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Pairwise Fuzzy Closed Sets in Fuzzy Biclosure Spaces

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Abstract

The purpose of this paper is to introduce the concept of pairwise fuzzy closed (fuzzy open) sets in fuzzy biclosure spaces and study their fundamental properties. We introduce the notion of preserve pairwise fuzzy closed (fuzzy open) maps by using pairwise fuzzy closed sets and investigate some of their characterizations.

Keywords : Fuzzy closure operator, Fuzzy biclosure space, Pairwise fuzzy closed sets, Preserve pairwise fuzzy closed maps.

Mathematics Subject Classification : 54A40

1.Introduction

Fuzzy closure spaces were first studied by Mashhour and Ghanim [1,2]. Recently, Chawalit Boonpok [3] introduced the notion of biclosure spaces. Such spaces are equipped with two arbitrary closure operators. He extended some of the standard results of separation axioms in closure space to a biclosure space. Thereafter a large number of papers have been written to generalize the concept of closure space to a biclosure space.The author [4] has introduced the notion of fuzzy biclosure spaces and generalized the concept of fuzzy closure space to fuzzy biclosure space.

In this paper we introduce and study the concept of pairwise fuzzy closed sets in fuzzy biclosure spaces. Using the notion of pairwise fuzzy closed sets, we introduce preserve pairwise fuzzy closed maps and discusses some of their properties.

2. Preliminaries

Definition 2.1. A *fuzzy biclosure space* is a triple (X, u_1, u_2) where X is a non empty set and u_1, u_2 are two fuzzy closure operators on X which satisfy the following properties:

(i) $u_1(0_X) = 0_X$ and $u_2(0_X) = 0_X$

(ii) $\mu \le u_1\mu$ and $\mu \le u_2\mu$ for all $\mu \le I^X$

(iii) $u_1(\mu \vee \upsilon) = u_1\mu \vee u_1\upsilon$ and $u_2(\mu \vee \upsilon) = u_2\mu \vee u_2\upsilon$ for all $\mu, \upsilon \le I^X$.

Definition 2.2[4]. A subset μ of a fuzzy biclosure space (X,u_1,u_2) is called fuzzy closed if. The complement of fuzzy closed set is called fuzzy open.

Definition 2.3[4]. A fuzzy closure space (Y,v_1,v_2) is said to be a subspace of (X,u_1,u_2) if $Y \le X$ and $v_1\mu = u_1\mu \wedge 1_Y$ or $v_2\mu = u_2\mu \wedge 1_Y$ for each fuzzy subset $\mu \le I^Y$. If 1_Y is fuzzy closed in (X,u_1,u_2), then the subspace (Y,v_1,v_2) of (X,u_1,u_2) is also fuzzy closed.

Definition 2.4[4]. Let (X,u_1,u_2) and (Y,v_1,v_2) be fuzzy biclosure spaces. A map $f:(X,u_1,u_2)\to(Y,v_1,v_2)$ is called fuzzy continuous if $f^{-1}(\mu)$ is a fuzzy closed subset of (X,u_1,u_2) for every fuzzy closed subset μ of (Y,v_1,v_2).

Clearly, it is easy to prove that a map $f:(X,u_1,u_2)\to(Y,v_1,v_2)$ is fuzzy continuous if and only if $f^{-1}(\upsilon)$ is a fuzzy open subset of (X,u_1,u_2) for every fuzzy open subset υ of (Y,v_1,v_2).

Definition 2.5[4]. Let (X,u_1,u_2) and (Y,v_1,v_2) be fuzzy biclosure spaces. A map $f:(X,u_1,u_2)\to(Y,v_1,v_2)$ is said to be fuzzy closed (resp. fuzzy open) if $f(\mu)$ is fuzzy closed (resp. fuzzy open) subset of (Y,v_1,v_2) whenever μ is a fuzzy closed (resp. fuzzy open) subset of (X,u_1,u_2).

Definition 2.6. The product of a family $\{(X_\alpha,u_\alpha^1,u_\alpha^2):\alpha\in J\}$ of fuzzy biclosure spaces denoted by $\prod_{\alpha\in J}(X_\alpha,u_\alpha^1,u_\alpha^2)$ is the fuzzy biclosure space $\left(\prod_{\alpha\in J}X_\alpha,u^1,u^2\right)$ where $\left(\prod_{\alpha\in J}X_\alpha,u^i\right)$ for $i\in\{1,2\}$ is the product of the family of fuzzy closure spaces $\{X_\alpha,u^i:\alpha\in J\}$.

