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Vertex Cover Polynomial of $K_n \times K_2$.

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ABSTRACT

The vertex cover Polynomial of a graph G of order n has been already introduced in [3]. It is defined as the polynomial, $C(G, x) = \sum_{i = \beta(G)}^{|v(G)|} c(G, i)x^i$, where c (G , i) is the number of vertex covering sets of G of size i and β(G) is the vertex covering number of G. In this paper, we derived a formula for finding the vertex cover polynomial of the $K_n \times K_2$.

Key words : Vetex covering set, vertex covering number, vertex cover polynomial.

Introduction 1:

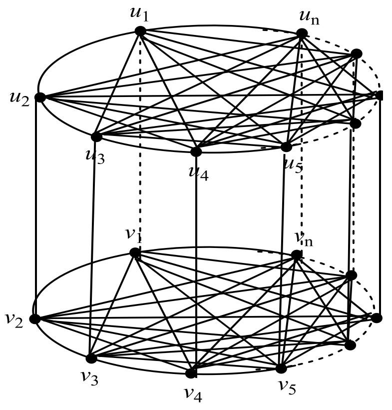
Let G = (V, E) be a simple graph. For any vertex $v \in V$, the open neighborhood of v is the set $N(v) = \{u \in V / uv \in E\}$ and the closed neighborhood of v is the set $N[v] = N(v) \chi \{v\}$. For a set S ϕV, the open neighborhood of S is $N(S) = \bigcup_{v \in S} N(v)$ and the closed neighborhood of S is $N[S] = N(S) \cup S$. A set S ϕV is a vertex covering of G if every edge uv 0 E is adjacent to at least one vertex in S. The vertex covering number β(G) is the minimum cardinality of the vertex covering sets in G. A vertex covering set with cardinality β(G) is called a β - set. Let C (G , i) be the family of vertex covering sets of G with cardinality i and let $c(G, i) = |C(G, i)|$. The polynomial, $C(G, x) = \sum_{i = \beta(G)}^{|v(G)|} c(G, i) x^i$ is defined as the vertex cover polynomial of G. In [3], many properties of the vertex cover polynomials have been studied.

2. Vertex Cover Polynomial:

Definition: 2.1

A graph G is said to be Complete if and only if every pair of vertices of G are adjacent in G. A Complete graph with n- vertices is denoted by K_n.

The graph $K_n \times K_2$ is obtained by two copies of K_n and the corresponding vertices are connected by spokes. The graph $K_n \times K_2$ is represented in figure (i) as follows:



(Figure 1)

Theorem : 2.2

The vertex cover polynomial of $K_n \times K_2$ is $C(K_n \times K_{2,} x) = x^{2n-2}[x^2 + 2nx + n(n-1)]$.

Proof :

Let the vertices of $G = K_n \times K_2$ be denoted by $\{u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n\}$. Let $S_1 = \{u_1, u_2, \ldots u_n\}$ and $S_2 = \{v_1, v_2, \ldots v_n\}$. The maximum independent sets of G are $S_{ij} = \{u_i, v_j\}$ where $i \neq j$, $i = 1, 2 \ldots n$, $j = 1, 2 \ldots n$.

Therefore, the minimum covering sets of G are

$(S_1 \cup S_2) - S_{ij}$ $i = 1, 2, \ldots n$, $j = 1, 2 \ldots n$

Therefore, the cardinality of minimum vertex covering set is 2n-2.

Since the number of maximum independent sets is equal to the number of minimum covering of G, for each vertex $u_i \in S_1$, there are n – 1 elements v_j, $j = 1, 2, \ldots n$, $i \neq j$ are independent to u_i.

Therefore, there are n(n – 1) minimum vertex covering sets with cardinality 2n-2.

Therefore, $c(K_n \times K_2, 2n-2) = n(n-1)$.

The vertex covering sets with cardinality 2n - 1 are $S_1 \cup S_2 - \{v_j\}$ for $j = 1 \ldots n$ and $S_1 \cup S_2 - \{u_i\}$ for $j=1 \ldots n$.

Therefore, the number of vertex covering sets with cardinality 2n – 1 is $c(K_n \times K_2, 2n-1) = 2n$, and the vertex covering set with cardinality 2n is $S_1 \cup S_2$.

Therefore, $c(K_n \times K_2, 2n) = 1$

Therefore, the vertex Cover polynomial is

$$c(K_n \times K_2, x) = n(n-1)x^{2n-2} + 2n\,x^{2n-1} + x^{2n}$$
$$= x^{2n-2}[x^2 + 2nx + n(n-1)]$$

Lemma : 2.3

The coefficients of the vertex cover polynomial $c(K_n \times K_2\ ,\ 2n{-}2)$ are connected by the relation $c(K_n \times K_2\ ,\ 2n - 2\ ) = c(K_{n-1} \times K_2\ ,\ 2n - 4\ ) + c(K_{n-1} \times K_2\ ,\ 2n - 3\ )$.

Proof:

$$
\begin{aligned}
\text{R.H.S} &= c(K_{n-1} \times K_2 , 2n - 4) + c(K_{n-1} \times K_2 , 2n - 3) \\
&= (n - 1)(n - 2) + 2(n - 1) \qquad [\text{ by theorem } 2.2] \\
&= (n - 1)[\, n - 2 + 2] \\
&= n(n - 1) \\
&= c(K_{n-1} \times K_2 , 2n - 2)
\end{aligned}
$$

Theorem : 2.4

The roots of the vertex cover polynomial of $K_n \times K_2$ are real.

Proof:

By theorem 2.3 the vertex cover polynomial of $K_n \times K_2$ is

$C(K_n \times K_2\ ,\ x\ ) = x^{2n-2}\,[\,x^2 + 2nx + n(n - 1)]$.

Therefore, $C(K_n \times K_2\ ,\ x\ ) = 0$

$\Rightarrow x^{2n-2}\,[\,x^2 + 2nx + n(n - 1)] = 0$

$\Rightarrow x^2 + 2nx + n(n - 1) = 0$

This is a quadratic equation in *n*

with a = 1; b = 2n and c = n(n □ 1).

We have $(2n)^2 > 4n\,(n - 1),\ \forall\ n > 3$.

That is, $b^2 > 4ac$.

Therefore, the roots of the vertex cover polynomial of $K_n \times K_2$ are always real.

Theorem : 2.5

The non – zero roots of the vertex cover polynomial of $K_n \times K_2$ are $-n \pm \sqrt{n}$.

Proof :

By theorem 2.3, the vertex Cover polynomial of $K_n \times K_2$ is

$$x^{2n-2}\,[\,x^2 + 2nx + n(n - 1)]$$

Its roots are given by $x^{2n-2}\,[\,x^2 + 2nx + n(n - 1)] = 0$

$\Rightarrow x^2 + 2nx + n(n - 1) = 0$

$$x = \frac{-\,2n \pm \sqrt{(2n)^2 - 4n\,(n-1)}}{2}$$

$$= \frac{-2n \pm \sqrt{4n}}{2}$$

$$= -n \pm \sqrt{n}\ .$$

Hence the result.

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