\ell n\phi}{\partial t}\eta K'$$

$$\frac{\partial\theta}{\partial y}=W\frac{\partial K}{\partial\eta}\frac{\partial\eta}{\partial y}=\phi WK'$$

$$\frac{\partial^2\theta}{\partial y^2}=\frac{\partial}{\partial y}\,(\phi\,W\,K')=\phi^2WK''$$

Substituting the above into equations (1.4), we have

$$\left(\frac{\partial\phi}{\partial t}\,H+\phi\frac{\partial H}{\partial t}\right)F'+\frac{\partial\phi}{\partial t}H\,\eta\,F''+\phi HF'\left[\left(\frac{\partial\phi}{\partial x}H+\phi\frac{\partial H}{\partial x}\right)F'+\frac{\partial\phi}{\partial x}H\,\eta\,F''\right]$$
$$-\phi^2\,HF''\left[\frac{\partial H}{\partial x}F+\frac{\partial\ell n\phi}{\partial x}H\,\eta\,F'\right]=\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}$$

$$+WK+\phi^3HF'''+\bar{\mu}_1\left\{\left(3\phi\frac{\partial\phi}{\partial t}H+\phi^3\frac{\partial H}{\partial t}\right)F'''\right.$$

$$+\phi^2\frac{\partial\phi}{\partial t}\eta\,F^{\text{iv}}+\phi FF'\left[\left(3\phi^2\frac{\partial\phi}{\partial x}H+\phi^3\frac{\partial H}{\partial x}\right)F'''\right.$$

$$\left.+\phi^2\frac{\partial\phi}{\partial x}H\eta\,F^{\text{iv}}\right]-\phi^4H\,F^{\text{iv}}\left[\frac{\partial H}{\partial x}F+\frac{\partial\ell n\phi}{\partial x}H\,\eta\,F'\right]$$

$$\left.+\phi^3HF'''\left[\left(\frac{\partial\phi}{\partial x}H+\phi\frac{\partial H}{\partial x}\right)F'+\frac{\partial\phi}{\partial x}H\eta\,F''\right]\right\}$$

$$+(3\bar{\mu}_1+2\bar{\mu}_2)\phi^2HF''\left[\left(2\phi\frac{\partial\phi}{\partial x}H+\phi^2\frac{\partial H}{\partial x}\right)F''+\phi\frac{\partial\phi}{\partial x}H\eta\,F'''\right]$$

and

$$\frac{\partial W}{\partial t}K+W\frac{\partial\ell n\phi}{\partial t}\eta K'+\phi HF'\left[\frac{\partial W}{\partial x}K+W\frac{\partial\ell n\phi}{\partial x}\eta K'\right]-\phi\,W\,K'\left[\frac{\partial H}{\partial x}F+\frac{\partial\ell n\phi}{\partial x}H\eta\,F'\right]=\sigma\phi^2wK''$$

Upon simplification, we get

$$\frac{1}{\phi^3H}\left(\frac{\partial\phi}{\partial t}H+\phi\frac{\partial H}{\partial t}\right)F'+\frac{1}{\phi^3}\frac{\partial\phi}{\partial t}\eta F''+\left(\frac{H}{\phi^2}\frac{\partial\phi}{\partial x}+\frac{1}{\phi}\frac{\partial H}{\partial x}\right)F'^2-\frac{1}{\phi}\frac{\partial H}{\partial x}\,FF''$$

$$=\frac{1}{\phi^3H}\left(\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}\right)+\frac{1}{\phi^3H}WK+F'''+\bar{\mu}_1\left\{\left(\frac{3}{\phi}\frac{\partial\ell n\phi}{\partial t}+\frac{\partial\ell nH}{\partial t}\right)F'''+\frac{\partial\ell n\phi}{\partial t}\eta\,F^{\text{iv}}\right.$$

$$\left.+2\left(\phi\frac{\partial H}{\partial x}+2H\frac{\partial\phi}{\partial x}\right)F'F'''-\phi\frac{\partial H}{\partial x}F\,F^{\text{iv}}\right\}+(5\bar{\mu}_1+2\bar{\mu}_2)H\frac{\partial\phi}{\partial x}\eta\,F''$$

$$+(3\bar{\mu}_1+3\bar{\mu}_2)\left(2H\frac{\partial\phi}{\partial x}+\phi\frac{\partial H}{\partial x}\right)F''^2 \qquad (1.6a)$$

and,

$$\left(\frac{1}{\phi^2W}\frac{\partial W}{\partial t}+\frac{H}{\phi W}\frac{\partial W}{\partial x}F'\right)K+\frac{1}{\phi^2}\frac{\partial\ell n\phi}{\partial t}\eta K'-\frac{1}{\phi}\frac{\partial H}{\partial x}FK'=\sigma K'' \qquad (1.6b)$$