Remark 2.7. Let $\prod_{\alpha\in J}(X_\alpha,u_\alpha^1,u_\alpha^2)=\left(\prod_{\alpha\in J}X_\alpha,u^1,u^2\right)$. Then for each $\mu\le\prod_{\alpha\in J}X_\alpha$, $u^1u^2\mu=\prod_{\alpha\in J}u_\alpha^1u_\alpha^2\pi_\alpha(\mu)$.

Proposition 2.8[4]. *Let $\{(X_\alpha,u_\alpha^1,u_\alpha^2):\alpha\in J\}$ be a family of fuzzy biclosure spaces. Then for each $\beta\in J$, the projection map $\pi_\beta:\prod_{\alpha\in J}(X_\alpha,u_\alpha^1,u_\alpha^2)\to(X_\beta,u_\beta^1,u_\beta^2)$ is fuzzy continuous.*

Proposition 2.9[4]. *Let $\{(X_\alpha,u_\alpha^1,u_\alpha^2):\alpha\in J\}$ be a family of fuzzy biclosure spaces and let $\beta\in J$. Then $\eta\le X_\beta$ is a fuzzy closed subset of $(X_\beta,u_\beta^1,u_\beta^2)$ if and only if $\eta\times\prod_{\substack{\alpha\ne\beta\\ \alpha\in J}}X_\alpha$ is a fuzzy closed subset of $\prod_{\alpha\in J}(X_\alpha,u_\alpha^1,u_\alpha^2)$.*

Proposition 2.10[4]. *Let $\{(X_\alpha,u_\alpha^1,u_\alpha^2):\alpha\in J\}$ be a family of fuzzy biclosure spaces and let $\beta\in J$. Then $\gamma\le X_\beta$ is a fuzzy open subset of $(X_\beta,u_\beta^1,u_\beta^2)$ if and only if $\gamma\times\prod_{\substack{\alpha\ne\beta\\ \alpha\in J}}X_\alpha$ is a fuzzy open subset of $\prod_{\alpha\in J}(X_\alpha,u_\alpha^1,u_\alpha^2)$.*

3. PAIRWISE FUZZY CLOSED SETS

The purpose of this section is to introduce the concept of pairwise fuzzy closed (pairwise fuzzy open) sets in fuzzy biclosure spaces and study some of their important properties. We also introduce the notion of preserve pairwise fuzzy closed (preserve pairwise fuzzy open) maps using pairwise fuzzy closed (pairwise fuzzy open) sets and investigate some of their characterizations.

Definition 3.1. A subset μ of a fuzzy biclosure space (X,u_1,u_2) is called pairwise fuzzy closed if $u_1u_2\mu=\mu=u_2u_1\mu$. The complement of pairwise fuzzy closed sets is called pairwise fuzzy open.

Remark 3.2. Every fuzzy closed set is pairwise fuzzy closed set.

Proposition 3.3. *Let (X,u_1,u_2) be a fuzzy biclosure space and let u_1, u_2 be additive. If μ and υ are pairwise fuzzy closed subsets of (X,u_1,u_2), then $\mu\vee\upsilon$ is pairwise fuzzy closed.*

Proof. Let μ and υ be pairwise fuzzy closed. Then $u_1u_2\mu=\mu=u_2u_1\mu$ and $u_1u_2\upsilon=\upsilon=u_2u_1\upsilon$. Since u_1 and u_2 are additive,

$$u_1u_2(\mu\vee\upsilon) = u_1(u_2\mu\vee u_2\upsilon) = u_1u_2\mu\vee u_1u_2\upsilon = \mu\vee\upsilon \text{ and}$$

$$u_2u_1(\mu\vee\upsilon) = u_2(u_1\mu\vee u_1\upsilon) = u_2u_1\mu\vee u_2u_1\upsilon = \mu\vee\upsilon$$

Consequently $u_1u_2(\mu\vee\upsilon)=\mu\vee\upsilon=u_1u_2\mu\vee\upsilon$. Hence, $\mu\vee\upsilon$ is pairwise fuzzy closed.

