[Volume 2 issue 10 October 2014]
Page No.674-678 ISSN :2320-7167

INTERNATIONAL JOURNAL OF MATHEMATICS AND COMPUTER RESEARCH

Z – Connectedness in Closure Space

***U.D.Tapi* [1] *Bhagyashri A. Deole* [2]**

M.Sc., Ph. D.

Professor and Head

Department of Applied Mathematics and Computational Science

Shri G. S. Institute of Technology and Science Indore (M.P.), India

E-mail: utapi@sgsits.ac.in

M. Sc.

Research Scholar

Department of Applied Mathematics and Computational Science

Shri G. S. Institute of Technology and Science (M.P.) Indore, India

E-mail: deolebhagyashri@gmail.com

Affiliated By D.A.V.V. Indore, (M. P.), India

Corresponding Author

Bhagyashri A. Deole [2]

Address: B-103, Ridhhi-Sidhhi apart. Telephone Nagar

6-Gyans park Indore (M.P.) India

Pin code: 452018

Abstract

A Čech closure space (X, u) is a set X with Čech closure operator u: P(X) → P(X) where P(X) is a power set of X, which satisfies $u\phi=\phi$, A ⊆u A for every A⊆X, u (A⋃B) = u A ⋃uB, for all A, B ⊆ X. Many properties which hold in topological space hold in Čech closure space as well. Let Z be a topological space with more than one point. A space X is Z-connected if and only if any continuous map from X to Z is constant. In this paper we introduce **Z-connectedness in Čech closure space** and study some of its properties.

Keywords: - Čech Closure space, connectedness in Čech closure space, Z-connectedness in topological space, Z-connectedness in Čech closure space.

Mathematics Subject Classification: 54A40

1. Introduction :-

The topological study of connectedness is heavily geometric. Intuitively, a space is connected if it does not consist of two separate pieces. Čech closure space was introduced by Čech E. [1] in 1963. The modern notion of connectedness was proposed by Jorden (1893) and Schoenfliesz, and put on firm footing by Riesz [2] with the use of subspace topology. The concept of Z-connectedness was introduced by Bo.Dai. and Yan-loi Wong[3].

Many mathematicians such as Eissa D. Habil, Khalid A. Elzenati[4], Eissa D. Habil[5], Stadler B.M.R. and Stadler P.F.[6] have extended various concepts of Z-connectedness. In this paper we introduce **Z-connectedness in Čech closure space** and study its properties.

2. Preliminaries:-

***Definition 2.1[7]*:-** An operator u: P(X) →P(X) defined on the power set P(X) of a set X satisfying the axioms:

1. $u\phi=\phi$,
2. $A \subseteq uA$, for every $A\subseteq X$,
3. $u(A\cup B)=uA \cup uB$, for all $A, B\subseteq X$.

is called a Čech closure operator and the pair (X, u) is a Čech closure space.

***Definition 2.2[8]*:-** A Čech closure space (X, u) is said to be connected if and only if any continuous map from X to the discrete space {0, 1} is constant .A subset A in a Čech closure space (X, u) is said to be connected if A with the subspace topology is a connected space.

***Definition 2.3[3]*:-** Let Z is a topological space with more than one point. A space X is called Z-connected if and only if any continuous map from X to Z is constant.

3. Z-CONNECTEDNESS IN CLOSURE SPACE:-

***Definition 3.1*: -** Let (Z, u_1) be a Čech closure space with more than one point .A Čech closure space (X, u_2) is called Z- connected Čech closure space if and only if any continuous map f from X to Z is constant.

***Example 3.2*:-** Consider a non empty set Z = {x, y}, we define a Čech closure operator

u_1: P (Z) → P (Z) such that

$u_1\{x\} = u_1\{X\} = X$, $u_1\{y\} = \{y\}$, $u_1\{\varnothing\} =\varnothing$.

Hence (Z, u_1) is a Čech closure space.

Consider a non empty set X= {a, b, c}, we define a Čech closure operator

u_2: P (X) → P (X) such that

$u_2\{a\} = \{a, b\}$, $u_2\{b\} = \{b, c\}$, $u_2\{c\} = \{c, a\}$,

$u_2\{a, b\} = u_2\{b, c\} = u_2\{c, a\} = u_2\{X\} = X$, $u_2\{\varnothing\} =\varnothing$.

Hence (X, u_2) is a Čech closure space.

Define a mapping f: X→Z such that

f{a}=f{b}=f{c}=f{a, b}=f{b, c}=f{c, a}=f{X}=x,

f{∅}=y.

Here function f is constant. Hence (X, u_2) is called Z-connected Čech closure space.

Proposition 3.3: -A Z-connected Čech closure space is a connected Čech closure space.

Proof: - Let (X, u) is a Z-connected Čech closure space i.e. there exist a function f: X→Z is constant, where Z is a Čech closure space having more than one element. If Z = {0, 1} a two point Čech closure space then function f: X→ {0, 1} is constant. Hence (X, u) is a Z-connected Čech closure space.

The Čech closure space (X, u) varies when different topologies are added to a two point set

{0, 1}. Then there are only three types of topologies on Z, namely, indiscrete topology, order topology and discrete topology.

For simplicity we write:

(2_i): The space {0, 1} with indiscrete topology, whose open sets are ∅ and {0, 1};

(2_o): The space {0, 1} with order Topology, whose open sets are ∅,{0} and {0, 1};

(2_d): The space {0, 1} with discrete topology, whose open sets are ∅,{0},{1}, {0, 1}.

Corollary 3.4:- A Čech closure space (X, u) is called 2_i-connected Čech closure space, if and only if X is a one point Čech closure space.