The above partial differential equations will become will ordinary differential equations provided the following conditions hold

$$\phi=c_0=1 \quad \text{(chosen)} \qquad (1.7a)$$

$$\frac{\partial\ell nH}{\partial H}=c_1 \qquad (1.7b)$$

$$\frac{\partial H}{\partial x}=c_2 \qquad (1.7c)$$

$$\frac{1}{H}\left(\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}\right)=c_3 \qquad (1.7d)$$

$$\frac{W}{H}=c_4 \qquad (1.7e)$$

$$\frac{\partial\ell nW}{\partial t}=c_5 \qquad (1.7f)$$

$$H\frac{\partial\ell nW}{\partial x}=c_6 \qquad (1.7g)$$

III. The Steady Case $\left(\frac{\partial}{\partial t}=0\right)$

From (1.7c), we get

$$H=c_2\,x+c_7 \qquad (1.8\text{ a})$$

Using (1.8c) in (1.7e), we have

$$W = c_4\,(c_2\,x + c_7\,) \quad (1.8\text{ b})$$

and (1.7g) is satisfied with $c_2 = c_6$.

From (1.7d), we obtain

$$U\,\frac{\partial U}{\partial x} = c_3(c_2 x + c_7)$$

which, upon integration, becomes

$$\frac{U^2}{2} = c_3\left(\frac{c_2}{2}x^2 + c_7 x + c_8\right)$$

or ,
$$U = \left[2c_3\left(\frac{c_2}{2}x^2 + c_7 x + c_8\right)\right]^{1/2}$$

We still have to worry about the restriction imposed on U by the boundary condition at ∞, that is $\frac{U}{\phi H}$ must be constant. Thus, we must have

$$c_7 = c_8 = 0$$

Therefore,

$$H = c_2\,x \quad (1.8\text{ c})$$

$$W = c_2\,c_4\,x \quad (1.8\text{ d})$$

$$U = \sqrt{c_2\,c_3\,x} \quad (1.8\text{ e})$$

And, consequently,

$$\frac{u}{\phi H} \to \frac{U}{\phi H} = \sqrt{\frac{c_3}{c_2}}$$

Equations (1.6) become

$$c_2\,(F'^2 - FF'') = c_3\; + c_4\,K + F''' + c_2\,\bar{\mu}_1(2F'F''' - F\,F^{\mathrm{iv}}) + c_2\,(3\,\bar{\mu}_1 + 2\bar{\mu}_2)F''^2 \quad (1.9\text{ a})$$

and

$$c_2\,(F'K - FK') = \sigma\,K'', \quad (1.9\text{ b})$$

subject to the boundary conditions :

At $\eta = 0, y = 0$

$$u = \phi HF' = 0 \Rightarrow F' = 0$$

$$v = -c_2\,F = 0 \Rightarrow F = 0$$

$$\theta = WK = W \Rightarrow K = 1$$

As $\quad \eta \to \infty, y \to \infty$ (1.10)

$$u \;\to \mathrm{U} = \sqrt{c_2\,c_3}\,x \Rightarrow F' \to \sqrt{\frac{c_3}{c_2}}$$

$$\theta = WK \to 0 \;\Rightarrow\; \mathrm{K} \to 0.$$

The above system is solved numerically, and the velocity and temperature distributions are obtained from the following (with $\eta = y$):

$$\psi \; = c_2\,xF(\eta) \quad (1.11\text{ a})$$

$$u(x,y) = \frac{\partial\psi}{\partial \mathrm{y}} = \; c_2\,x\,F'^{(\eta)} \quad (1.11\text{ b})$$

$$v(x,y) = -\frac{\partial \psi}{\partial y} = -\, c_2\, F(\eta) \quad (1.11\ c)$$

$$\theta(x,y) = c_2\, c_4\;\, xK\,(\eta) \quad (1.11\ d)$$

If $\bar{\mu}_1 = \bar{\mu}_2 = 0$ in equation (1.9), we obtain the purely viscous case

$$c_2\,(F'^2 - FF'') = c_3 + c_4 K + F''' \quad (1.12\ a)$$

For the viscous fluid the order is three. For the viscoelastic fluid the order is four.

Returning to the free stream velocity, it should be noted that the case $U = 0$ is also permissible. This is the case of free convection, where $c_3 = 0$ and equations (1.8 a,b) are retained. Equations (1.9) are the same except that $c_3 = 0$ now.