Proposition 3.4. *Let (X,u_1,u_2) be a fuzzy biclosure space and let (Y,v_1,v_2) be a fuzzy closed subspace of (X,u_1,u_2). If μ is a pairwise fuzzy closed subset of (Y,v_1,v_2), then μ is a pairwise fuzzy closed subset of (X,u_1,u_2).*

Proof. Let μ be a pairwise fuzzy closed subset of (Y,v_1,v_2). Then $v_1v_2\mu=\mu$ and $v_2v_1\mu=\mu$. Since Y is both a fuzzy closed subset of (X,u_1) and (X,u_2), $u_1\mu=\mu$ and $u_2\mu=\mu$. Therefore

$$F=v_1v_2\mu=v_1(u_2\mu\wedge 1_Y)=v_1(u_2(\mu\wedge 1_Y))=v_1(u_2\mu)=u_1(u_2\mu)\wedge 1_Y=u_1(u_2\mu\wedge 1_Y)$$
$$=u_1(u_2(\mu\wedge 1_Y))=u_1u_2\mu \quad \textit{and}$$
$$F=v_2v_1\mu=v_2(u_1\mu\wedge 1_Y)=v_2(u_1(\mu\wedge 1_Y))=v_2(u_1\mu)=u_2(u_1\mu)\wedge 1_Y=u_2(u_1\mu\wedge 1_Y)$$
$$=u_2(u_1(\mu\wedge 1_Y))=u_2u_1\mu.$$

Consequently, $u_1u_2\mu=\mu=u_2u_1\mu$. Hence, μ is a pairwise fuzzy closed subset of (X,u_1,u_2).

The following statement is obvious.

Proposition 3.5. *Let (X,u_1,u_2) be a fuzzy biclosure space and let $\mu\le X$. Then*

(i) *μ is pairwise fuzzy open if and only if $\mu=1_X-u_1u_2(1_X-\mu)=1_X-u_2u_1(1_X-\mu)$.*

(ii) *If υ is pairwise fuzzy open and $\upsilon\le\mu$, then $\upsilon\le 1_X-u_1u_2(1_X-\mu)=1_X-u_2u_1(1_X-\mu)$.*

Proposition 3.6. *Let $\{(X_\alpha,u_\alpha^1,u_\alpha^2):\alpha\in J\}$ be a family of fuzzy biclosure spaces and let $\beta\in J$. Then $\eta\le X_\beta$ is a pairwise fuzzy closed subset of $(X_\beta,u_\beta^1,u_\beta^2)$ if and only if $\eta\times\prod_{\substack{\alpha\ne\beta\\ \alpha\in J}}X_\alpha$ is a pairwise fuzzy closed subset of $\prod_{\alpha\in J}(X_\alpha,u_\alpha^1,u_\alpha^2)$.*

Proof. Let $\beta\in J$ and let η be a pairwise fuzzy closed subset of $(X_\beta,u_\beta^1,u_\beta^2)$. Then η is both a fuzzy closed subset of (X_β,u_β^1) and (X_β,u_β^2). Since $\pi_\beta:\prod(X_\alpha,u_\alpha^1)\to(X_\beta,u_\beta^1)$ is fuzzy continuous, $\pi_\beta^{-1}(\eta)=\eta\times\prod_{\substack{\alpha\ne\beta\\ \alpha\in J}}X_\alpha$ is a fuzzy closed subset of $\prod_{\alpha\in I}(X_\alpha,u_\alpha^1)$. Since $\pi_\beta:\prod(X_\alpha,u_\alpha^2)\to(X_\beta,u_\beta^2)$ is fuzzy continuous, $\pi_{\beta}^{-1}(\eta)=\eta\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_{\alpha}$ is a fuzzy closed subset of $\prod_{\alpha\in J}\left(X_{\alpha},u_{\alpha}^{2}\right)$. Consequently, $\eta\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_{\alpha}$ is a fuzzy closed subset of $\prod_{\alpha\in J}\left(X_{\alpha},u_{\alpha}^{1},u_{\alpha}^{2}\right)$. Then $\eta\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_{\alpha}$ is a pairwise fuzzy closed subset of $\prod_{\alpha\in J}\left(X_{\alpha},u_{\alpha}^{1},u_{\alpha}^{2}\right)$.