Proof: Consider a Čech closure space Z= {0, 1}.If X is a one point Čech closure space, for any continuous map f: X →Z, f(X) is constant. Hence (X, u) is 2_i-connected Čech closure space.

Conversely, if Čech closure space (X, u) has more than one point,

X = U∪V where U and V are nonempty and disjoint sets. Define f: X→Z such that f[U] = 0 and

f [V] = 1 this function is continuous but not constant .Thus X is not 2_i-connected Čech closure space. Therefore X is not Z-connected Čech closure space except that X is one point Čech closure space.

Corollary 3.5:- A Čech closure space (X, u) is called 2_o-connected Čech closure space if and only if X is indiscrete Čech closure space.

Proof: Let X is indiscrete Čech closure space. Consider a continuous map f from X to the

Z= {0, 1}. Since {0} is open in the 2_o -space. So $f^{-1}(0)$ is open in indiscrete Čech closure space X, thus $f^{-1}(0)$ = X or ∅. If $f^{-1}(0)$ =X, f(X) =0 if $f^{-1}(0)$ =∅, f(X) =1. In either case, f is constant.

Conversely, if X is not indiscrete, there exists a proper open set S of X. Define f: X → {0, 1} by f[S] =0 and f[X-S] =1. Then $f^{-1}(0)$ = S, $f^{-1}(\{0, 1\})$ =X, thus f is continuous but not constant. Therefore, X is 2_o - connected if and only if X is indiscrete.

Corollary 3.6:- X is 2_d-connected Čech closure space if and only if X is connected.

The following proposition is a summary of the above corollaries.

***Proposition 3.7*:-** Let Z is a two point space. Then

1. X is 2_i-connected Čech closure space if and only if X is one point space.

2. X is 2_o-connected Čech closure space if and only if X is indiscrete.

3. X is 2_d-connected Čech closure space if and only if X is connected.

***Proposition 3.8*:-** A continuous image of Z-connected Čech closure space is Z-connected.

***Proof*:** Let X is any Z-connected Čech closure space. By definition, there exists a continuous map from X to Z is constant. Let f: X →f(X) is a continuous surjective map and g: f(X) →Z is continuous. But the function gof: X→Z is continuous and constant, so g is constant. Therefore f(X) is Z-connected, i.e. the continuous image of X is Z-connected.

***Proposition 3.9*:-** If $\{X_\alpha\}$ is a collection of Z-connected subspaces of a Čech closure space X such that $\cap_\alpha X_\alpha \neq \emptyset$ then $\cup_\alpha X_\alpha$ is Z-connected.

***Proof*:** For any continuous map f: $\cup_\alpha X_\alpha \to Z$, let map i: Xα $\to \cup_\alpha X_\alpha$ be the inclusion map and let

f: $\cup_\alpha X_\alpha \to Z$ be any continuous map. Since each X_α is Z-connected Čech closure space,

foi: $X_\alpha \to Z$ is continuous and thus constant and $\cap_\alpha X_\alpha \neq \emptyset$, so there exists a point p such that

$p \in \cap_\alpha X_\alpha$ i.e. $p \in X_\alpha$ for all α. Then function foi is constant and equal to f (p). Therefore f is constant and $\cup_\alpha X_\alpha$ is Z-connected.

***Proposition 3.10:*-** Let A and B are subsets of a connected Čech closure space X such that

$A \subseteq B \subseteq \overline{A}$. If A is Z-connected then B is Z-connected.

***Proof*:** Let f: B→Z be any continuous map where $A \subseteq B \overline{\subseteq} A$ and let $f|_A$: A→Z be the restriction of f. Since A is Z-connected and $f|_A$ is continuous, $f|_A$ (A) =f (A) is constant. Z is a T_1 space, thus

f (A) is closed. Note that $\overline{A}^B = \overline{A} \cap B = B$, therefore, f (B) =f $(\overline{A}^B) \subseteq f\overline{(A)}$ = f(A).Thus f(B) is constant and B is Z-connected.

***Proposition 3.11*:-** A Čech closure space (X, u) is connected if and only if for all T_1-Čech closure doubleton space Y= {0, 1}, any continuous function f: X→Y is constant.

Conclusion*: - *In this paper the idea of Z-connectedness was introduced and relationship between the Z-connectedness and Čech closure space were explained.

References

1. Čech E. Topological space, Topological Papers of Eduard Čech, Academia, Prague, (1968) 436-472.
2. Riesz F. Stetigkeitsbegriff und abstrakte Mengenlehre, in Atti del IV Congress. Internationale di Mat. I(1909) pp. 18-24.
3. Bo. Dai and Yan-loi Wong, Strongly connected spaces. Department of Mathematics, National University of Singapore, (1999).

4. Eissa D. Habil , Elzenati A. Khalid, Connectedness in Isotonic spaces, Turk J. Math Tubitak, (2006) 247-262,30.
5. Eissa D. Habil , Hereditary properties in Isotonic spaces, Department of Mathematics, Islamic University of Gaza, P.O. Box 108, Gaza, Palestine.
6. Stadler B.M.R. and Stadler P.F., Basic properties of closure spaces.J.Chem.Inf.Comput.Sci.42, (2002) p: 577.
7. Boonpok C. On Continuous Maps in Closure Spaces, General Mathematics (2009) Vol. 17 No. 2.
8. Tapi U.D. and Deole Bhagyashri A., Connectedness in closure space, International journal of Math. Sci. & Eng. Appls. (IJMSEA), (2014) Vol. 7, No.4, pp. 147-150.