Also, the boundary conditions are the same except as $\eta \to \infty, F' = 0$.

The velocity and temperature profiles are, then

$$\psi = (c_2\, x + c_7)F(y) \quad (1.12\ b)$$

$$u(x,y) = (c_2\, x + c_7)F(y) \quad (1.12\ c)$$

$$v(x,y) = -c_2\, F(y) \quad (1.12\ d)$$

$$\theta(x,y) = c_4(c_2\, x + c_7)\, K(y) \quad (1.12\ e)$$

where $\eta = y$

IV. Unsteady Free Convection

From (1.7b), we get

$$\ell n H = c_1 t + a_1(x)$$

or, $\quad H \;=\; A_1(x)e^{c_1 t}$

where $A_1(x) = e^{a_1(x)}$ and $c_1 < 0$.

Employing that in (1.7c), we obtain

$$A_1'(x)e^{C_1 t} = c_2$$

$$A_1'(x) = 0, \text{and } c_2 = 0$$

Therefore, $A_1(x) = c_9 = \text{constant, and}$

$$H = c_9 e^{C_1 t} \quad (1.13\ a)$$

Combining (1.13a) with (1.7e), we get

$$W = c_4 c_9 e^{C_1 t} \quad (1.13\ b)$$

and Equation (1.7f) is satisfied if $c_5 = c_1$.

In Equation (1.7g), c_6 must be zero.

The similarity transformations (1.5) become

$$\left.\begin{aligned} \eta &= y \\ \psi &= c_9 e^{C_1 t}\, F(\eta) \\ \phi &= c_4 c_9 e^{C_1 t}\;\, K(\eta) \end{aligned}\right\} \quad (1.14\ )$$

Equations (1.6) become

$$c_1 c_9 e^{C_1 t} \frac{1}{c_9{}^{C_1 t}} F' + c_4 c_9 e^{C_1 t} \frac{1}{c_9 e^{C_1 t}} K = F''' + \bar{\mu}\left[c_1 c_9 e^{C_1 t} \frac{1}{c_9 e^{C_1 t}}\right] F'''$$

and

$$c_1 c_4 c_9 e^{C_1 t} \frac{1}{c_4 c_9 e^{C_1 t}} K = \sigma K''$$

or, upon simplification,

$$cF' + c_4 K = (1 + c_1\bar{\mu}_1)F''' \tag{1.15 a}$$

$$c_1 K = \sigma K'' \tag{1.15 b}$$

with boundary conditions:

At, $\eta = 0$, $F' = 0$, $K = 1$ (1.16 a)

At, $\eta \to \infty$, $F' \to 0$, $K \to 0,$ (1.16 b)

The solution of (1.15 b) is

$$K = c_{10} \cos \sqrt{\frac{-c_1}{\sigma}}\, \eta + c_{11} \sin\left(\sqrt{\frac{-c_1}{\sigma}}\, \eta\right).$$

From the first boundary condition, $c_{10} = 1$. Thus,

$$K = \cos\left(\sqrt{\frac{-c_1}{\sigma}}\, \eta\right) + c_{11} \sin\left(\sqrt{\frac{-c_1}{\sigma}}\, \eta\right).$$

The other boundary condition at ∞ cannot be satisfied , so it seems that this solution is valid only near the plate or within the boundary layer.

Equation (1.15a) becomes

$$(1 + c_1\bar{\mu}_1)\, F''' - c_1 F' = c_4 \cos\left(\sqrt{\frac{-c_1}{\sigma}}\, \eta\right) + c_4 c_{11} \sin\left(\sqrt{\frac{-c_1}{\sigma}}\, \eta\right).$$

or ,

$$(1 + c_1\bar{\mu}_1)\, G'' - c_1 G = c_4 \left[\cos\left(\sqrt{\frac{-c_1}{\sigma}}\, \eta\right) + c_{11} \sin\left(\sqrt{\frac{-c_1}{\sigma}}\, \eta\right)\right] \tag{1.17}$$

Where $G = F'$

The homogeneous solution is

$$G_H = c_{12} \cos\left(\sqrt{\frac{-c_1}{1 + c_1\bar{\mu}_1}}\, \eta\right) + c_{13} \sin\left(\sqrt{\frac{c_1}{1 + c_1\bar{\mu}_1}}\, \eta\right)$$