Conversely, let $\eta\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_{\alpha}$ be a pairwise fuzzy closed subset of $\prod_{\alpha\in J}\left(X_{\alpha},u_{\alpha}^{1},u_{\alpha}^{2}\right)$. Then $\eta\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_{\alpha}$ is both a fuzzy closed subset of $\prod_{\alpha\in I}\left(X_{\alpha},u_{\alpha}^{1}\right)$ and $\prod_{\alpha\in J}\left(X_{\alpha},u_{\alpha}^{2}\right)$. Since $\pi_{\beta}:\prod_{\alpha\in I}(X_{\alpha},u_{\alpha}^{1})\rightarrow(X_{\beta},u_{\beta}^{1})$ is fuzzy closed, $\pi_{\beta}\left(\eta\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_{\alpha}\right)=\eta$ is a fuzzy closed subset of $\left(X_{\beta},u_{\beta}^{1}\right)$. Since $\pi_{\beta}:\prod_{\alpha\in J}\left(X_{\alpha},u_{\alpha}^{2}\right)\rightarrow\left(X_{\beta},u_{\beta}^{2}\right)$ is fuzzy closed, $\pi_{\beta}\left(\eta\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_{\alpha}\right)=\eta$ is a fuzzy closed subset of $\left(X_{\beta},u_{\beta}^{2}\right)$. Consequently, η is a fuzzy closed subset of $\left(X_{\beta},u_{\beta}^{1},u_{\beta}^{2}\right)$. Then η is a pairwise fuzzy closed subset of $\left(X_{\beta},u_{\beta}^{1},u_{\beta}^{2}\right)$.

Definition 3.7. Let (X,u_1,u_2) and (Y,v_1,v_2) be fuzzy biclosure spaces. A map $f:(X,u_1,u_2)\rightarrow(Y,v_1,v_2)$ is said to be preserve pairwise fuzzy closed (resp. preserve pairwise fuzzy open) if $f(\mu)$ is a pairwise fuzzy closed (resp. pairwise fuzzy open) set in (Y,v_1,v_2) whenever μ is a pairwise fuzzy closed (resp. pairwise fuzzy open) set in (X,u_1,u_2).

The following statement is evident.

Proposition 3.8. *Let (X,u_1,u_2), (Y,v_1,v_2) and (Z,w_1,w_2) be fuzzy biclosure spaces. If $f:(X,u_1,u_2)\rightarrow(Y,v_1,v_2)$ and $h:(Y,v_1,v_2)\rightarrow(Z,w_1,w_2)$ are preserve pairwise fuzzy closed, then $h\,o\,f:(X,u_1,u_2)\rightarrow(Z,w_1,w_2)$ is preserve pairwise fuzzy closed.*

Proof. Obivious.

Proposition 3.9. *Let $\left\{\left(X_{\alpha},u_{\alpha}{}^{1},u_{\alpha}{}^{2}\right):\alpha\in J\right\}$ be a family of fuzzy biclosure spaces. Then for each $\beta\in J$, the projection map $\pi_{\beta}:\prod_{\alpha\in J}\left(X_{\alpha},u_{\alpha}^{1},u_{\alpha}^{2}\right)\rightarrow\left(X_{\beta},u_{\beta}^{1},u_{\beta}^{2}\right)$ is preserve pairwise fuzzy closed.*

Proof. Let η be a pairwise fuzzy closed subset of $\prod_{\alpha\in J}\left(X_{\alpha},u_{\alpha}^{1},u_{\alpha}^{2}\right)$. Then η is both a fuzzy closed subset of $\prod_{\alpha\in I}\left(X_{\alpha},u_{\alpha}^{1}\right)$ and $\prod_{\alpha\in I}\left(X_{\alpha},u_{\alpha}^{2}\right)$. Since the map $\pi_{\beta}:\prod_{\alpha\in I}\left(X_{\alpha},u_{\alpha}^{1}\right)\rightarrow\left(X_{\beta},u_{\beta}^{1}\right)$ is fuzzy closed, $\pi_{\beta}(\eta)$ is a fuzzy closed subset of $\left(X_{\beta},u_{\beta}^{1}\right)$. Since, the map $\pi_{\beta}:\prod_{\alpha\in J}\left(X_{\alpha},u_{\alpha}^{2}\right)\rightarrow\left(X_{\beta},u_{\beta}^{2}\right)$ is fuzzy closed, $\pi_{\beta}(\eta)$ is a fuzzy closed subset of $\left(X_{\beta},u_{\beta}^{2}\right)$. Consequently, $\pi_{\beta}(\eta)$ is a fuzzy closed subset of $\prod_{\alpha\in J}\left(X_{\alpha},u_{\alpha}^{1},u_{\alpha}^{2}\right)$. Then $\pi_{\beta}(\eta)$ is a pairwise fuzzy closed subset of $\prod_{\alpha\in J}\left(X_{\alpha},u_{\alpha}^{1},u_{\alpha}^{2}\right)$. Hence, the map π_{β} is preserve pairwise fuzzy closed.

Proposition 3.10. *Let* (X,u_1,u_2) *be a fuzzy biclosure space,* $\{(Y_\alpha,v_\alpha^1,v_\alpha^2):\alpha\in J\}$ *be a family of fuzzy biclosure spaces and* $f:X\to\prod_{\alpha\in J}Y_\alpha$ *be a map. Then* $f:(X,u_1,u_2)\to\prod_{\alpha\in J}(Y_\alpha,v_\alpha^1,v_\alpha^2)$ *is preserve pairwise fuzzy closed if and only if* $\pi_\alpha\circ f:(X,u_1,u_2)\to(Y_\alpha,v_\alpha^1,v_\alpha^2)$ *is preserve pairwise fuzzy closed for each* $\alpha\in J$.

Proof. Let f be preserve pairwise fuzzy closed. Since π_α is preserve pairwise fuzzy closed for each $\alpha\in J$, it follows that $\pi_\alpha\circ f$ is preserve pairwise fuzzy closed for each $\alpha\in J$.

Conversely, let the map $\pi_\alpha\circ f$ be preserve pairwise fuzzy closed for each $\alpha\in J$. Suppose that f is not preserve pairwise fuzzy closed. Therefore, there exists a pairwise fuzzy closed subset η of (X,u_1,u_2) such that $\prod_{\alpha\in J}v_\alpha^1v_\alpha^2\,\pi_\alpha(f(\eta))\not\leq f(\eta)$ or $\prod_{\alpha\in J}v_\alpha^2v_\alpha^1\,\pi_\alpha(f(\eta))\not\leq f(\eta)$. If $\prod_{\alpha\in J}v_\alpha^2v_\alpha^1\,\pi_\alpha(f(\eta))\not\leq f(\eta)$. Then, there exists $\beta\in J$ such that $v_\beta^1v_\beta^2\pi_\beta(f(\eta))\not\leq\pi_\beta(f(\eta))$. But the map $\pi_\beta\circ f$ is preserve pairwise fuzzy closed, $\pi_\beta(f(\eta))$ is a pairwise fuzzy closed subset of $(Y_\beta,v_\beta^1,v_\beta^2)$. This is a contradiction. If $\prod_{\alpha\in J}v_\alpha^2v_\alpha^1\,\pi_\alpha(f(\eta))\not\leq f(\eta)$. Then, there exists $\beta\in J$ such that $v_\beta^2v_\beta^1\pi_\beta(f(\eta))\not\leq\pi_\beta(f(\eta))$. But the map $\pi_\beta\circ f$ is preserve pairwise fuzzy closed, therefore $\pi_\beta(f(\eta))$ is a pairwise fuzzy closed subset of $(Y_\beta,v_\beta^1,v_\beta^2)$. This is a contradiction. Therefore the map f is preserve pairwise fuzzy closed.

Proposition 3.11. *Let* $\{(X_\alpha,u_\alpha{}^1,u_\alpha{}^2):\alpha\in J\}$ *and* $\{(Y_\alpha,v_\alpha^1,v_\alpha^2):\alpha\in J\}$ *be families of fuzzy biclosure spaces. For each* $\alpha\in J$, *let* $f_\alpha:X_\alpha\to Y_\alpha$ *be a surjection and let the map* $f:\prod_{\alpha\in J}X_\alpha\to\prod_{\alpha\in J}Y_\alpha$ *be defined by* $f((x_\alpha)_{\alpha\in J})=(f_\alpha(x_\alpha))_{\alpha\in J}$. *Then the map* $f:\prod_{\alpha\in J}(X_\alpha,u_\alpha^1,u_\alpha^2)\to\prod_{\alpha\in J}(Y_\alpha,v_\alpha^1,v_\alpha^2)$ *is preserve pairwise fuzzy closed if and only if the map* $f_\alpha:(X_\alpha,u_\alpha^1,u_\alpha^2)\to(Y_\alpha,v_\alpha^1,v_\alpha^2)$ *is preserve pairwise fuzzy closed for each* $\alpha\in J$