A particular integral of (1.17) is obtained as follows:

$$G_P = c_{14} \cos\left(\sqrt{\frac{-c_1}{\sigma}}\, \eta\right) + c_{15} \sin\left(\sqrt{\frac{-c_1}{\sigma}}\, \eta\right)$$

$$G_P'' = -c_{14}\sqrt{\frac{-c_1}{\sigma}} \sin\left(\sqrt{\frac{-c_1}{\sigma}}\, \eta\right) + c_{15}\sqrt{\frac{-c_1}{\sigma}} \cos\left(\sqrt{\frac{-c_1}{\sigma}}\, \eta\right)$$

$$C_P'' = \frac{-c_1}{\sigma} c_{14} \cos\left(\sqrt{\frac{-c_1}{\sigma}}\, \eta\right) - \frac{c_1}{\sigma}\, c_{15} \sin\left(\sqrt{\frac{-c_1}{\sigma}}\, \eta\right)$$

Substituting into (1.17), we get

$$-(1 + c_1\bar{\mu}_1)\, \frac{c_1}{\sigma}\left[c_{14} \cos\left(\sqrt{\frac{-c_1}{\sigma}}\, \eta\right) + c_{15} \sin\left(\sqrt{\frac{-c_1}{\sigma}}\, \eta\right)\right]$$

$$-c_1\left[c_{14} \cos\left(\sqrt{\frac{-c_1}{\sigma}}\, \eta\right) + c_{15} \sin\left(\sqrt{\frac{-c_1}{\sigma}}\, \eta\right)\right]$$

$$= c_4\left[\cos\left(\sqrt{\frac{-c_1}{\sigma}}\, \eta\right) + c_{11} \sin\left(\sqrt{\frac{-c_1}{\sigma}}\, \eta\right)\right]$$

Comparing terms on both sides, we find that

$$(1 + c_1\bar{\mu}_1)\,\frac{c_1}{\sigma}\;c_{14} + c_1\,c_{14} = -\,c_4\;, \quad \text{and}$$

$$(1 + c_1\bar{\mu}_1)\,\frac{c_1}{\sigma}\;c_{15} + c_1\,c_{15} = -\,c_4\;c_{11}$$

or, $$c_{14} = \frac{-\,c_4\sigma}{c_1(1+c_1\bar{\mu}_1+\sigma)}\;, \quad \text{and} \quad c_{15} = \frac{-\,c_4\;c_{11}\sigma}{c_1(1+c_1\bar{\mu}_1+\sigma)}$$

Therefore,

$$G = c_{12}\cos\left(\sqrt{\frac{-c_1}{1 + c_1\bar{\mu}_1}}\;\eta\right) + c_{13}\sin\left(\sqrt{\frac{-c_1}{1 + c_1\bar{\mu}_1}}\;\eta\right) - \frac{c_4\,\sigma}{c_1(1 + c\bar{\mu}_1 + \sigma)}\left[\cos\left(\sqrt{\frac{-c_1}{\sigma}}\;\eta\right) + c_{11}\sin\left(\sqrt{\frac{-c_1}{\sigma}}\;\eta\right)\right]$$

At, $\eta = 0\;,\; G = 0\;,\; so$

$$c_{12} = \frac{c_4\sigma}{c_1(1 + c_1\bar{\mu}_1 + \sigma)}$$

Again, the condition at ∞ cannot be satisfied, so the constants c_{11} and c_{13} will still be unknown.

Now

$$F' = \frac{c_4\sigma}{c_1(1 + c_1\bar{\mu}_1 + \sigma)}\left[\cos\left(\sqrt{\frac{-c_1}{1 + c_1\bar{\mu}_1}}\;\eta\right) - \cos\left(\sqrt{\frac{-c_1}{\sigma}}\;\eta\right)\,c_{11}\sin\left(\sqrt{\frac{-c_1}{\sigma}}\;\eta\right)\right]$$
$$+\,c_{13}\sin\left(\sqrt{\frac{-c_1}{1 + c_1\bar{\mu}_1}}\;\eta\right)$$

Therefore,

$$F = \frac{-\,c_4\sigma}{c_1(1 + c_1\bar{\mu}_1 + \sigma)}\left[\left(\frac{-c_1}{1 + c_1\bar{\mu}_1}\right)^{\frac{1}{2}}\sin\left(\sqrt{\frac{-c_1}{1 + c_1\bar{\mu}_1}}\;\eta\right) - \left(\frac{-c_1}{\sigma}\right)^{\frac{1}{2}}\sin\left(\frac{\sqrt{-c_1}}{\sigma}\eta\right)\right.$$
$$\left.+\left(\frac{-c_1}{\sigma}\right)^{\frac{1}{2}}\cos\left(\sqrt{\frac{-c_1}{\sigma}}\;\eta\right)\right] + c_{13}\left(\frac{-c_1}{1 + c_1\bar{\mu}_1}\right)^{\frac{1}{2}}\cos\left(\sqrt{\frac{-c_1}{1 + c_1\bar{\mu}_1}}\;\eta\right)$$