Proof. Let $\beta\in J$ and let η be a pairwise fuzzy closed subset of $(X_\beta,u_\beta^1,u_\beta^2)$. Then $\eta\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_\alpha$ is a pairwise fuzzy closed subset of $\prod_{\alpha\in J}(X_\alpha,u_\alpha^1,u_\alpha^2)$. Since f is preserve pairwise fuzzy closed, $f\left(\eta\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_\alpha\right)$ is a pairwise fuzzy closed subset of $\prod_{\alpha\in J}(Y_\alpha,v_\alpha^1,v_\alpha^2)$. But $f\left(\eta\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}X_\alpha\right)=f_\beta(\eta)\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}Y_\alpha$, hence $f_\beta(\eta)\times\prod_{\substack{\alpha\neq\beta\\ \alpha\in J}}Y_\alpha$ is a pairwise fuzzy closed subset of $\prod_{\alpha\in J}(Y_\alpha,v_\alpha^1,v_\alpha^2)$. By Proposition 3.6, $f_\beta(\eta)$ is a pairwise fuzzy closed subset of $(Y_\beta,v_\beta^1,v_\beta^2)$. Hence, the map f_β is preserve pairwise fuzzy closed.

Conversely, let the map f_β be preserve pairwise fuzzy closed for each $\beta\in J$. Suppose that the map f is not preserve pairwise fuzzy closed. Therefore, there exists a pairwise fuzzy closed subset η of $\prod_{\alpha\in J}(X_\alpha,u_\alpha^1,u_\alpha^2)$ such that

$\prod_{\alpha\in J} v_\alpha^1 v_\alpha^2\, \pi_\alpha\left(f(\eta)\right) \not\leq f(\eta)$ or $\prod_{\alpha\in J} v_\alpha^2 v_\alpha^1\, \pi_\alpha\left(f(\eta)\right) \not\leq f(\eta)$. If $\prod_{\alpha\in J} v_\alpha^1 v_\alpha^2\, \pi_\alpha\left(f(\eta)\right) \not\leq f(\eta)$. Then, there exists $\beta \in J$ such that $v_\beta^1 v_\beta^2 \pi_\beta\left(f(\eta)\right) \not\leq \pi_\beta\left(f(\eta)\right)$. But $\pi_\beta(\eta)$ is a pairwise fuzzy closed subset of $\left(X_\beta, u_\beta^1, u_\beta^2\right)$ and f_β is preserve pairwise fuzzy closed, $f_\beta\left(\pi_\beta(\eta)\right)$ is a pairwise fuzzy closed subset of $\left(Y_\beta, v_\beta^1, v_\beta^2\right)$. This is a contradiction. If $\prod_{\alpha\in J} v_\alpha^2 v_\alpha^1\, \pi_\alpha\left(f(\eta)\right) \not\leq f(\eta)$. Then, there exists $\beta \in J$ such that $v_\beta^2 v_\beta^1 \pi_\beta\left(f(\eta)\right) \not\leq \pi_\beta\left(f(\eta)\right)$. But $\pi_\beta(\eta)$ is a pairwise fuzzy closed subset of $\left(X_\beta, u_\beta^1, u_\beta^2\right)$ and f_β is preserve pairwise fuzzy closed, $f_\beta\left(\pi_\beta(\eta)\right)$ is a pairwise fuzzy closed subset of $\left(Y_\beta, v_\beta^1, v_\beta^2\right)$. This is a contradiction. Therefore, the map f is preserve pairwise fuzzy closed.

4. Conclusion

In this paper we have introduced the concept of pairwise fuzzy closed (pairwise fuzzy open) sets in fuzzy biclosure spaces and studied some of their important properties. We have also introduced the notion of preserve pairwise fuzzy closed (preserve pairwise fuzzy open) maps using pairwise fuzzy closed (pairwise fuzzy open) sets and investigated some of their characterizations.

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