The velocities and temperature are obtained from

$$u = \left\{\frac{c_4\sigma}{c_1(1 + c_1\bar{\mu}_1 + \sigma)}\left[\cos\left(\sqrt{\frac{-c_1}{1 + c_1\bar{\mu}_1}}\;\eta\right) - \cos\left(\sqrt{\frac{-c_1}{\sigma}}\;\eta\right)\right.\right.$$
$$\left.\left.-c_{11}\sin\left(\sqrt{\frac{-c_1}{\sigma}}\eta\right)\right] + c_{13}\,sin\left(\sqrt{\frac{-c_1}{1 + c_1\bar{\mu}_1}}\;\eta\right)\right\}\,c_9e^{c_1t} \tag{1.17a}$$

$$v = 0 \tag{1.17b}$$

$$\theta = c_4\,c_4\left[\cos\left(\sqrt{\frac{-c_1}{\sigma}}\;\eta\right) + c_{11}\sin\left(\frac{\sqrt{-c_1}}{\sigma}\eta\right)\right]e^{c_1t} \tag{1.17c}$$

The following observations are in order:

First, (1.17) represents an asymptotic solution that is valid at large distances away from the leading edge where no changes in the *x*- direction are taking place.

Secondly, this solution is not uniformly valid everywhere, it is only valid near the plate for small y.

Finally, since $c_1 < 0$ in (1.17), it is obvious that as $t \to \infty, u \to 0$ and $\theta \to 0$ which is to be expected. For, as $t \to \infty$, the temperature of the fluid and the temperature of the wall would become equal $(\theta = 0)$. The temperature difference will disappear and, consequently, the buoyancy forces that have been the only cause of the fluid's motion will also disappear, hence $u = 0$ and $v = 0$.

V. Steady Two-Dimensional Flow Over A Flat Plate

From (1.7c), we get

$$H = c_2 x + c_{14}$$

(1.7d) yields,

$$U \frac{dU}{dx} = c_2 \; c_3 x + c_{14}$$

or, $$\frac{dU^2}{dx} = 2 \, c_2 c_3 x + C_{14}$$

Upon integration, we have

$$U^2 = c_2 \; c_3 x^2 + c_{14} x + c_{15}$$

Then, $$\mathrm{U} = (\mathrm{C}_2 \, \mathrm{C}_3 \, x^2 + \mathrm{C}_{14} x + C_{15})$$

But the boundary condition at infinity implies that

$$F' \longrightarrow \frac{U}{\phi H} = \frac{(c_2 c_3 x^2 + c_{14} x + c_{15})^{\frac{1}{2}}}{c_2 x + c_{14}}$$

$$\therefore c_{14} + c_{15} = 0$$

Hence, $$H = c_2 x \qquad (1.18a)$$

$$U = \sqrt{c_2 c_3} \; x \qquad (1.18b)$$

The equation of motion is

$$\frac{1}{c_2} F''' - F'^2 + FF'' + \bar{\mu}_1 (2F'F''' - FF^{\mathrm{iv}} + 3F''^2) + 3\bar{\mu}_2 F''^2 + \frac{c_3}{c_2} = 0 \qquad (1.19)$$

Subject to the boundary conditions

At, $$\eta = 0, \; F' \; = F = 0 \qquad (1.19a)$$

At, $$\eta \to 0, \; F' \; \to \sqrt{\frac{c_3}{c_2}} \qquad (1.19b)$$

$$\left.\begin{aligned} \psi &= c_2 x \, F(\eta) \\ u &= c_2 x \, F'(\eta) \\ v &= -c_2 \, F(\eta) \end{aligned}\right\} \qquad (1.20)$$

The flow described by (1.18b) corresponds to flow near a stagnation point represented.

For $\overline{\mu_1} = \overline{\mu_2} = 0,$ we obtain

$$\frac{1}{c^2} F''' - F'^2 + FF'' + \frac{c_3}{c_2} = 0 \qquad (1.21)$$

VI. Conclusion

The case (1.21) has been studied in some detail. Again, we see that the equation for the viscoelastic case (1.19) is one order higher than the purely viscous one (1.21). It has been reported that for viscoelastic fluids whose constitutive equation is very similar to the one we used in our analysis, the effect of the fluid's elasticity is that it increases the velocity of the fluid in the boundary layer and it also increases the stress on the solid boundary. This is true for both two-dimensional and three-dimensional flows.

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